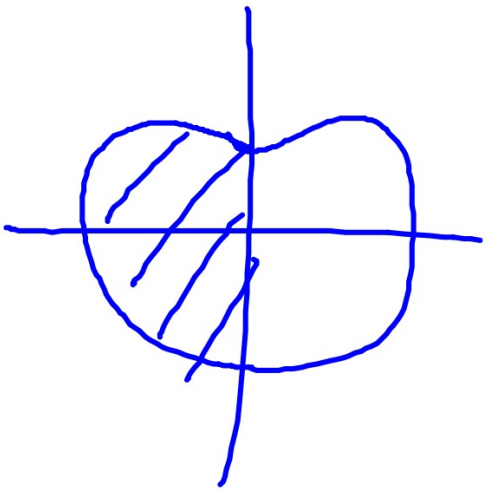
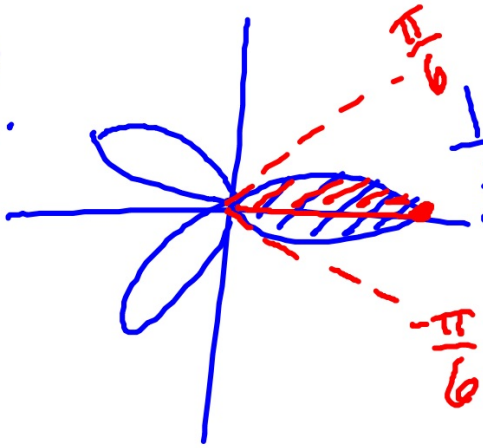
3.



$$\frac{1}{2}\left[\frac{1}{2}\int_0^{2\pi}(3-2\sin\theta)^2\,d\theta\right]$$

$$\frac{1}{2}\int_0^{\pi} r^2\,d\theta$$

9.



$$\frac{1}{2}\int_0^{\pi}(2\cos 3\theta)^2\,d\theta$$

$r = 2\cos 3\theta$

$1 = 2\cos 3\theta$

$1 = \cos 3\theta$

$0 = 3\theta$

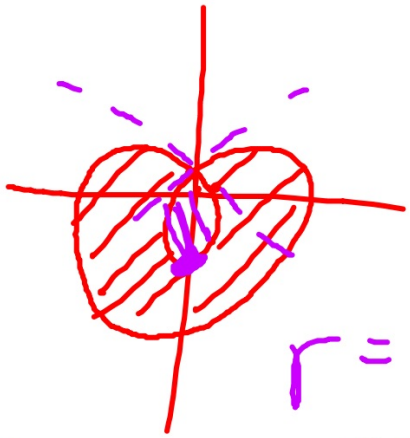
$0 = \theta$

$0 = 2\cos 3\theta$

$\frac{\pi}{2} = 3\theta$

$\frac{\pi}{6} = \theta$

25.)



$$\frac{1}{2}\int_0^{2\pi} r^2\,d\theta - 2\cdot\frac{1}{2}\int_{\pi/6}^{5\pi/6} r^2\,d\theta$$

$r = 3 - 6\sin\theta$

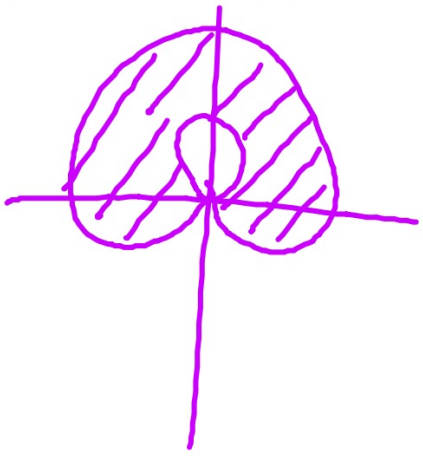
$\sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

$\left(-3, \frac{\pi}{2}\right)$

9.5 Polar Area - Cont.

ex: Find the area bounded between the loops of $r = 1 + 2\sin\theta$.



$$\frac{1}{2}\int_{0}^{2\pi}(1+2\sin\theta)^2 d\theta - 2\cdot\frac{1}{2}\int_{\frac{7\pi}{6}}^{\frac{11\pi}{6}}(1+2\sin\theta)^2 d\theta$$

$$8.338$$

Intersections

ex: Find all intersections.

a) $r = 3 + \sin\theta \qquad r = 3 - \sin\theta$

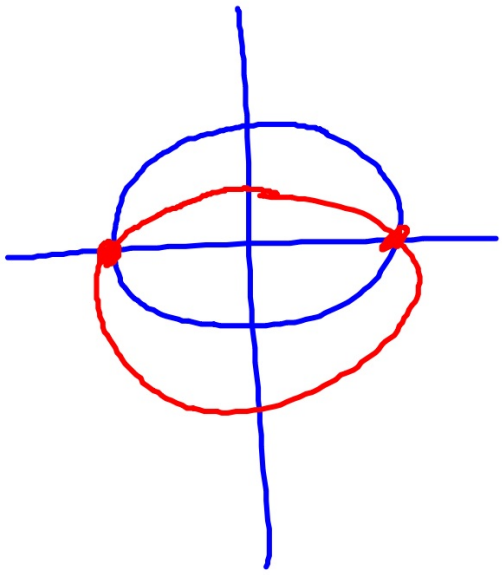
$3 + \sin\theta = 3 - \sin\theta$

$2\sin\theta = 0$

$\theta = 0, \pi, \ldots$

$(3, 0)$

$(3, \pi)$



ex: Find all intersections.

b) $r = 2 - 2\cos\theta \qquad r = 2 + 2\cos\theta$

$2 - 2\cos\theta = 2 + 2\cos\theta$

$4\cos\theta = 0$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$

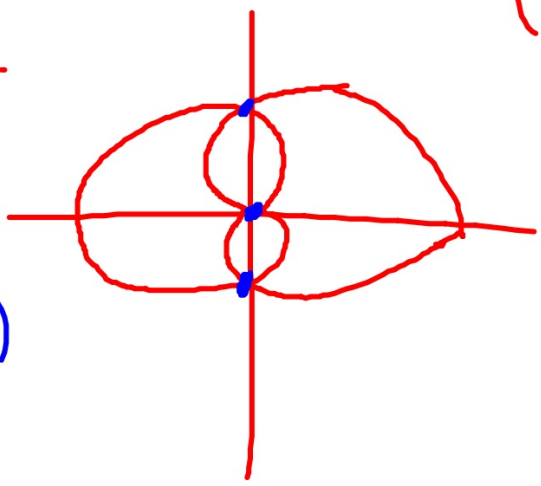
$\left(2, \frac{\pi}{2}\right)$

$\left(2, \frac{3\pi}{2}\right)$

$(0,0)$

$\left(0, \frac{\pi}{3}\right) = (0,0)$

$\left(0, \frac{\pi}{2}\right)$



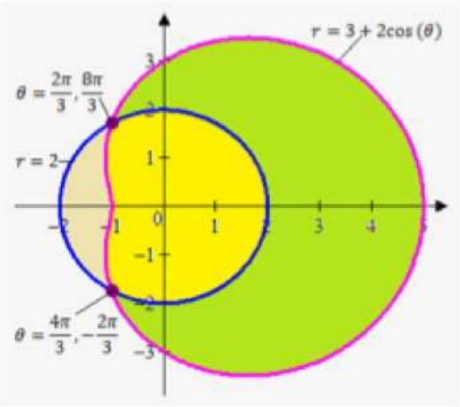
ex: Find all intersections.

c) $r = 2\cos\theta \qquad r = 1$

Review: Area Bounded by ONE Curve

$$A = \frac{1}{2}\int_a^b r^2 \, d\theta$$

Area Bounded by TWO Curves



Two types of problems:

1. "Outer inner"
2. common interior

ex: Find the indicated area.

a)

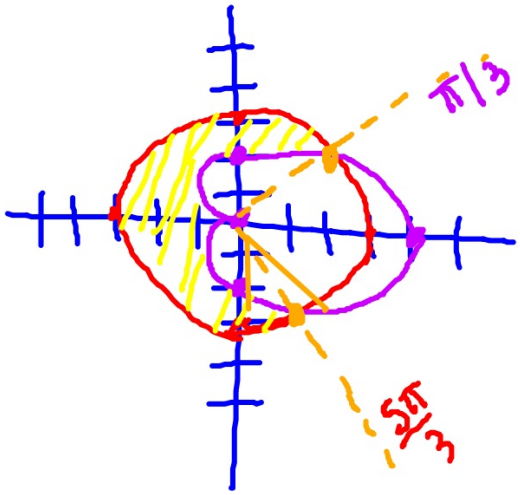
inside: $r = 3$

outside: $r = 2 + 2\cos\theta$

$3 = 2 + 2\cos\theta$

$\frac{1}{2} = \cos\theta$

$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$



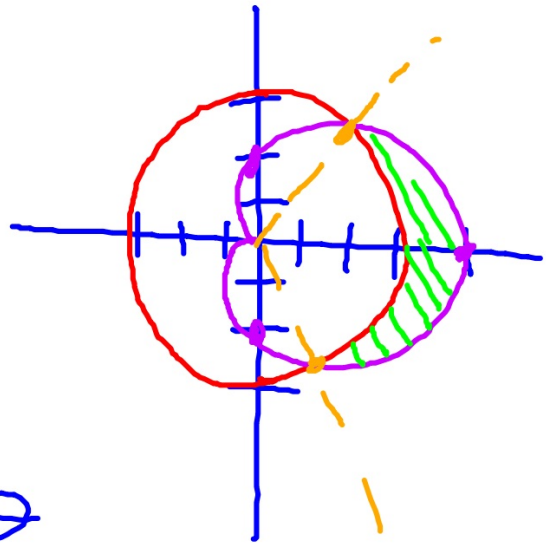
$$A = \frac{1}{2}\int_{\pi/3}^{5\pi/3}\left(3^2 - (2+2\cos\theta)^2\right)d\theta$$

ex: Find the indicated area.

b)

inside: $r = 2 + 2\cos\theta$

outside: $r = 3$



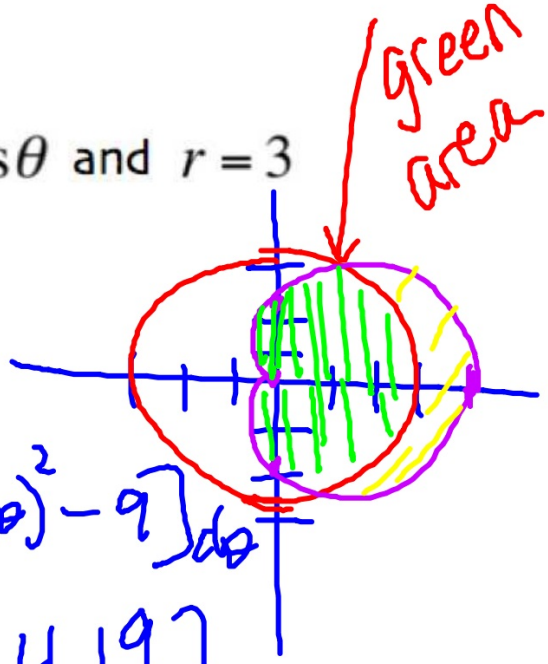
$$\frac{1}{2}\int_{-\pi/3}^{\pi/3}\left((2+2\cos\theta)^2 - 9\right)d\theta$$

4.65?

ex: Find the indicated area.

c)

common interior of: $r = 2 + 2\cos\theta$ and $r = 3$



cardioid −

$$\frac{1}{2}\int_{0}^{2\pi}(2+2\cos\theta)^2\,d\theta - \frac{1}{2}\int_{-\pi/3}^{\pi/3}\left[(2+2\cos\theta)^2 - 9\right]d\theta$$

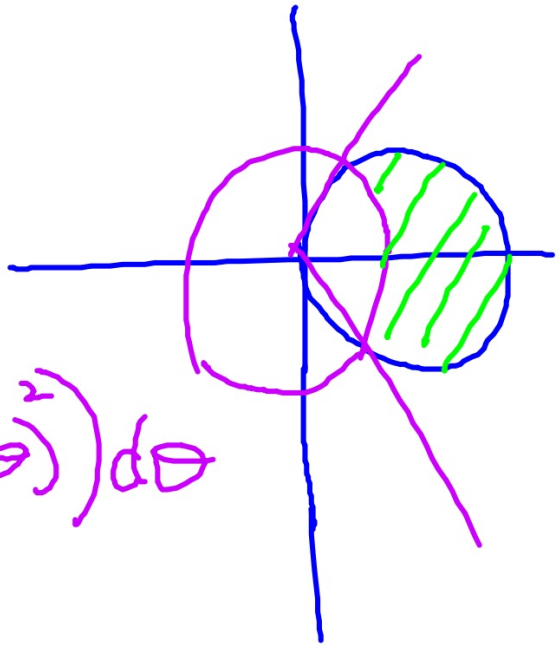
14.197

ex: Find the indicated area.

d)

inside: $r = 3\cos\theta$

outside: $r = 2 - \cos\theta$



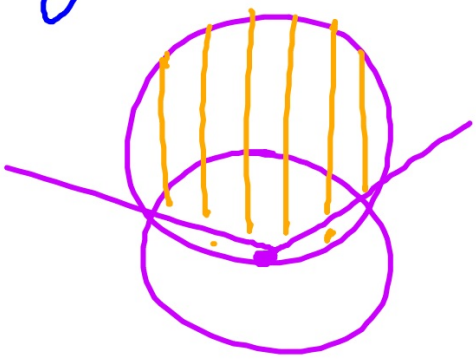
$$\frac{1}{2}\int_{-\pi/3}^{\pi/3}\left((3\cos\theta)^2 - (2-\cos\theta)^2\right)d\theta$$

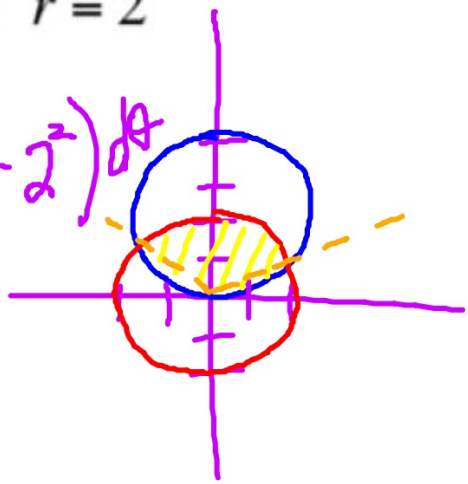
ex: Find the indicated area.

e) #41

common interior of: $r = 4\sin\theta$ and $r = 2$

$$\frac{1}{2}\int_{0}^{\pi/6}(4\sin\theta)^2\,d\theta - \frac{1}{2}\int_{\pi/6}^{5\pi/6}\left((4\sin\theta)^2 - 2^2\right)d\theta$$





ex: Find the indicated area.

e) common interior of: $r = 3 + 3\sin\theta$ and $r = 3 - 3\sin\theta$

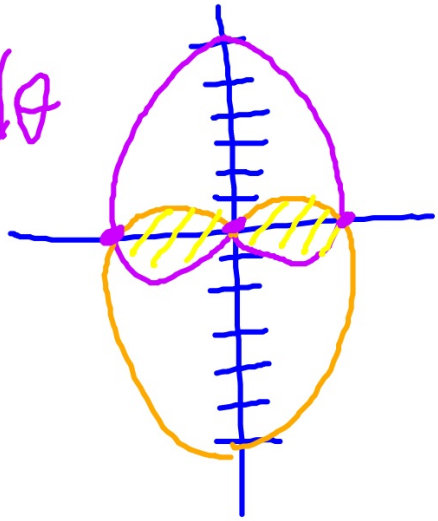
$3 + 3\sin\theta = 3 - 3\sin\theta$

$6\sin\theta = 0$

$\sin\theta = 0;\ \theta = 0, \pi$

points: $(0, 0)$, $(3, 0)$, $(3, \pi)$

$$\frac{1}{2}\int_{\pi}^{3\pi/2} (3 + 3\sin\theta)^2\, d\theta + \frac{1}{2}\int_{0}^{\pi/2} (3 - 3\sin\theta)^2\, d\theta$$



ex: Find the indicated area.

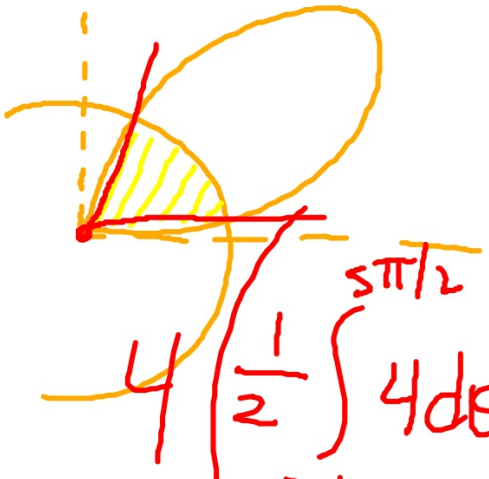
f) #37

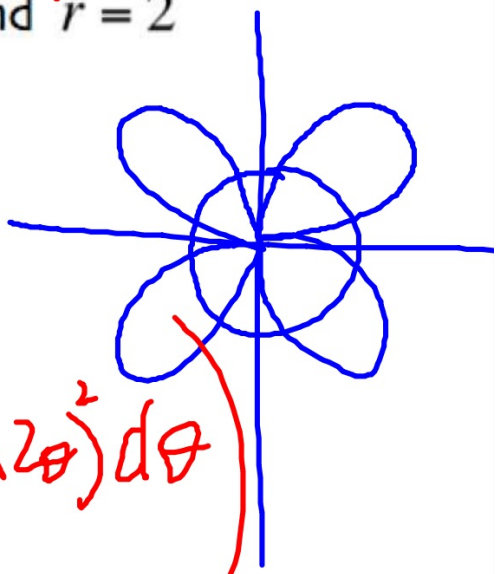
common interior of: $r = 4\sin 2\theta$ and $r = 2$

$2 = 4\sin 2\theta$

$\frac{1}{2} = \sin 2\theta$

$\frac{5\pi}{12}, \frac{\pi}{12} = \theta$





$$4\left(\frac{1}{2}\int_{\pi/12}^{5\pi/12} 4\,d\theta + 2\cdot\frac{1}{2}\int_{0}^{\pi/12} (4\sin 2\theta)^2\,d\theta\right)$$