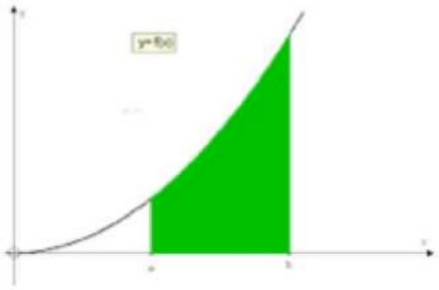
9.5 Polar Area

- Rectangular Area

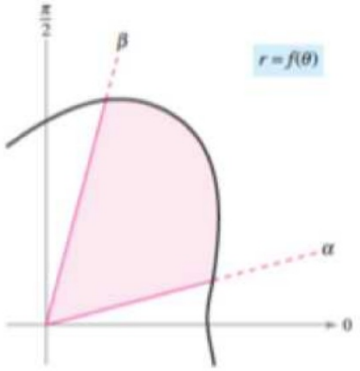
$A = \frac{1}{2} r^2 \theta$

sector



$$\int_a^b f(x)\,dx$$

- Polar Area



$$\frac{1}{2}\int_a^b r^2\,d\theta$$

$a < b$

Polar Area - Bounded by ONE Curve

THEOREM 9.12 Area in Polar Coordinates

If f is continuous and nonnegative on the interval $[\alpha, \beta]$ where

$$0 < \beta - \alpha \leq 2\pi$$

then the area of the region bounded by the graph of $r = f(\theta)$ between the radial lines $\theta = \alpha$ and $\theta = \beta$ is

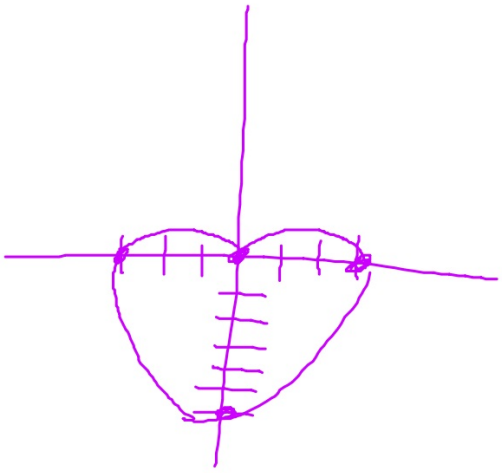
$$A =$$

$$\alpha < \beta$$

ex: Find the area bounded by the polar curve.

a) $r = 3 - 3\sin\theta$

traces once $[0, 2\pi)$



r	θ
3	0
0	$\pi/2$
3	π
6	$3\pi/2$

$$\frac{1}{2}\int_0^{2\pi} (3-3\sin\theta)^2 \, d\theta \approx 42.412$$

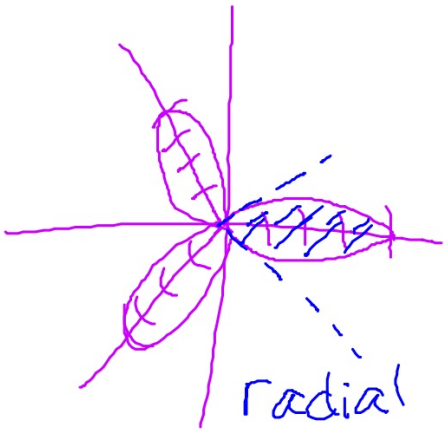
$$2 \cdot \frac{1}{2}\int_{-\pi/2}^{\pi/2} (3-3\sin\theta)^2 \, d\theta$$

ex: Find the area bounded by the polar curve.

b) $r = 4\cos 3\theta$

traces once $[0, \pi)$

$$\frac{1}{2}\int_0^{\pi} (4\cos 3\theta)^2 \, d\theta$$



$4\cos 3\theta = 0$

$\cos 3\theta = \frac{\pi}{2}$

$3\theta = \frac{\pi}{2}$; $\theta = \frac{\pi}{6}$

OR

$$3 \cdot \frac{1}{2}\int_{-\pi/6}^{\pi/6} (4\cos 3\theta)^2 \, d\theta$$

ex: Find the area bounded by the polar curve.

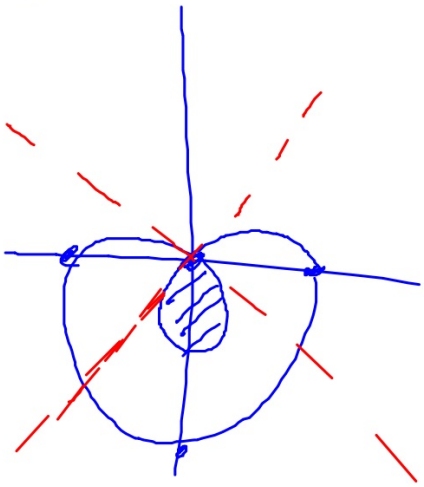
c) $r = 4 + 2\cos\theta$

$$\frac{1}{2}\int_0^{2\pi} (4+2\cos\theta)^2 \, d\theta$$

$$56.549$$

ex: $r = 1 - 2\sin\theta$

a) Find the area bounded by the inner loop.



.5435

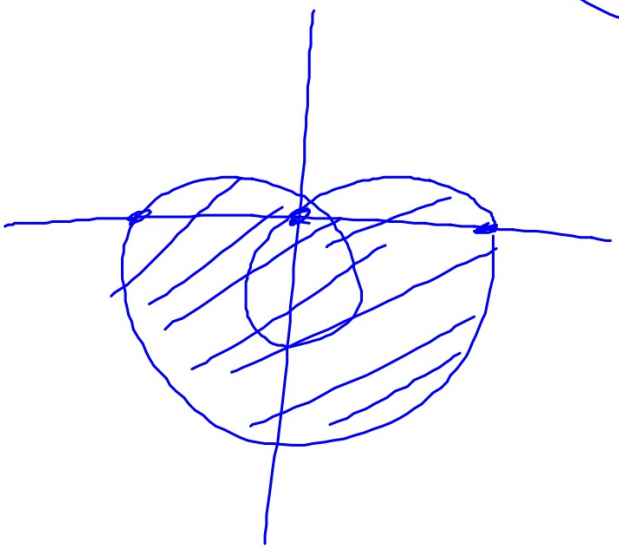
$0 = 1 - 2\sin\theta$

$\sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

ex: $r = 1 - 2\sin\theta$

b) Find the area bounded by the outer loop.



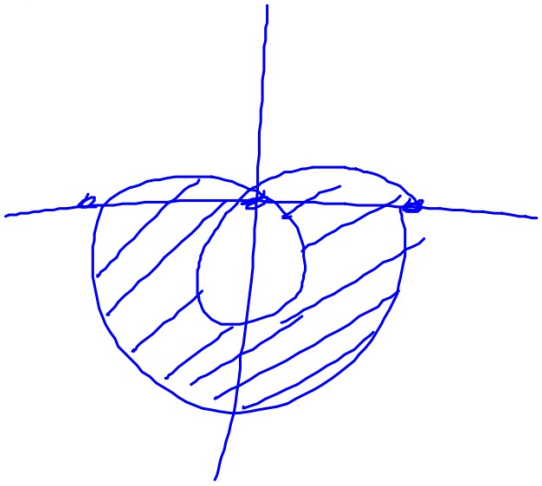
$$\frac{1}{2}\int_{0}^{2\pi}(1-2\sin\theta)^2 d\theta - \frac{1}{2}\int_{\pi/6}^{5\pi/6}(1-2\sin\theta)^2$$

$9.429 - .5435$

8.886

ex: $r = 1 - 2\sin\theta$

c) Find the area bounded between the loops.

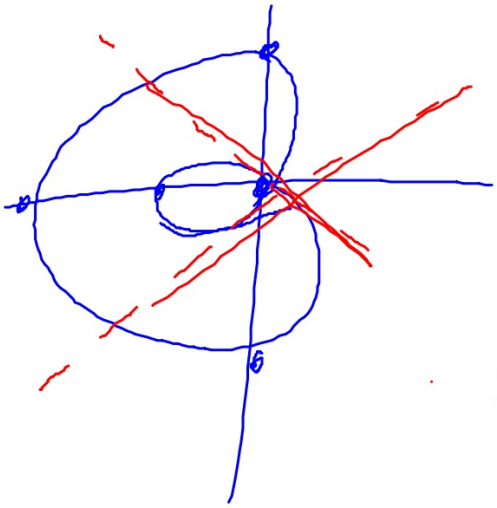


$$\frac{1}{2}\int_{0}^{2\pi}(1-2\sin\theta)^2\,d\theta - 2\cdot\frac{1}{2}\int_{\frac{\pi}{6}}^{5\pi/6}(1-2\sin\theta)^2\,d\theta$$

8.337

ex: Find the area enclosed by the inner loop of

$r = 1 - 2\cos\theta.$



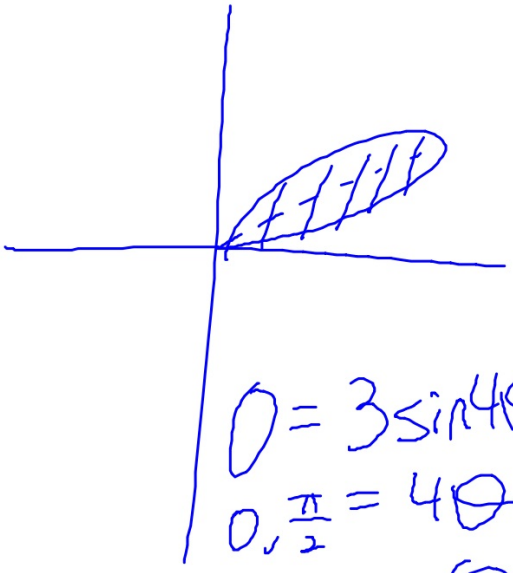
$0 = 1 - 2\cos\theta$

$\cos\theta = \frac{1}{2}$

$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$

$$\frac{1}{2}\int_{-\pi/3}^{\pi/3}(1-2\cos\theta)^2\,d\theta = .544$$

ex: Find the area enclosed by one petal of $r = 3\sin 4\theta$.



$0 = 3\sin 4\theta$

$0, \frac{\pi}{2} = 4\theta$

$0, \frac{\pi}{4} = \theta$

$$\frac{1}{2}\int_0^{\pi/4} (3\sin 4\theta)^2 \, d\theta$$

1.767

OR

$$\frac{1}{8} \cdot \frac{1}{2}\int_0^{2\pi} (3\sin 4\theta)^2 \, d\theta$$