13. $D: \{x \mid x > 0\}$

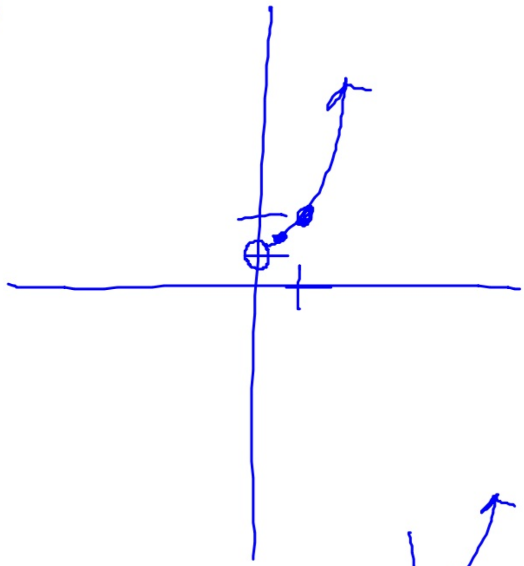
$R: \{y \mid y > 1\}$

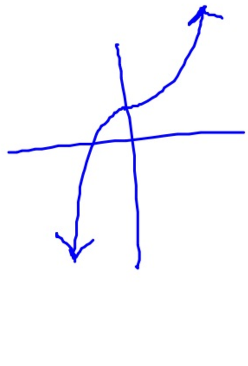
$$y = x^3 + 1$$

t	x	y
0	1	2
$\ln\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{8}+1$

$$x = e^t$$

$$y = e^{3t} + 1$$





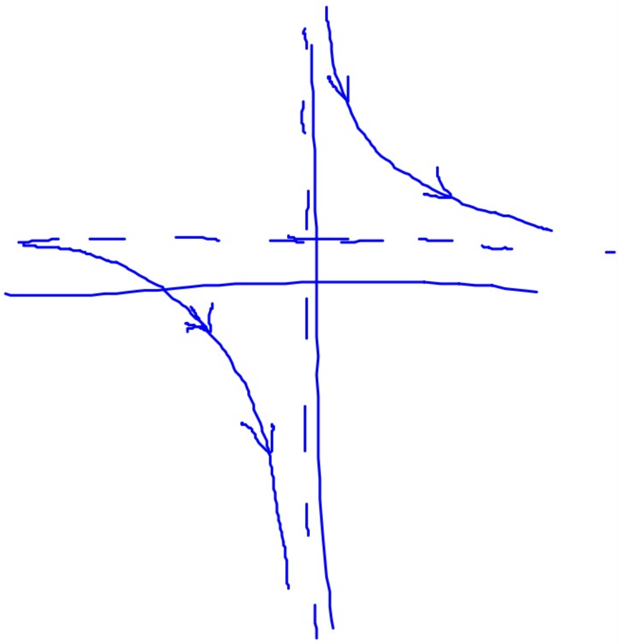
9. D: $\{x \mid x \neq 0\}$
R: $\{y \mid y \neq 1\}$

$t \neq 3$

$x = t - 3$

$y = \dfrac{t}{t-3}$

$y = \dfrac{x+3}{x} = 1 + \dfrac{3}{x}$



31.) $x = 4 - t$

$y = t^2$

$$\frac{dy}{dx} = \frac{2t}{-1} = \frac{-2t}{1}$$

$t = 4 - x$

$y = (4-x)^2$

$\underline{HT}$

$y = 0$

$\underline{VT}$

None

4 3.)

$$\frac{4t}{(2t+1)^3}$$

C. $x = \dfrac{t+2}{t^2-1}$ $y = t - 4$

$t = \pm 1$

$y = -3$

$y = -5$

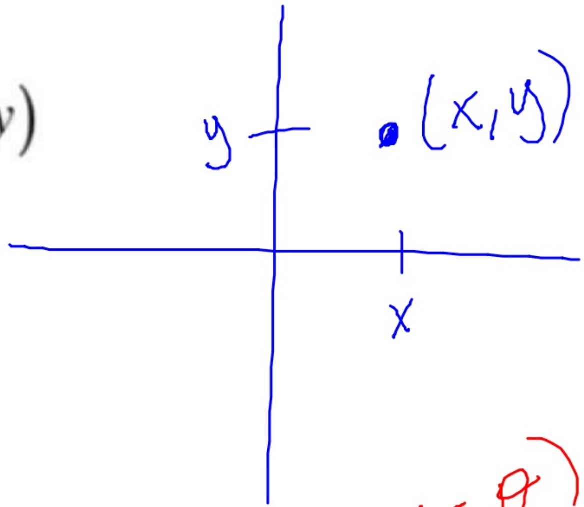
$t = y + 4$

$\lim_{x \to \infty} (y+4)$

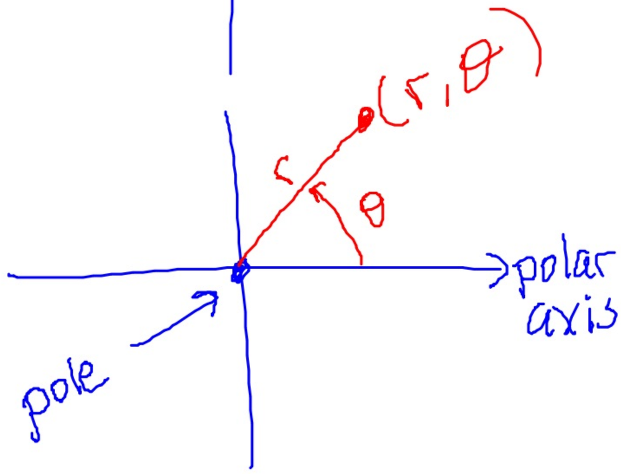
$x = 0$

9.4 Polar Coordinates

- Rectangular Coordinates: (x, y)



- Polar Coordinates: (r, θ)



ex: Plot.

a) $\left(1, \dfrac{\pi}{4}\right)$

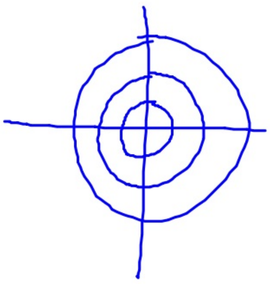
b) $\left(3, \dfrac{7\pi}{6}\right)$

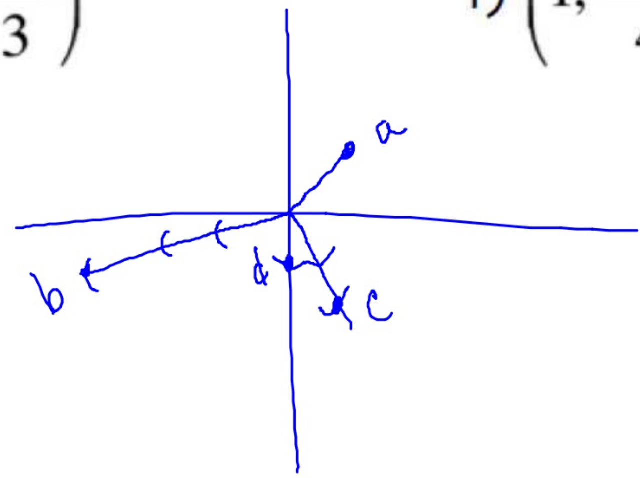
c) $\left(2, \dfrac{-\pi}{3}\right)$

d) $\left(-1, \dfrac{\pi}{2}\right)$

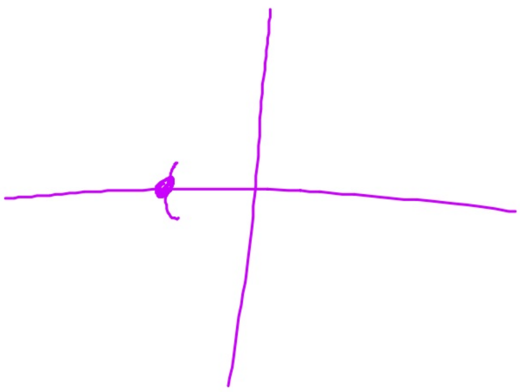
e) $\left(-2, \dfrac{-4\pi}{3}\right)$

f) $\left(1, \dfrac{-7\pi}{4}\right)$





ex: Find 3 other coordinates with the same location as $(1,\pi)$, given $\underbrace{-2\pi \le \theta \le 2\pi}$.



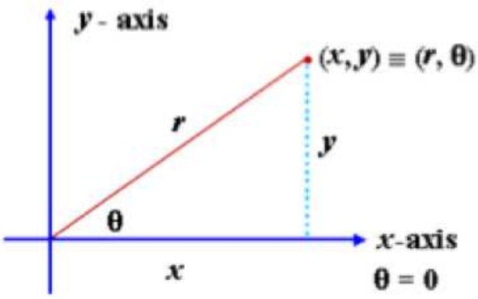
$(-1, 2\pi)$

$(-1, 0)$

$(1, -\pi)$

Converting Coordinates

1. From Polar (r,θ) to Rectangular (x,y)



Using SOHCAHTOA:

$$\cos\theta = \frac{x}{r} \qquad\qquad \sin\theta = \frac{y}{r}$$

$$\boxed{x = r\cos\theta \qquad\qquad y = r\sin\theta}$$

ex: Convert from polar to rectangular.

$$\left(5, -\frac{\pi}{3}\right)$$

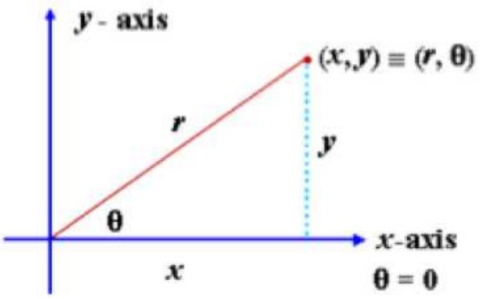
$x = r\cos\theta \qquad y = r\sin\theta$

$x = 5\left(\frac{1}{2}\right) \qquad y = 5\left(-\frac{\sqrt{3}}{2}\right)$

$$\left(\frac{5}{2}, \frac{-5\sqrt{3}}{2}\right)$$

Converting Coordinates

2. From Rectangular $\left(x, y\right)$ to Polar $\left(r, \theta\right)$

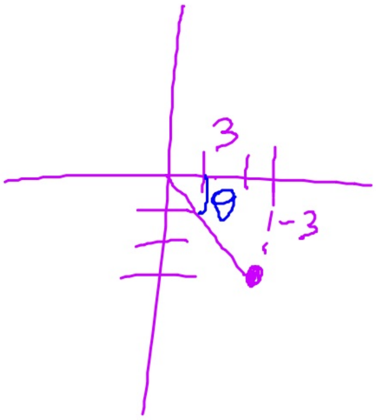


Using SOHCAHTOA and the Pythag. Theorem:

$$\boxed{\tan\theta = \frac{y}{x} \qquad\qquad x^2 + y^2 = r^2}$$

ex: Convert from rectangular to polar, $0 \le \theta \le 2\pi$.

a) $(3,-3)$



$r = 3\sqrt{2}$

$\theta = 45^\circ$

$\left(3\sqrt{2}, \frac{\pi}{4}\right)$

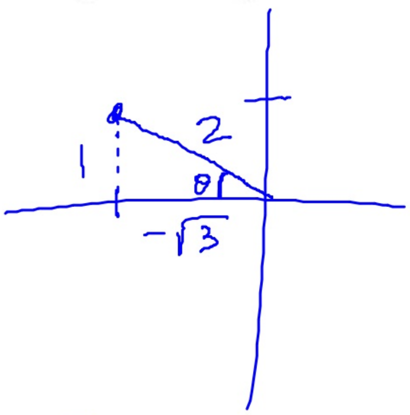
$\tan\theta = \frac{3}{3}$

$\tan\theta = 1$

$\theta = 45^\circ$

ex: Convert from rectangular to polar, $0 \le \theta \le 2\pi$.

b) $\left(-\sqrt{3}, 1\right)$



$\tan\theta = \frac{1}{\sqrt{3}}$

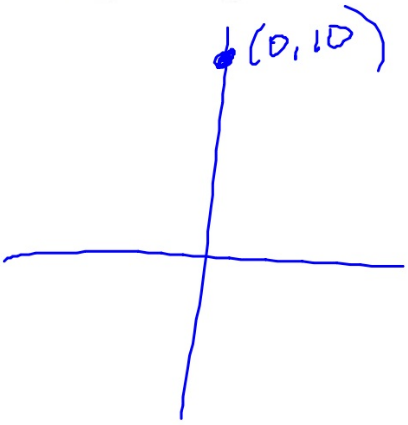
$\theta = \frac{\pi}{6}$

$\left(2, \frac{5\pi}{6}\right)$

$r = 2$

ex: Convert from rectangular to polar, $0 \le \theta \le 2\pi$.

c) $(0,10)$



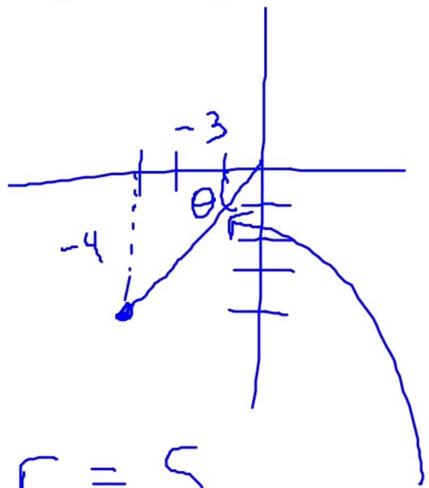
$\left(10, \frac{\pi}{2}\right)$

ex: Convert from rectangular to polar, $0 \le \theta \le 2\pi$.

d) $(-3,-4)$

$\tan\theta = \frac{4}{3}$

$\theta = .927$



$\pi + .927$

$(5,\ 4.069)$

$r = 5$

ex: Convert from rectangular to polar, $0 \le \theta \le 2\pi$.

e) $(-3,4)$

Converting Equations

You will need:

$$x = r\cos\theta \qquad y = r\sin\theta$$

$$\tan\theta = \frac{y}{x} \qquad x^2 + y^2 = r^2$$

*and possibly some trig identities too! :)

ex: Convert to polar.

a) $\underbrace{x^2 + y^2}_{} = 9$

$r^2 = 9$

$r = \pm 3$

$r = 3 \quad r = -3$

ex: Convert to polar.

$x = r\cos\theta \quad y = r\sin\theta$

b) $x^2 - y^2 = 9$

$(r\cos\theta)^2 - (r\sin\theta)^2 = 9$

$r^2(\cos^2\theta - \sin^2\theta) = 9$

$r = \pm\sqrt{\dfrac{9}{\cos^2\theta - \sin^2\theta}}$

ex: Convert to rectangular.

a) $r = -5\sin\theta$

eliminate θ first

$y = r\sin\theta$

$\frac{y}{r} = \sin\theta$

$r = -5\left(\frac{y}{r}\right)$

$r^2 = -5y$

$\boxed{x^2 + y^2 = -5y}$

ex: Convert to rectangular.

b) $r = 2$

$r^2 = 4$

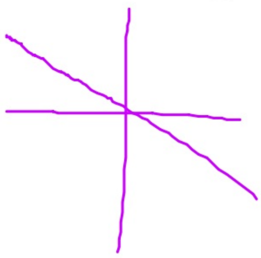
$x^2 + y^2 = 4$

c) $\theta = \dfrac{7\pi}{4}$

$\tan\theta = \tan\frac{7\pi}{4}$

$\frac{y}{x} = -1$

$y = -x$



ex: Convert to rectangular.

d) $r = -5\csc\theta$

$$y = r\sin\theta$$

$$\frac{y}{r} = \sin\theta$$

$$r = \frac{-5}{\sin\theta}$$

$$r = \frac{-5}{\frac{y}{r}}$$

$$\cancel{r} = \frac{-5 \cdot \cancel{r}}{y}$$

$$y = -5$$

Sketching Polar Curves

Circles

Type	Equation	Traces Once On The Interval	Symmetry
Centered at the Pole	$r = \#$		
Center on the x-axis, but not at the Pole	$r = d\cos\theta$		
Center on the y-axis, but not at the Pole	$r = d\sin\theta$		

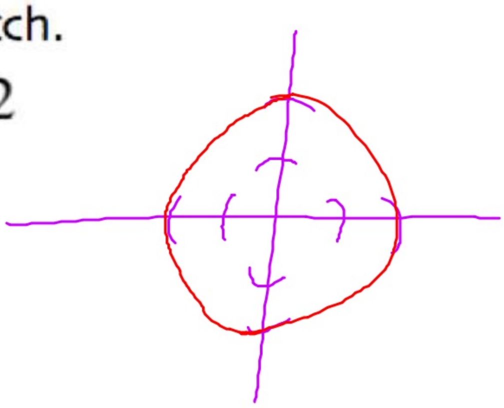
d = diameter

*See printout.

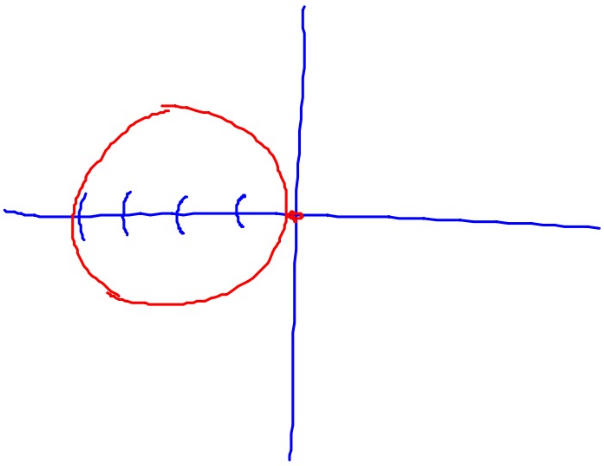
Circles

ex: Sketch.

a) $r = 2$



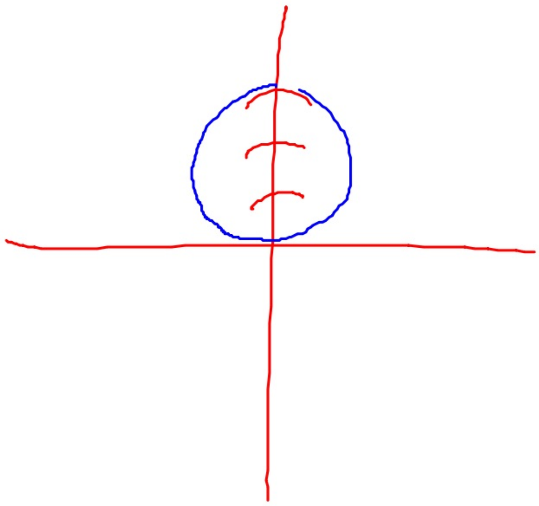
b) $r = -4\cos\theta$

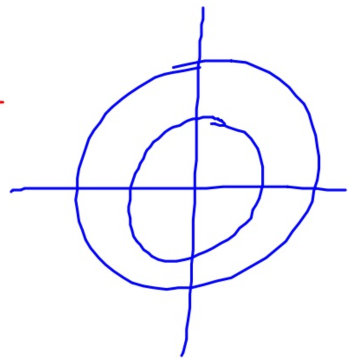


Circles

ex: Sketch.

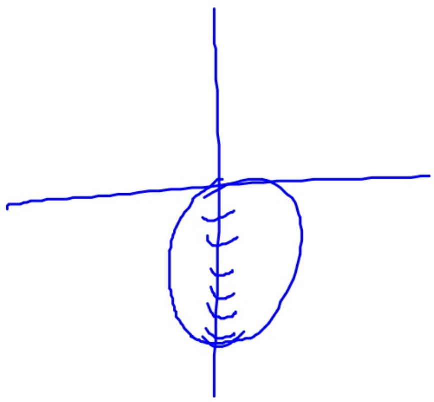
c) $r = 3\sin\theta$





d) $r\csc\theta = -7$

$r = -7\sin\theta$



Sketching Polar Curves

Lines

Type	Equation	Traces Once On The Interval	Symmetry
Oblique Line Through The Pole	θ = angle		
Vertical Line	# = $r\cos\theta$		
Horizontal Line	# = $r\sin\theta$		

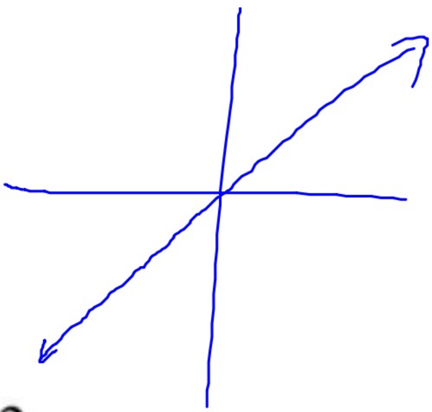
$r = -5\csc\theta$

$r\sin\theta = -5$

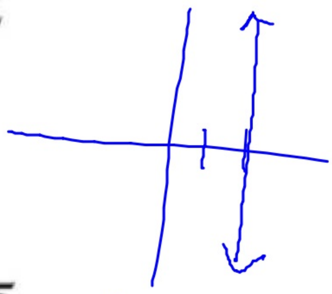
<u>Lines</u>

ex: Sketch.

e) $\theta = \dfrac{\pi}{4}$



f) $r\cos\theta = 2$



g) $r\sin\theta = -5$

Sketching Polar Curves

Leminiscates

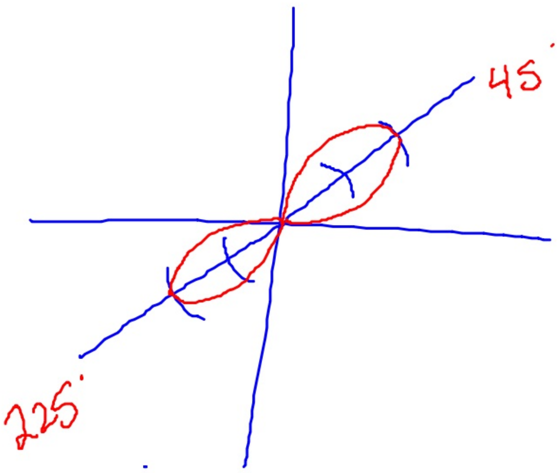
Type	Equation	Traces Once On The Interval	Symmetry
Petals On 45° and 225°	$r^2 = a^2 \sin 2\theta$		
Petals On 135° and 315°	$r^2 = -a^2 \sin 2\theta$		
Petals on the x-axis	$r^2 = a^2 \cos 2\theta$		
Petals on the y-axis	$r^2 = -a^2 \cos 2\theta$		

Lemniscates

ex: Sketch.

h) $r^2 = 4\sin 2\theta$

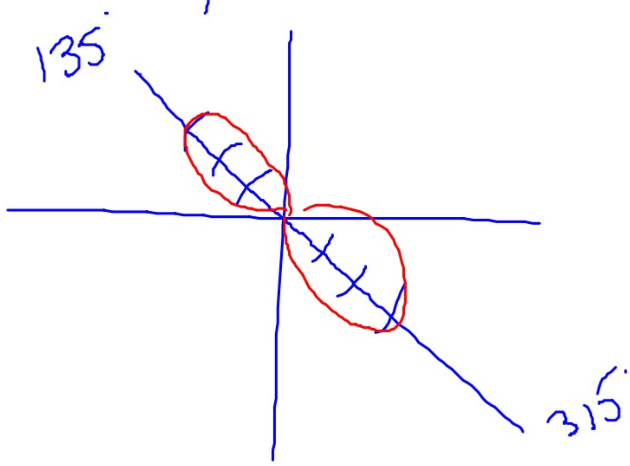
$a = 2$



+ slope

i) $r^2 = -9\sin 2\theta$

$a = 3$



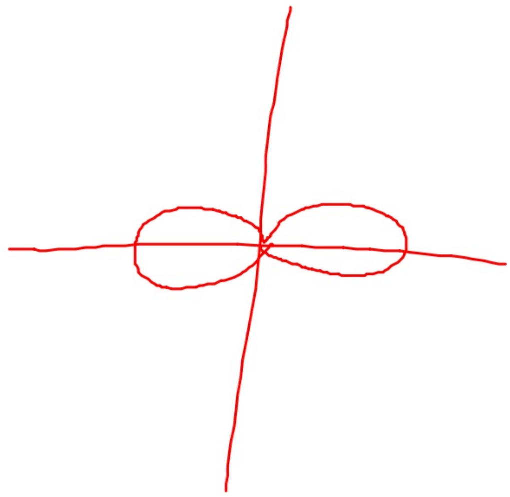
− slope

Lemniscates

ex: Sketch.

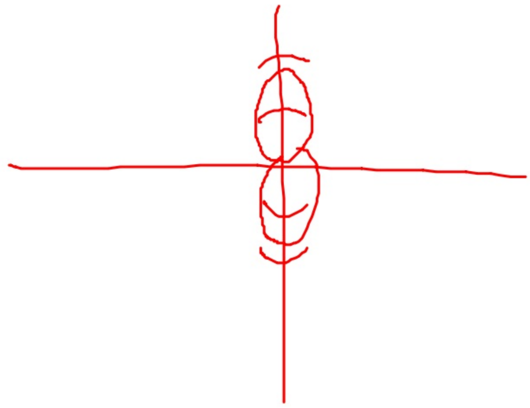
j) $r^2 = \cos 2\theta$

$a = 1$



k) $r^2 = -3\cos 2\theta$

$a = \sqrt{3}$



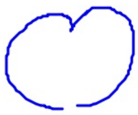
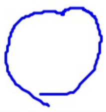
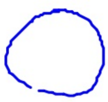
Sketching Polar Curves

Limacons

Equation	Traces Once On The Interval	Symmetry
$r = a \pm b\cos\theta$		
$r = a \pm b\sin\theta$		

Sketching Polar Curves

Limacons

Type	Condition
 Cardioid Through the Pole	$a = b$
 Limacon with inner loop	$a < b$
 Dimpled Limacon	$1 < \frac{a}{b} < 2$
 Convex Limacon	$\frac{a}{b} \geq 2$

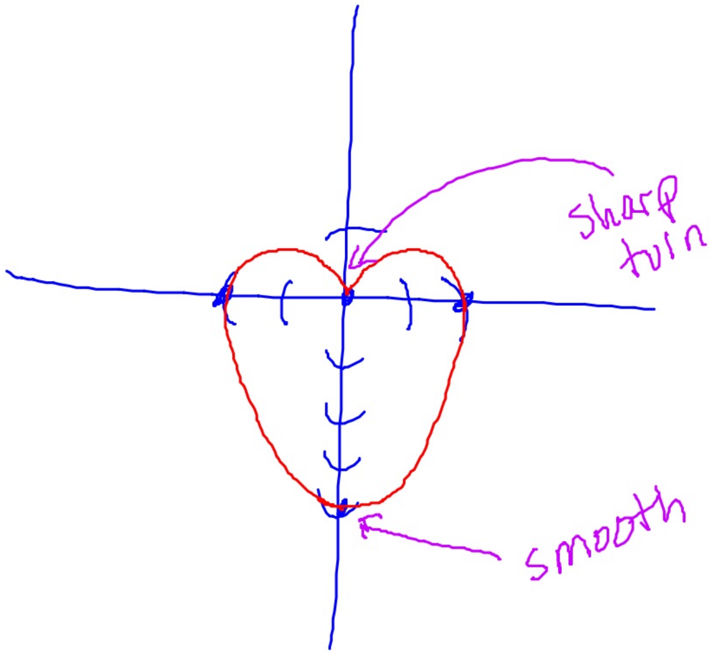
Limacons - Cardioid

ex: Sketch.

I) $r = 2 - 2\sin\theta$

$a = b$

r	θ
2	0
0	$\pi/2$
2	π
4	$3\pi/2$



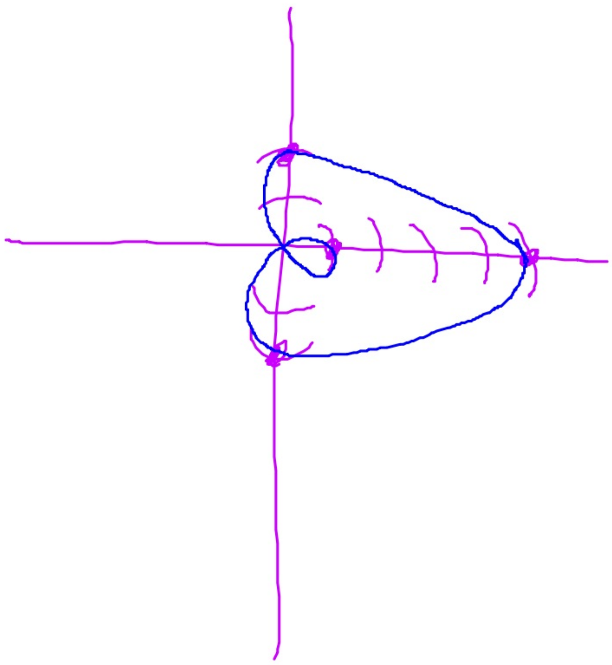
Limacons - Limacon With an Inner Loop

ex: Sketch.

m) $r = 2 + 3\cos\theta$

$a < b$

r	θ
5	0
2	$\pi/2$
-1	π
2	$3\pi/2$



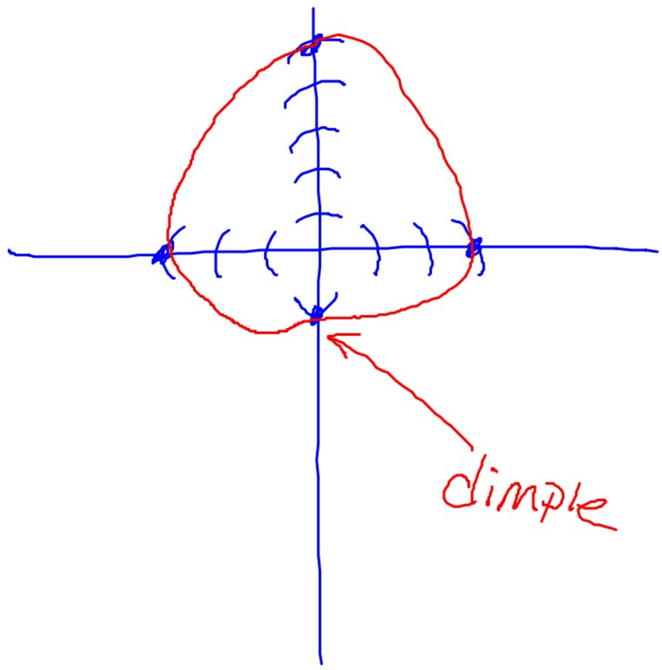
Limacons - Dimpled Limacon

ex: Sketch.

n) $r = 3 + 2\sin\theta$

$1 < \frac{a}{b} < 2$

r	θ
3	0
5	$\pi/2$
3	π
1	$3\pi/2$



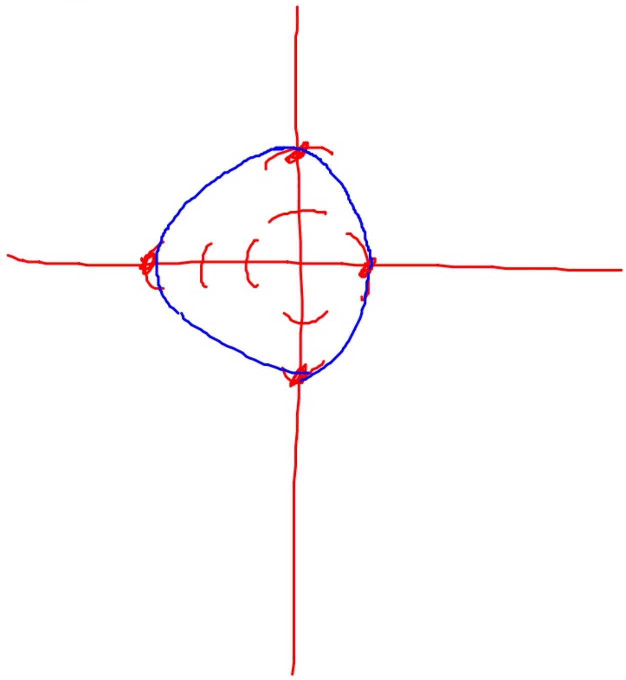
Limacons - Convex Limacon

ex: Sketch.

o) $r = 2 - \cos\theta$

$$\frac{a}{b} \geq 2$$

r	θ
1	0
2	$\pi/2$
3	π
2	$3\pi/2$



Sketching Polar Curves

Roses

Equation	Traces Once On The Interval	Symmetry
$r = a\cos(n\theta)$		
$r = a\sin(n\theta)$		

n is even $2n$

n is odd n

Roses

ex: Sketch.

p) $r = 3\sin 2\theta$

1.) petal length: 3

2.) # petals 4

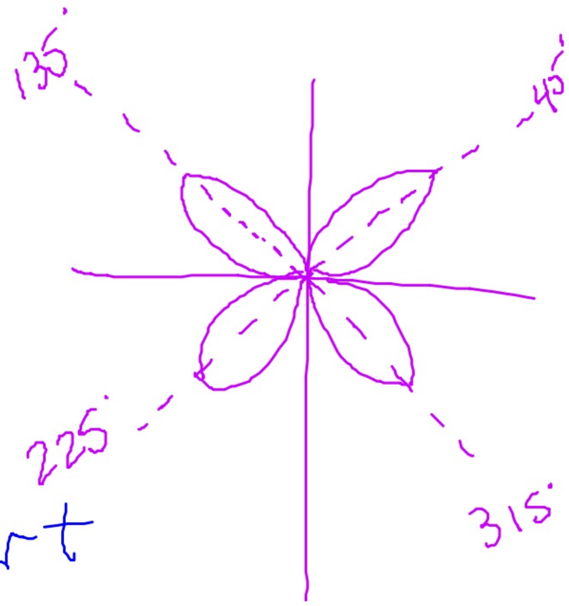
3. $\frac{360^\circ}{4} = 90^\circ$ apart

4. 1st petal

$\sin 2\theta = 1$

$2\theta = \frac{\pi}{2}$

$\theta = \frac{\pi}{4}$ ← first petal



Roses

ex: Sketch.

q) $r = 2\cos 3\theta$