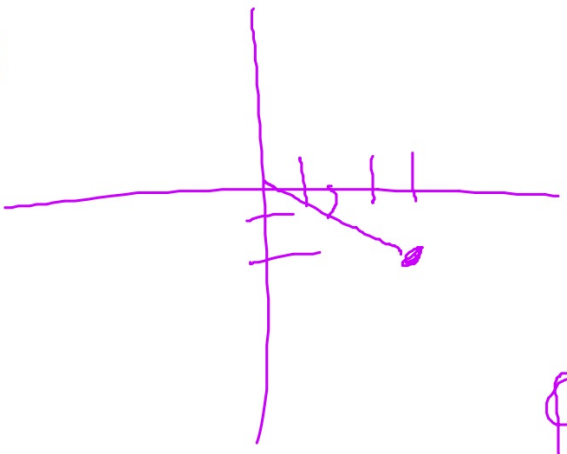
17.)



$r = \sqrt{13}$

$\theta = \tan^{-1}\left(\frac{2}{3}\right)$

$\theta = .588$

$(\sqrt{13}, 5.695)$

29) $3x - y + 2 = 0$

$3(r\cos\theta) - r\sin\theta + 2 = 0$

$r(3\cos\theta - \sin\theta) = -2$

$r = \dfrac{-2}{3\cos\theta - \sin\theta}$

9.4 Polar Coordinates - cont.

- Slope: $\frac{dy}{dx} = ?$

$$x = r\cos\theta$$
$$y = r\sin\theta$$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$$

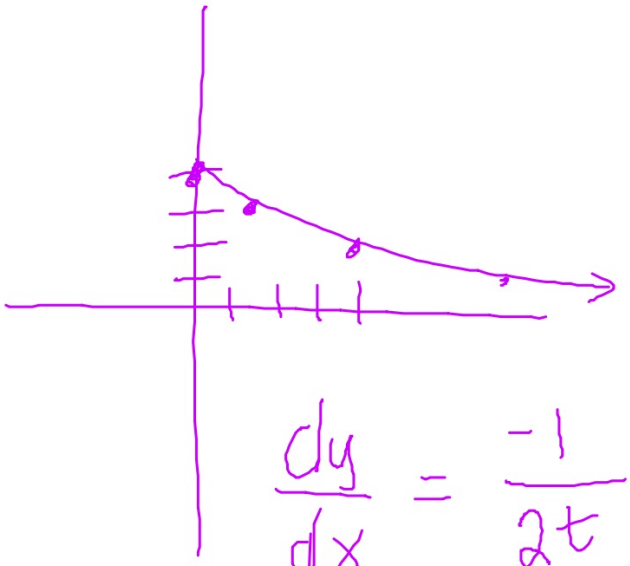
$x = t^2$

$y = 4 - t$

t	x	y
0	0	4
1	1	3
2	4	2
3	9	1
4	16	0

D: $[0, \infty)$

R: $(-\infty, 4]$



$$\frac{dy}{dx} = \frac{-1}{2t}$$

Roses

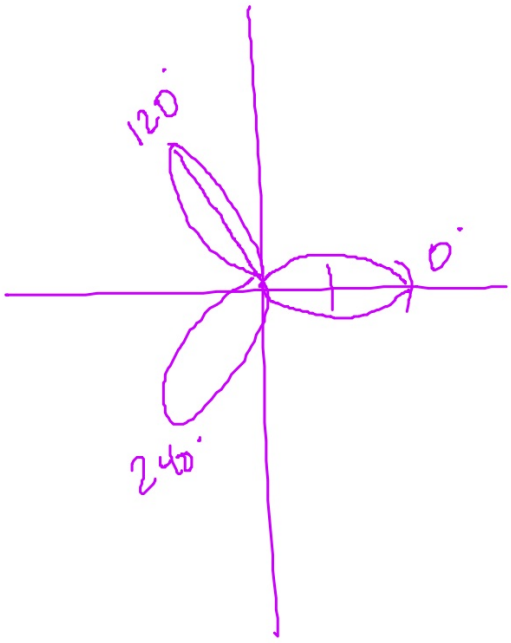
ex: Sketch.

q) $r = 2\cos 3\theta$

1. petal length: 2
2. # petals: 3
3. $\frac{360}{3} = 120$
4. $\cos 3\theta = 1$

$3\theta = 0$

$\theta = 0$



ex: Find the slope of $r = 5\cos 4\theta$ at $\theta = \frac{\pi}{4}$.

$x = r\cos\theta = 5(\cos 4\theta)(\cos\theta)$

$y = r\sin\theta = 5(\cos 4\theta)(\sin\theta)$

$$\frac{dy}{dx} = \frac{5\cos 4\theta\cos\theta + 5\sin\theta(-4\sin 4\theta)}{5\cos 4\theta(-\sin\theta) + 5\cos\theta(-4\sin 4\theta)}$$

$$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{4}} = -1$$

ex: Write the equation of the tangent line to $r = 5\cos 4\theta$ at $\theta = \frac{\pi}{4}$.

$\left(\frac{-5\sqrt{2}}{2}, \frac{-5\sqrt{2}}{2}\right)$ $m = -1$

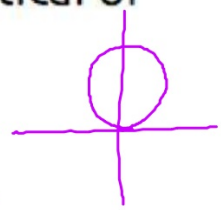
$y + \frac{5\sqrt{2}}{2} = -1\left(x + \frac{5\sqrt{2}}{2}\right)$

$x = 5\cos 4\theta \cos\theta = -\frac{5\sqrt{2}}{2}$

$x = r\cos\theta$
$y = r\sin\theta$

ex: Find the point(s) at which $r = \sin\theta$ has a vertical or horiztonal tangent.

$x = \sin\theta\cos\theta$
$y = \sin^2\theta$



HT

$\frac{dy}{d\theta} = 0, \quad \frac{dx}{d\theta} \neq 0$

$y = \sin^2\theta$

$\frac{dy}{d\theta} = 2\sin\theta\cos\theta$ HT

$0 = 2\sin\theta\cos\theta$

$\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

$(0, 0)$
$(0, 1)$

VT

$\frac{dx}{d\theta} = 0, \quad \frac{dy}{d\theta} \neq 0$

$x = \sin\theta\cos\theta$

$\frac{dx}{d\theta} = \cos^2\theta - \sin^2\theta$

$0 = \cos 2\theta$

$\frac{\pi}{2}, \frac{3\pi}{2} = 2\theta$

$\frac{\pi}{4}, \frac{3\pi}{4} = \theta$ VT

$\left(\frac{1}{2}, \frac{1}{2}\right) \quad \left(-\frac{1}{2}, \frac{1}{2}\right)$

ex: $r = 2 + 2\sin\theta$

a) If x=-1, what is θ?

$$x = r\cos\theta$$

$$-1 = (2 + 2\sin\theta)\cos\theta$$

$$-1 = 2\cos\theta + 2\sin\theta\cos\theta$$

$$\theta = 1.828,\ 3.6$$

ex: $r = 2 + 2\sin\theta$

b) If y=3, what is θ?

$3 = (2 + 2\sin\theta)\sin\theta$

$3 = 2\sin^2\theta + 2\sin\theta$

$[0, 2\pi)$

$0 = 2\sin^2\theta + 2\sin\theta - 3$

$\theta = .966, 2.175$

ex: On what intervals do each polar curve trace once?

Circles

Type	Traces Once On The Interval
Centered at the Pole	$[0, 2\pi)$
Center on the x-axis, but not at the Pole	$[0, \pi)$
Center on the y-axis, but not at the Pole	$[0, \pi)$

*See printout.

ex: On what intervals do each polar curve trace once?

Lines

Type	Traces Once On The Interval
Oblique Line Through The Pole	$[0, 2\pi)$
Vertical Line $3 = r\cos\theta$, $\frac{3}{\cos\theta} = r$	$[0, \pi)$
Horizontal Line	$[0, \pi)$

ex: On what intervals do each polar curve trace once?

Lemniscates

Type	Traces Once On The Interval
Petals On 45° and 225°	$[0, \pi)$
Petals On 135° and 315°	$[0, \pi)$
Petals on the x-axis	$[0, \pi)$
Petals on the y-axis	$[0, \pi)$

ex: On what intervals do each polar curve trace once?

Limacons

Equation	Traces Once On The Interval
$r = a \pm b\cos\theta$	$(0, 2\pi)$
$r = a \pm b\sin\theta$	$[0, 2\pi)$

ex: On what intervals do each polar curve trace once?

Roses

Type	Traces Once On The Interval
$r = a\cos(n\theta)$	If n is ODD: $[0, \pi)$ If n is EVEN: $[0, 2\pi)$
$r = a\sin(n\theta)$	If n is ODD: $[0, \pi)$ If n is EVEN: $[0, 2\pi)$

Polar Arc Length

$$s = \int_{\alpha}^{\beta} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

*r must be differentiable on $[\alpha, \beta]$

ex: Find the length of the curve: $r = 1 + \sin\theta$

$$S = \int_0^{2\pi} \sqrt{(1+\sin\theta)^2 + (\cos\theta)^2}\, d\theta$$

ex: Find the length of one petal: $r = 4\cos 3\theta$

$$S = \int_0^{\pi} \sqrt{(4\cos 3\theta)^2 + (-12\sin 3\theta)^2}\, d\theta$$

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