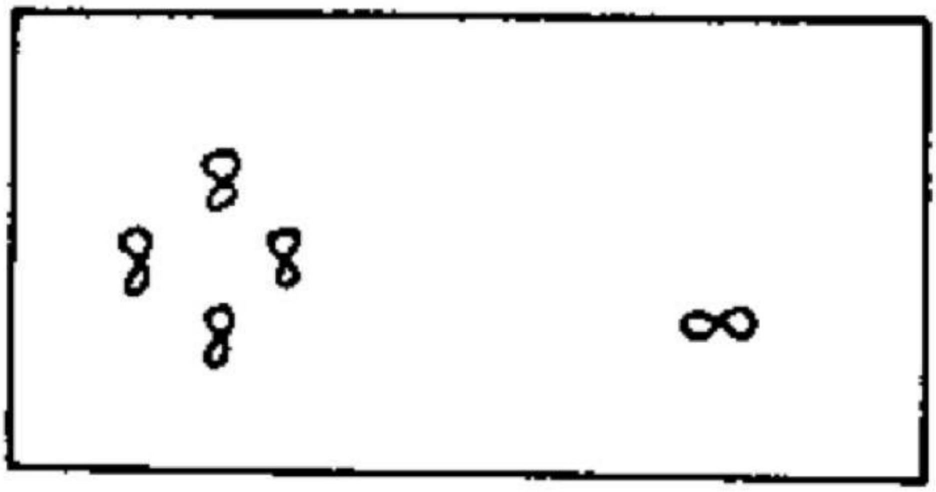
8.6 Ratio & Root Tests



Now that he's fallen down you can see how large he is.

The Ratio Test

*Look for a series that contains factorials or is "messy"

THEOREM 8.17 Ratio Test

Let $\Sigma\, a_n$ be a series with nonzero terms.

1. The series $\Sigma\, a_n$ converges absolutely when $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| < 1$.
2. The series $\Sigma\, a_n$ diverges when $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| > 1$ or $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \infty$.
3. The Ratio Test is inconclusive when $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 1$.

ex: Determine the convergence or divergence of the series.

a) $\sum_{n=1}^{\infty} \frac{n^2}{n!}$

$$\lim_{n\to\infty} \left| \frac{\frac{(n+1)^2}{(n+1)!}}{\frac{n^2}{n!}} \right| = \lim_{n\to\infty} \left| \frac{(n+1)^2}{(n+1)!} \cdot \frac{n!}{n^2} \right|$$

$(n+1)n!$

$$= \lim_{n\to\infty} \frac{n+1}{n^2} = 0 < 1$$

convergence by ratio test

ex: Determine the convergence or divergence of the series.

b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n!}{2^n}$

$$\lim_{n\to\infty} \left| \frac{(n+1)!}{2^{n+1}} \cdot \frac{2^n}{n!} \right|$$

$(n+1)n!$

$$\lim_{n\to\infty} \left| \frac{n+1}{2} \right| = \infty > 1$$

diverges by ratio test

ex: Determine the convergence or divergence of the series.

c) $\sum_{n=1}^{\infty} \frac{n^n}{n!}$

$$\lim_{n\to\infty} \left| \frac{(n+1)^{n+1}}{(n+1)!} \cdot \frac{n!}{n^n} \right| = \lim_{n\to\infty} \left| \frac{(n+1)^n}{n^n} \right|$$

$$= \lim_{n\to\infty} \left| \left(\frac{n+1}{n} \right)^n \right| = \lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n = e > 1$$

1^{∞}

divergence by ratio test

The Root Test

*Look for a series that contains nth powers.

THEOREM 8.18 Root Test

1. The series Σa_n converges absolutely when

$$\lim_{n\to\infty} \sqrt[n]{|a_n|} < 1.$$

2. The series Σa_n diverges when

$$\lim_{n\to\infty} \sqrt[n]{|a_n|} > 1 \quad \text{or} \quad \lim_{n\to\infty} \sqrt[n]{|a_n|} = \infty.$$

3. The Root Test is inconclusive when

$$\lim_{n\to\infty} \sqrt[n]{|a_n|} = 1.$$

ex: Determine the convergence or divergence of the series.

d) $\sum_{n=1}^{\infty} \frac{e^{2n}}{n^n}$

$$\sum_{n=1}^{\infty} \left(\frac{e^2}{n}\right)^n$$

$$\lim_{n\to\infty} \sqrt[n]{\left(\frac{e^2}{n}\right)^n} = \lim_{n\to\infty} \frac{e^2}{n} = 0 < 1$$

convergence by root test

There are so many tests, HOW DO I KNOW WHICH SERIES TEST TO USE?!?!?!

I like to use the tests in this order...

1. geometric/n^{th} term/ p-series, AST
2. LCT/ DCT
3. Ratio/Root
4. Integral

ex: Determine the convergence or divergence of the series. Show the work that leads to your conclusion. Use ALL tests exactly once.

a) $\sum_{n=1}^{\infty} \frac{1}{3n+1}$ LCT $\frac{1}{3n}$

b) $\sum_{n=1}^{\infty} \left(\frac{\pi}{e}\right)^n$ geo.

c) $\sum_{n=1}^{\infty} ne^{-n^2}$

d) $\sum_{n=1}^{\infty} \frac{(-1)^n (n+1)}{3n+1}$ nth term

e) $\sum_{n=1}^{\infty} \left(\frac{n+1}{2n+1}\right)^n$ root

f) $\sum_{n=6}^{\infty} 7n^{-5}$ p-series

g) $\sum_{n=0}^{\infty} \frac{n!}{10^n}$ ratio

h) $\sum_{n=1}^{\infty} \frac{(-1)^n}{3n+1}$ Alt. series

i) $\sum_{n=1}^{\infty} \frac{\cos 2n}{n^3}$ DCT

ex: Determine the convergence or divergence of the series. Show the work that leads to your conclusion. Use ALL tests exactly once.

a) $\sum_{n=1}^{\infty} \frac{1}{3n+1}$

ex: Determine the convergence or divergence of the series. Show the work that leads to your conclusion. Use ALL tests exactly once.

b) $\sum_{n=1}^{\infty} \left(\frac{\pi}{e}\right)^n$

ex: Determine the convergence or divergence of the series. Show the work that leads to your conclusion. Use ALL tests exactly once.

c) $\sum_{n=1}^{\infty} ne^{-n^2}$

ex: Determine the convergence or divergence of the series. Show the work that leads to your conclusion. Use ALL tests exactly once.

d) $\sum_{n=1}^{\infty} \frac{(-1)^n (n+1)}{3n+1}$

ex: Determine the convergence or divergence of the series. Show the work that leads to your conclusion. Use ALL tests exactly once.

e) $\sum_{n=1}^{\infty}\left(\frac{n+1}{2n+1}\right)^n$

ex: Determine the convergence or divergence of the series. Show the work that leads to your conclusion. Use ALL tests exactly once.

f) $\sum_{n=6}^{\infty} 7n^{-5}$

ex: Determine the convergence or divergence of the series. Show the work that leads to your conclusion. Use ALL tests exactly once.

g) $\sum_{n=0}^{\infty} \frac{n!}{10^n}$

ex: Determine the convergence or divergence of the series. Show the work that leads to your conclusion. Use ALL tests exactly once.

h) $\sum_{n=1}^{\infty} \frac{(-1)^n}{3n+1}$

ex: Determine the convergence or divergence of the series. Show the work that leads to your conclusion. Use ALL tests exactly once.

i) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos 2n}{n^3}$

ex: Write an equivalent series with the index of summation beginning at n=o.

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} = \frac{-1}{1} + \frac{1}{4} - \frac{1}{9} + \cdots$$

$$\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{(n+1)^2}$$

ex: Write an equivalent series with the index of summation beginning at n=2.

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{3n+1}$$

ex:

If the series $\sum_{n=1}^{\infty} a_n$ converges and $a_n > 0$ for all n, which of the following must be true?

(A) $\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = 0$

(B) $|a_n| < 1$ for all n

(C) $\sum_{n=1}^{\infty} a_n = 0$

(D) $\sum_{n=1}^{\infty} n a_n$ diverges.

(E) $\sum_{n=1}^{\infty} \frac{a_n}{n}$ converges.

$\frac{5}{n^3}$

$\sum_{n=1}^{\infty} \frac{1}{n^2}$

$n \cdot \frac{1}{n^3}$

Fill in your convergence chart...

Test	Series	Condition(s) of Convergence	Condition(s) of Divergence	Comment
Ratio				

Test	Series	Condition(s) of Convergence	Condition(s) of Divergence	Comment
Root				

76.) $\sum_{n=0}^{\infty} \frac{(-1)^n}{n+4}$

$\lim_{n \to \infty} \frac{1}{n+4} = 0$ ✓

$a_{n+1} \leq a_n$ ✓

A.S.T.

(convergence)

conditionally

$\sum_{n=0}^{\infty} \frac{1}{n+4}$

diverges

(general harmonic)