$$\lim_{b\to\infty}\int_1^b 2^{-n}\,dn = \lim_{b\to\infty} -\frac{2^{-n}}{\ln 2}\Bigg|_1^b$$

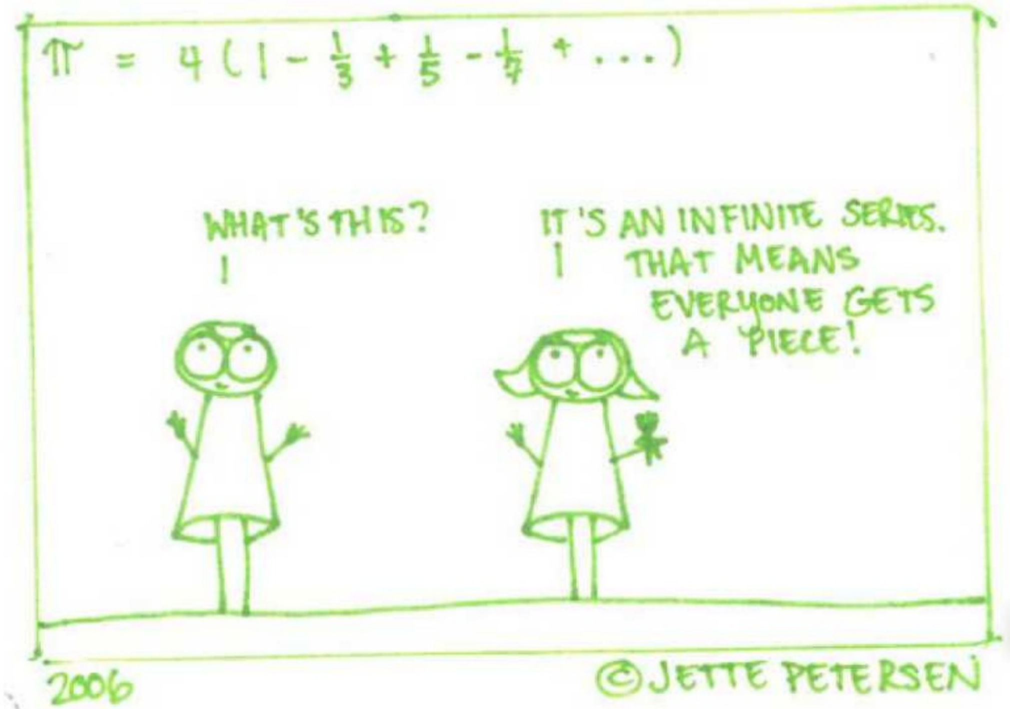
$$= -\lim_{b\to\infty}\left(\frac{2^{-b}}{\ln 2} - \frac{2^{-1}}{\ln 2}\right)$$

35.) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}} = \sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$

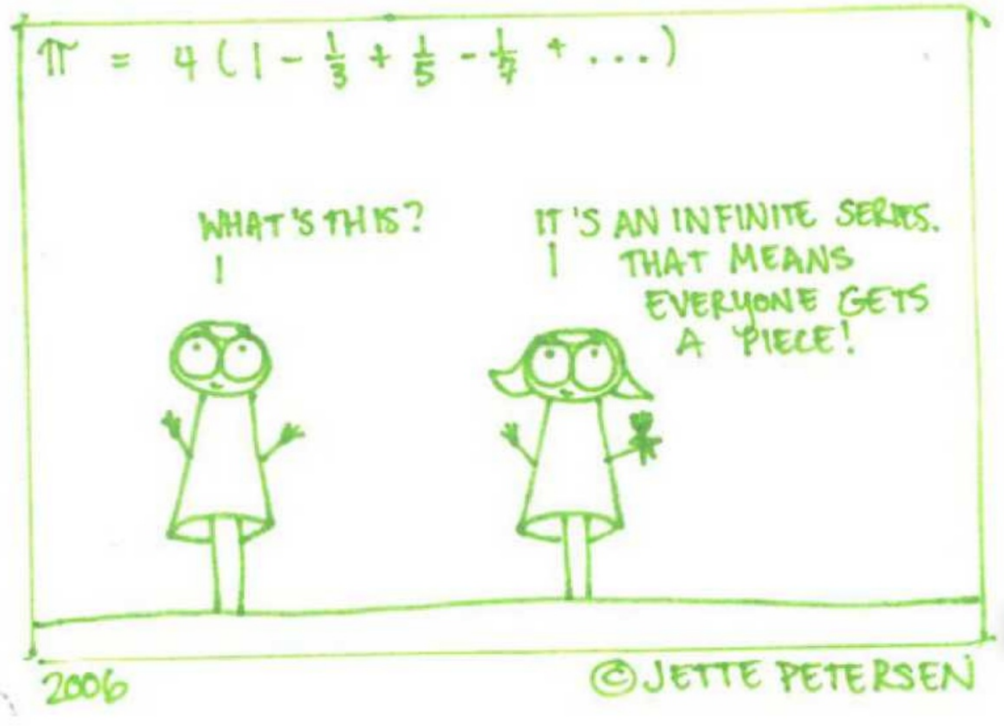
27.) $\dfrac{2+\sin 1}{1} \qquad \dfrac{2+\sin 2}{2}$

$$\int_{\#}^{\infty} a_n \, dn$$

8.4 Comparison Tests



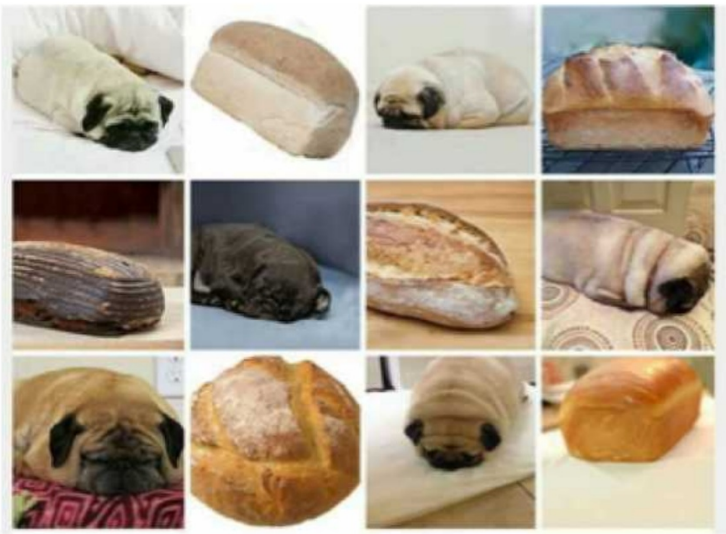
8.4 Comparison Tests



Lets have some fun...

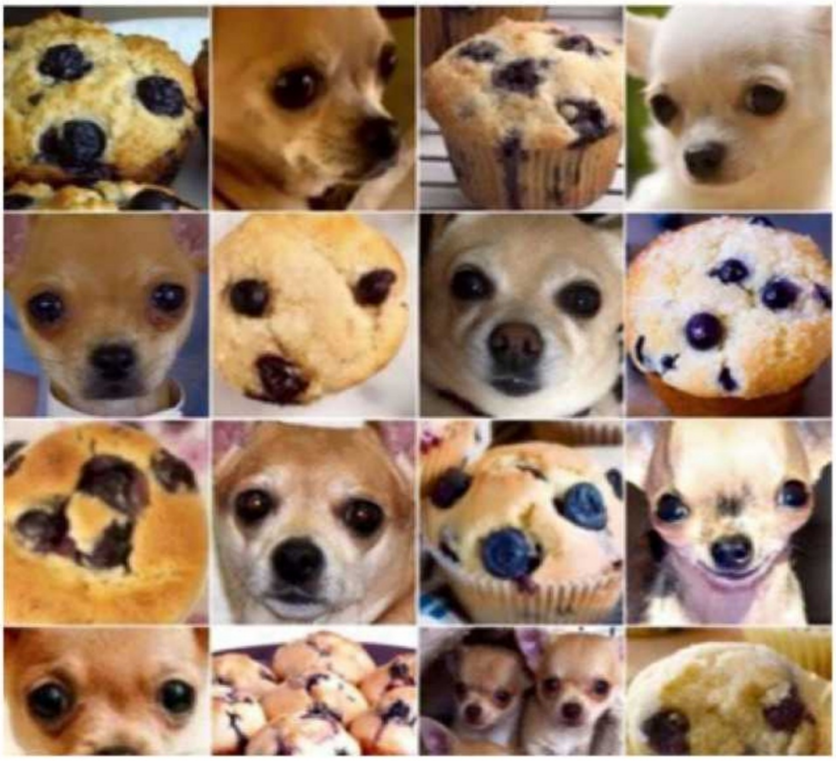
Puppies or
Fried
Chicken?!?!

How about this one?



Pug or Loaf?!?!

Last one!



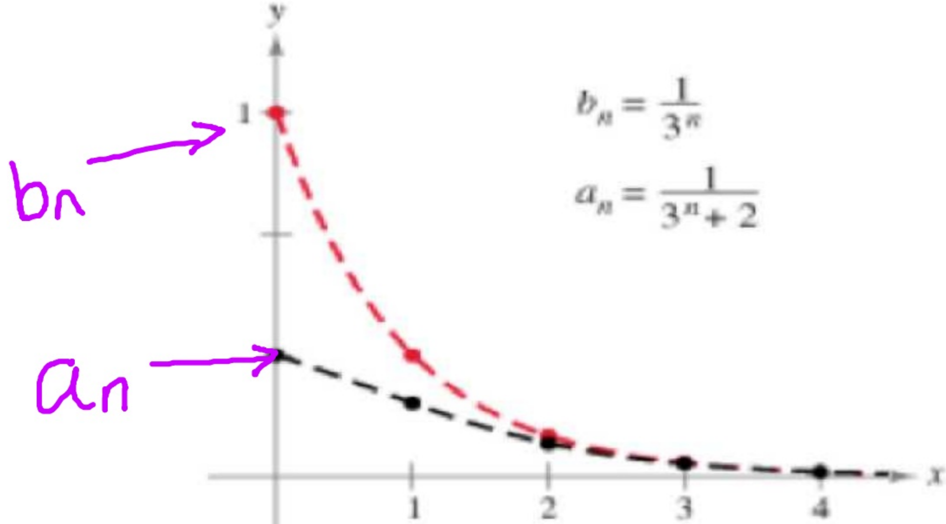
Dog or Muffin?!?!

Consider the series: $\sum_{n=0}^{\infty} \frac{1}{3^n + 2}$

We may not know about this series but we should know something about: $\sum_{n=0}^{\infty} \frac{1}{3^n}$

Reminder: If a series <u>converges</u>, then the sum of said series is finite.

Graphical Perpective



Since, $\frac{1}{3^n+2} < \frac{1}{3^n}$ for all n, then $\sum_{n=0}^{\infty} \frac{1}{3^n+2}$

must converge too!

The Direct Comparison Test (DCT)

*Look for a series that looks "similar" to a p-series or a geometric series.

*The numerator and/or denominator must contain multiple terms.

- Looks "like" a geometric series: $\sum_{n=1}^{\infty} \frac{3}{4+5^n}$ $\left(\frac{1}{5}\right)^n$

- Looks "like" a p-series: $\sum_{n=1}^{\infty} \frac{n^2+n-5}{n^3+1}$ $\frac{1}{n}$

The Direct Comparison Test (DCT)

THEOREM 8.12 Direct Comparison Test

Let $0 < a_n \leq b_n$ for all n.

1. If $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges.
2. If $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} b_n$ diverges.

Using The Direct Comparison Test (DCT)

Process:

1. Pick a comparision test - keep the term in the numerator AND denominator that has the largest magnitude.
2. Test the comparison series.
3. Set up an inequality.

   If the comparison converges show:

   $$\text{given} \leq \text{comparison}$$

   $<$ "C"

   If the comparison diverges show:

   $$\text{given} \geq \text{comparison}$$

   $>$ "D"

* If the inequality is FALSE, the DCT fails. In this case use the LCT.

ex: Determine the convergence or divergence of the series.

a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2+3^n}$

compare: $\left(\frac{1}{3}\right)^n$ $\frac{1}{3^n}$

$\frac{1}{2+3^n} \leq \frac{1}{3^n}$

$2+3^n \geq 3^n$

true

$3^n \leq 2+3^n$

converges by D.C.T.

ex: Determine the convergence or divergence of the series.

b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{3\sqrt{n}-2}$

Compare : $\dfrac{1}{3\sqrt{n}}$ (divergent)

$\dfrac{1}{3\sqrt{n}-2} \geq \dfrac{1}{3\sqrt{n}}$

$3\sqrt{n}-2 \leq 3\sqrt{n}$ ✓

Divergent D.C.T.

ex: Determine the convergence or divergence of the series.

c) $\displaystyle\sum_{n=0}^{\infty} \frac{5}{3\sqrt{n}+2}$

Compare $\frac{5}{3\sqrt{n}}$

Div.

$\frac{5}{3\sqrt{n}+2} \overset{?}{\geq} \frac{5}{3\sqrt{n}}$ X

$15\sqrt{n} \geq 5(3\sqrt{n}+2)$

$15 \geq 5(3+2)$

$15 \geq 25$ X

D.C.T. fails.

use L.C.T.

The Limit Comparison Test (LCT)

*Also works well with "p-like series" and "geometric-like series" or "messy" series

*The numerator and/or denominator must contain multiple terms.

- "messy" series: $\sum_{n=1}^{\infty} \frac{n5^n}{4n^3+1}$

The Limit Comparison Test (LCT)

THEOREM 8.13 Limit Comparison Test

If $a_n > 0$, $b_n > 0$, and

$$\lim_{n \to \infty} \frac{a_n}{b_n} = L$$

where L is *finite and positive,* then

$$\sum_{n=1}^{\infty} a_n \quad \text{and} \quad \sum_{n=1}^{\infty} b_n$$

either both converge or both diverge.

Using The Limit Comparison Test (LCT)

Process:

1. Pick a comparision test - keep the term in the numerator AND denominator that has the largest magnitude.
2. Test the comparison series.
3. Set up a limit.

* When used appropriately this test rarely fails! :)

ex: Determine the convergence or divergence of the series.

d) $\displaystyle\sum_{n=0}^{\infty} \frac{5}{3\sqrt{n}+2}$

$\dfrac{5}{3\sqrt{n}}$ div. p-series

$$\lim_{n\to\infty} \frac{\frac{5}{3\sqrt{n}+2}}{\frac{5}{3\sqrt{n}}} = \lim_{n\to\infty} \frac{\cancel{5}}{3\sqrt{n}+2} \cdot \frac{3\sqrt{n}}{\cancel{5}} = 1$$

$\therefore$ series diverges by LCT

ex: Determine the convergence or divergence of the series.

compare

e) $\sum_{n=1}^{\infty} \frac{n5^n}{4n^3+1}$

$\frac{5^n}{4n^2}$

div.

$\lim_{n \to \infty} \frac{n\cancel{5^n}}{4n^3+1} \cdot \frac{4n^2}{\cancel{5^n}} = 1$

Divergence L.C.T.

ex: Determine the convergence or divergence of the series.

Compare $\frac{3}{n^2}$

(conv.)

p-series

$p>1$

f) $\displaystyle\sum_{n=1}^{\infty} \frac{3}{n\sqrt{n^2+4}}$

$$\lim_{n\to\infty} \frac{\cancel{3}}{n\sqrt{n^2+4}} \cdot \frac{n^2}{\cancel{3}}$$

$$\lim_{n\to\infty} \frac{n}{\sqrt{n^2+4}} = 1$$

converges by L.C.T.

ex: Determine the convergence or divergence of the series.

Compare: $\frac{1}{n}$ div.

g) $\sum_{n=1}^{\infty} \tan\left(\frac{1}{n}\right)$

L'Hop

$$\lim_{n\to\infty} \frac{\tan\left(\frac{1}{n}\right)}{\frac{1}{n}} \stackrel{\text{L'Hop}}{=} \lim_{n\to\infty} \frac{\sec^2\left(\frac{1}{n}\right)\cdot\cancel{\left(-\frac{1}{n^2}\right)}}{\cancel{\left(-\frac{1}{n^2}\right)}} = 1$$

divergence by L.C.T.

ex:

Which of the following series cannot be shown to converge using the limit comparison test with the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$?

(A) $\sum_{n=1}^{\infty} \frac{4}{3n^2 - n}$

(B) $\sum_{n=1}^{\infty} \frac{15}{\sqrt{n^4 + 5}}$

(C) $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n^2}\right)$

(D) $\sum_{n=1}^{\infty} \frac{\ln n}{n^2}$

$$\lim_{n\to\infty} \frac{\ln n}{\cancel{n^2}} \cdot \frac{\cancel{n^2}}{1} = \infty$$

$$\lim_{n\to\infty} \frac{4}{3n^2 - n} \cdot \frac{n^2}{1} = \frac{4}{3}$$

ex:

Which of the following series converges?

(A) $\displaystyle\sum_{n=1}^{\infty} \frac{3n}{n+2}$

(B) $\displaystyle\sum_{n=1}^{\infty} \frac{3n}{n^2+2}$ $\longrightarrow$ $\displaystyle\lim_{n\to\infty} \frac{3n}{n^2+2} \cdot \frac{n}{3} = 1$

(C) $\displaystyle\sum_{n=1}^{\infty} \frac{3n}{n^2+2n}$

(D) $\displaystyle\sum_{n=1}^{\infty} \frac{3n^2}{n^3+2n}$

$$\lim_{n\to\infty} \frac{\cancel{3}n^2}{n^4+2n} \cdot \frac{n^2}{\cancel{3}} = 1$$

(E) $\displaystyle\sum_{n=1}^{\infty} \frac{3n^2}{n^4+2n}$

Fill in your convergence chart...

Compare $\sum_{n=\#}^{\infty} b_n$

Test	Series	Condition(s) of Convergence	Condition(s) of Divergence	Comment
Direct Comparison like pseries or geo.	$\sum_{n=\#}^{\infty} a_n$	$a_n \leq b_n$	$a_n \geq b_n$	if test fails use LCT

Compare $\sum_{n=\#}^{\infty} b_n$

Test	Series	Condition(s) of Convergence	Condition(s) of Divergence	Comment
Limit Comparison "messy"	$\sum_{n=\#}^{\infty} a_n$	comp. conv. $\lim_{n \to \infty} \frac{a_n}{b_n}$	comp. div. $\lim_{n \to \infty} \frac{a_n}{b_n}$	

finite, pos.