8.1 Sequences

sequence - a function whose domain is the set of nonnegative integers

ex: 1, 2, 3, 4

ex: 2, -4, 8, -16, 32...

ex: $1, x, x^2, x^3, x^4 \ldots$

Sequence Terms

$$\begin{array}{cccccccc} 1, & 2, & 3, & 4, & \ldots, & n, & \ldots & \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \text{Sequence} \\ a_1, & a_2, & a_3, & a_4, & \ldots, & a_n, & \ldots & \end{array}$$

ex: Write the 1st 5 terms of the sequence and sketch the graph.

a) $a_n = 2n + 3$

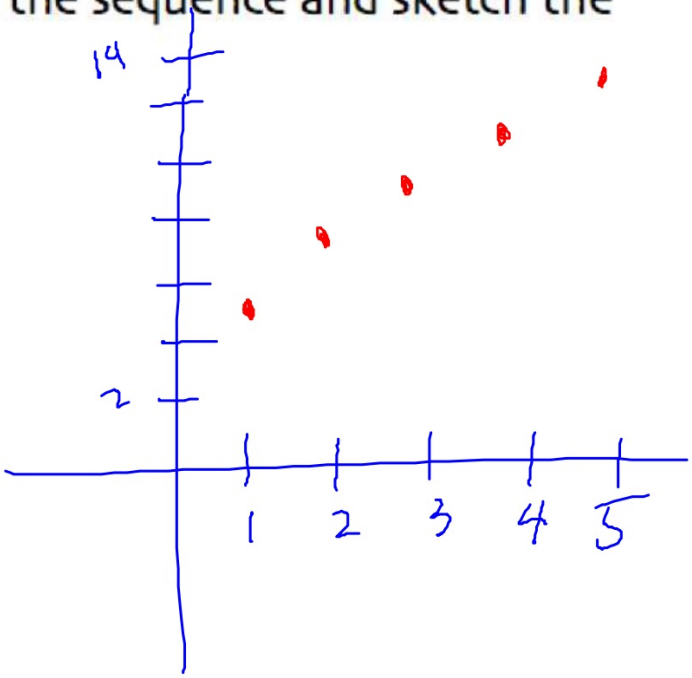
$a_1 = 5$

$a_2 = 7$

$a_3 = 9$

$a_4 = 11$

$a_5 = 13$



ex: Write the 1st 5 terms of the sequence and sketch the graph.

Recursive

b) $a_{n+1} = -2a_n, \quad a_1 = 1$

$a_1 = 1$

$a_2 = a_{1+1} = -2(1) = -2$

$a_3 = -2(-2) = 4$

$a_4 = -2(4) = -8$

$a_5 = -2(-8) = 16$

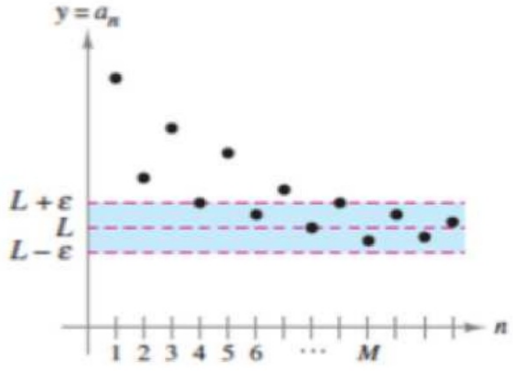
$a_n = (-2)^{n-1}$

The Limit Of A Sequence

Definition of the Limit of a Sequence

Let L be a real number. The **limit** of a sequence $\{a_n\}$ is L, written as

$$\lim_{n\to\infty} a_n = L$$



*If L is finite, the sequence converges to L

*If L is infinite or nonexistent, the sequence diverges.

ex: Determine if the sequence converges or diverges.

a) $a_n = \dfrac{5n}{\sqrt{n^2+4}}$ $\lim_{n\to\infty} a_n = 5$ converges to 5

b) $a_n = \sin(n)$ $\lim_{n\to\infty} \sin n =$ dne diverges

ex: Determine if the sequence converges or diverges.

c) $a_n = 3 + (-1)^n$ $\lim_{n \to \infty} a_n = \text{dne}$ $\therefore$ diverges

d) $a_n = \dfrac{3 + (-1)^n}{n}$ $\lim_{n \to \infty} a_n = 0$ $\therefore$ converges to 0

ex: Determine if the sequence converges or diverges.

e) $a_n = \arctan(n)$ $\lim_{n\to\infty} a_n = \frac{\pi}{2}$ Conv.

f) $a_n = \left(1+\frac{1}{n}\right)^n$ $\lim_{n\to\infty} a_n = e$ Conv.

g) $a_n = \frac{1}{n!}$ $\lim_{n\to\infty} a_n = 0$ Conv.

Factorials!

$$n! = n \cdot (n-1) \cdot (n-2) \cdots 3 \cdot 2 \cdot 1$$

ex: Evaluate.

$0! =$ 1

$1! =$ 1

$2! =$ 2

$3! =$ 6

$4! =$ 24

$5! =$ 120

$6! =$ 720

ex: Simplify.

a) $\dfrac{10!}{8!} = \dfrac{10 \cdot 9 \cdot 8!}{8!} = 90$

b) $\dfrac{n!}{(n+2)!} = \dfrac{n!}{(n+2)(n+1)n!} = \dfrac{1}{(n+2)(n+1)}$

c) $\dfrac{n!}{(n-1)!} = \dfrac{n(n-1)!}{(n-1)!} = n$

Writing Explicit Nth Terms

$$a_n = ?$$

(Its easiest to start at n=1)

Arithmetic Sequences

ex: Write the nth term explicity.

a) 7, 11, 15, 19, 23 . . .

$m = 4$

$a_n = 4n + 3$

Arithmetic Sequences

ex: Write the nth term explicity.

b) 3, -4, -11, -18 . . .

$a_n = -7n + 10$

c) 17, 13, 9, 5 . . .

$a_n = -4n + 21$

Geometric Sequences

ex: Write the nth term explicity.

d) 2, 8, 32, 128 . . .

common ratio: 4 $\quad \frac{a_2}{a_1} = r$

$$a_n = a_1 \cdot r^{n-1}$$

$$a_n = 2 \cdot 4^{n-1}$$

$$= 2 \cdot 4^n \cdot 4^{-1}$$

$$a_n = \frac{4^n}{2}$$

Geometric Sequences

ex: Write the nth term explicity.

e) -1, 1, -1, 1 . . .

$$a_n = (-1)^n$$

$$= -(-1)^{n-1}$$

f) 1, -1, 1, -1 . . .

$$a_n = -(-1)^n$$

$$= (-1)^{n-1}$$

Quadratic Sequences

ex: Write the nth term explicity.

g) 0, 3, 8, 15, 24 . . .

$$a_n = n^2 - 1$$

Other Sequences

ex: Write the nth term explicity.

h) 1, -4, 9, -16, 25 . . .

$a_n = (-1)^{n-1} \cdot n^2$

i) $1, \frac{1}{2}, \frac{1}{6}, \frac{1}{24}, \frac{1}{120}, \ldots$

$a_n = \frac{1}{n!}$

Other Sequences

ex: Write the nth term explicity.

j) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}\ldots$ $a_n = \frac{n}{n+1}$

k) $1+\frac{1}{2}, 1+\frac{3}{4}, 1+\frac{7}{8}, 1+\frac{15}{16}\ldots$

$a_n = 1 + \frac{2^n - 1}{2^n}$

CHALLENGE!

$$a_n = n!$$

Rewrite a_n using a recursive definition.