ex: Evaluate.

e) $\int e^x \sin x \, dx$

$u = \sin x \qquad dv = e^x$

$du = \cos x \qquad v = e^x$

$e^x \sin x - \int e^x \cos x \, dx$

$u = \cos x \qquad dv = e^x$

$du = -\sin x \qquad v = e^x$

$$\int e^x \sin x \, dx = e^x \sin x - \left[e^x \cos x + \int e^x \sin x \, dx \right]$$

$$\int e^x \sin x \, dx = e^x \sin x - e^x \cos x - \int e^x \sin x \, dx$$

$$\frac{1}{2}\left(e^x \sin x - e^x \cos x\right) + C$$

7.5 Partial Fractions

Partial Fractions is an integration technique necessary when an integrand contains a <u>rational function</u> whose denominator is factorable, u-sub fails AND inverse trig fails.

ex: Whi didat ons?

- $\int \frac{1}{x^2-5x+6}dx$
- $\int \frac{2x+4}{x^2+4x+3}dx$
- $\int \frac{7}{x^2+9}dx$
- $\int \frac{(2x+\cancel{3}4)+1}{x^2+4x+3}dx$

(1/2)

Review:

$$\frac{3^{(x+1)}}{x-6} + \frac{7^{(x-6)}}{x+1}$$

$$\frac{3x+3 \quad + \quad 7x-42}{(x-6)(x+1)}$$

$$\frac{10x-39}{(x-6)(x+1)}$$

ex: Decompose into partial fractions.

$$\frac{10x-39}{x^2-5x-6}$$

$$\frac{10x-39}{(x-6)(x+1)} = \frac{\overset{3}{A}}{x-6} + \frac{\overset{7}{B}}{x+1} \quad (x-6)(x+1)$$

$$10x-39 = A(x+1) + B(x-6)$$

$x=6$	$x=-1$
$21=7A$	$-49=-7B$
$3=A$	$7=B$

$$\int \frac{10x-39}{x^2-5x-6}\,dx = \int\left(\frac{3}{x-6}+\frac{7}{x+1}\right)dx$$

$$3\ln|x-6|+7\ln|x+1|+C$$

Distinct Linear Factors

ex: Integrate.

a) $\int \frac{1}{x^2-5x+6}dx$ $= \int\left(\frac{1}{x-3} - \frac{1}{x-2}\right)dx$

$\ln|x-3| - \ln|x-2| + C$

$\ln\left|\frac{x-3}{x-2}\right| + C$

Distinct Linear Factors

ex: Integrate.

b) $\displaystyle\int \frac{x^3+1}{x^2-4}\,dx$

$$\begin{array}{r} x \\ x^2-4\,\overline{\big)\,x^3+1} \\ -x^3\quad+4x \\ \hline 4x+1 \end{array}$$

$$\int\left(x+\frac{4x+1}{x^2-4}\right)dx$$

$$\int\left(x+\frac{9}{4}\left(\frac{1}{x-2}\right)+\frac{7}{4}\left(\frac{1}{x+2}\right)\right)dx$$

$$\frac{x^2}{2}+\frac{9}{4}\ln|x-2|+\frac{7}{4}|x+2|+C$$

Repeated Linear Factors

ex: Integrate.

a) $\int \frac{5x^2+20x+6}{x(x+1)^2}dx = \int \left(\frac{6}{x} + \frac{-1}{x+1} + \frac{9}{(x+1)^2}\right)dx$

$$\left(\frac{5x^2+20x+6}{x(x+1)^2} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}\right) x(x+1)^2$$

$$5x^2+20x+6 = A(x+1)^2 + Bx(x+1) + Cx$$

$x=-1$	$x=0$	$x=1$
$-9=-C$ $9=C$	$6=A$	$B=-1$

$6\ln|x| - \ln|x+1| - \frac{9}{x+1} + C$

Distinct Quadratic Factors

ex: Integrate.

a) $\int \frac{18}{(x+3)(x^2+9)}dx = \int \left(\frac{1}{x+3} + \frac{-x+3}{x^2+9}\right)dx$

$$\frac{18}{(x+3)(x^2+9)} = \frac{A}{x+3} + \frac{Bx+C}{x^2+9} \qquad A=1,\ C=3,\ B=-1$$

$$\ln|x+3| - \frac{1}{2}\ln|x^2+9| + \arctan\frac{x}{3} + C$$

$$\int\left(\frac{1}{x+3}+\frac{-x+3}{x^2+9}\right)dx$$

$$\int\left(\frac{1}{x+3}-\frac{x}{x^2+9}+\frac{3}{x^2+9}\right)dx$$

$$\ln|x+3|-\frac{1}{2}\ln|x^2+9|+\arctan\frac{x}{3}+C$$

Repeated Quadratic Factors

ex: Integrate.

a) $\int \frac{8x^3+13x}{(x^2+2)^2}dx = \int \left(\frac{8x}{x^2+2} + \frac{-3x}{(x^2+2)^2} \right) dx$

$$\frac{8x^3+13x}{(x^2+2)^2} = \frac{Ax+B}{x^2+2} + \frac{Cx+D}{(x^2+2)^2}$$

$$\boxed{\begin{aligned} A &= 8 \\ B &= 0 \\ C &= -3 \\ D &= 0 \end{aligned}}$$

$$8x^3+13x = (Ax+B)(x^2+2) + Cx + D$$

$$4\ln|x^2+2| + \frac{3}{2(x^2+2)} + C$$

ex:

$$\int \frac{12}{(x-1)(x-5)}\,dx =$$

(A) $-3\ln|x-1| + 3\ln|x-5| + C$

(B) $-2\ln|x-1| + 2\ln|x-5| + C$

(C) $3\ln|x-1| - 3\ln|x-5| + C$

(D) $12\ln|x-1| + 12\ln|x-5| + C$

$$\int \frac{12}{(x-1)(x-5)}\,dx$$

ex:

$$\int \frac{dx}{(x-1)(x+2)} =$$

(A) $\frac{1}{3}\ln\left|\frac{x-1}{x+2}\right| + C$

(B) $\frac{1}{3}\ln\left|\frac{x+2}{x-1}\right| + C$

(C) $\frac{1}{3}\ln|(x-1)(x+2)| + C$

(D) $(\ln|x-1|)(\ln|x+2|) + C$

(E) $\ln\left|(x-1)(x+2)^2\right| + C$

$$\int \frac{dx}{(x-1)(x+2)}$$

$$\frac{12}{(x-1)(x-5)} = \frac{\overset{-3}{A}}{x-1} + \frac{\overset{3}{B}}{x-5}$$

$$12 = A(x-5) + B(x-1)$$

$$12 = Ax - 5A + Bx - B$$

$$\underline{0x} + 12 = \underline{(A+B)x} + (-5A - B)$$

$0 = A + B$ $\qquad$ $12 = -5A - B$

$-A = B$ $\qquad$ $12 = -5A + A$

$12 = -4A \qquad A = -3$

$$\int \frac{1}{\sin x - 1} \frac{(\sin x + 1)}{(\sin x + 1)} dx$$

$$\int \frac{\sin x + 1}{\sin^2 x - 1} dx$$

$$\int \tan x \sec x \, dx$$

$$-\int \frac{\sin x + 1}{\cos^2 x} dx = -\left[\int \frac{\sin x}{\cos^2 x} dx + \int \sec^2 x \, dx\right]$$

$$-\sec x - \tan x + C$$

or

$$-\frac{1}{\cos x} - \tan x + C$$

$\int \tan^2 x\,dx$ ✓

$\int \sec^2 x\,dx$ ✓

$\int \cot^2 x\,dx$ ✓

$\int \csc^2 x\,dx$ ✓

$\int \sin^2(3x)\,dx = \int \frac{1}{2}(1-\cos 6x)\,dx$

$\int \sin^2 x\,dx$

$\sin^2 x = \int \frac{1}{2}(1-\cos 2x)\,dx$

$\cos 2x = 1 - 2\sin^2 x$
$= 2\cos^2 x - 1$

$\frac{1}{2}(1-\cos 2x) = \sin^2 x$

$\frac{1}{2}(1+\cos 2x) = \cos^2 x$