$\int x^2 \cos x \, dx$

$u = x^2 \qquad dv = \cos x$

$du = 2x \qquad v = \sin x$

$x^2 \sin x - \int 2x \sin x \, dx$

$h(x)$

B

10.) $\displaystyle\int_1^{e/2} \frac{\ln 2x}{x}\,dx = \int_1^{e/2} \ln 2x \cdot \frac{1}{x}\,dx = \left.\frac{(\ln 2x)^2}{2}\right]_1^{e/2}$

$u = \ln 2x$

$du = \frac{1}{x}\,dx$

$\displaystyle\frac{1}{2} - \frac{(\ln 2)^2}{2}$

12.) $\int (\ln x)^2 dx$

$u = (\ln x)^2 \quad dv = 1$

$du = \frac{2\ln x}{x} \quad v = x$

$x(\ln x)^2 - \int 2\ln x \, dx$

$\therefore x \cdot (\ln x)^2 - 2 \cdot (x\ln x - x) + C$

26. $\dfrac{3}{(x-1)(x+2)} = \dfrac{A^{(1)}}{x-1} + \dfrac{B^{(-1)}}{x+2}$

$$\frac{2x}{(x+2)(x+1)} = \frac{A}{x+2} + \frac{B}{x+1}$$

$$2x = A(x+1) + B(x+2)$$

$x=-1$	$x=-2$
$-2 = B$	$-4 = -A$
	$4 = A$

$$\int \left(\frac{4}{x+2} + \frac{-2}{x+1} \right) dx$$

$$4\ln|x+2| - 2\ln|x+1| + C$$

D

$$\int a^x dx = \frac{1}{\ln a} \cdot a^x + C$$

$$\int 4^x dx = \frac{1}{\ln 4} \cdot 4^x + c$$

24.) $\dfrac{6}{x^2+8x+25}$

$$\int \frac{6}{(x+4)^2+9}\,dx$$

$u = x+4$

$du = 1$

$a = 3$

$x^2+8x+16 \quad +25-16$

$(x+4)^2+9$

$$\frac{6}{3}\arctan\frac{x+4}{3}+C$$

14.) $\int \tan x \ln(\cos x)\,dx$

$u = \ln \cos x$

$du = \frac{-\sin x}{\cos x}$

$-\int u\,du$

18.) $\int \arctan 5x \, dx$

$u = \arctan 5x \quad dv = 1$

$du = \frac{5}{1+25x^2} \quad v = x$

$x\arctan 5x - 5\int \frac{x}{1+25x^2} dx$

$x\arctan 5x - \frac{5}{50} \ln|1+25x^2| + C$

22) $\displaystyle\int \frac{5}{x^2(x+1)}\,dx = \int\left(\frac{-5}{x} + \frac{5}{x^2} + \frac{5}{x+1}\right)dx$

$$\frac{5}{x^2(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1} \quad (\text{or}) \quad \frac{Ax+B}{x^2} + \frac{C}{x+1}$$

$$5 = Ax(x+1) + B(x+1) + Cx^2$$

$x=-1$	$x=0$	$x=1$
$5=C$	$5=B$	$5=A(2)+5(2)+5(1)$
		$5=2A+15$
		$-5=A$

25.) $\int \frac{x-5}{x+1}\,dx = \int \left(1 - \frac{6}{x+1}\right) dx$

$$\begin{array}{r|rr} -1 & 1 & -5 \\ & & -1 \\ \hline & 1 & -6 \end{array}$$

7.3 Trigonometric Integrals

$$\int \sin^a x \cos^b dx \qquad \int \tan^a x \sec^b dx$$

where a or b must be a positive integer.

$$\int \sin^a x \cos^b dx$$

4 Cases:

1. The power of sine is odd and positive.
2. The power of cosine is odd and positive.
3. Both powers are odd and positive.
4. Both powers are even and positive.

Guidelines for Evaluating Integrals Involving Powers of Sine and Cosine

1. When the power of the sine is odd and positive, save one sine factor and convert the remaining factors to cosines. Then, expand and integrate.

$$\int \overbrace{\sin^{2k+1}}^{\text{Odd}} x \cos^n x \, dx = \int \overbrace{(\sin^2 x)^k}^{\text{Convert to cosines}} \cos^n x \overbrace{\sin x \, dx}^{\text{Save for } du} = \int (1 - \cos^2 x)^k \cos^n x \sin x \, dx$$

2. When the power of the cosine is odd and positive, save one cosine factor and convert the remaining factors to sines. Then, expand and integrate.

$$\int \sin^m x \overbrace{\cos^{2k+1}}^{\text{Odd}} x \, dx = \int \sin^m x \overbrace{(\cos^2 x)^k}^{\text{Convert to sines}} \overbrace{\cos x \, dx}^{\text{Save for } du} = \int \sin^m x \, (1 - \sin^2 x)^k \cos x \, dx$$

3. When the powers of both the sine and cosine are even and nonnegative, make repeated use of the identities

$$\sin^2 x = \frac{1 - \cos 2x}{2} \quad \text{and} \quad \cos^2 x = \frac{1 + \cos 2x}{2}$$

to convert the integrand to odd powers of the cosine. Then proceeded as in the second guideline.

Case 1: The power of sine is odd and positive.

ex: Integrate.

$$\sin^3 x = \sin^2 x \cdot \sin x$$
$$= (1-\cos^2 x)\sin x$$

a) $\int \underbrace{\sin^3 x} \cos^4 x \, dx$

$$\int (1-\cos^2 x) \sin x \cos^4 x \, dx$$

$$\int (\cos^4 x - \cos^6 x) \sin x \, dx = \int \cos^4 x \sin x \, dx - \int \cos^6 x \sin x \, dx$$

$$= -\frac{\cos^5 x}{5} + \frac{\cos^7 x}{7} + C$$

Case 2: The power of cosine is odd and positive.

ex: Integrate.

b) $\int \sin^4(2x)\cos^5(2x)\,dx$

$$\cos^5 2x = \cos^4 2x \cos 2x$$
$$= (\cos^2 2x)^2 \cos 2x$$
$$= (1-\sin^2 2x)^2 \cos 2x$$

$$\int \sin^4 2x \,(1-\sin^2 2x)^2 \cos 2x\, dx$$

$$\int \sin^4 2x\,(1-2\sin^2 2x+\sin^4 2x)\cos 2x\, dx$$

$$\frac{\sin^5 2x}{10} - \frac{\sin^7 2x}{7} + \frac{\sin^9 2x}{18} + C$$

Case 3: Both powers are odd and positive.

ex: Integrate.

$\cos^3 x = \cos^2 x \cdot \cos x$

c) $\int \sin^{15} x \cos^3 x \, dx$

$\int \sin^{15} x (1-\sin^2 x) \cos x \, dx = \dfrac{\sin^{16} x}{16} - \dfrac{\sin^{18} x}{18} + C$

ex: Integrate.

d) $\int \cos^{27} x \sin x \, dx$

$u = \cos x$
$du = -\sin x \, dx$

$-\dfrac{\cos^{28} x}{28} + C$

Case 4: Both powers are even and positive.

*Need - Power Reduction Identities:

$$\boxed{\sin^2 x = \frac{1-\cos 2x}{2}} \qquad \boxed{\cos^2 x = \frac{1+\cos 2x}{2}}$$

ex: Rewrite in the first power of cosine.

$$\sin^2 3x = \frac{1-\cos 6x}{2} = \frac{1}{2}(1-\cos 6x)$$

ex: Integrate.

e) $\int \cos^2 x\, dx$ $= \frac{1}{2}\int (1+\cos 2x)\, dx$

$\frac{1}{2}\left(x + \frac{1}{2}\sin 2x\right) + C$

ex: Integrate.

f) $\int \cos^2 x \sin^2 x \, dx$

$$\int \frac{1}{2}(1+\cos 2x) \cdot \frac{1}{2}(1-\cos 2x)\, dx$$

$$\frac{1}{4}\int (1-\cos^2 2x)\, dx = \frac{1}{4}\int \sin^2 2x \, dx = \frac{1}{4}\int \frac{(1-\cos 4x)}{2}\, dx$$

$$= \frac{1}{8}\int (1-\cos 4x)\, dx = \frac{1}{8}\left(x - \frac{1}{4}\sin 4x\right) + C$$