19.) $\displaystyle\int \frac{x^2}{x-1}\,dx$

$$\begin{array}{c|ccc} 1 & 1 & 0 & 0 \\ & & 1 & 1 \\ \hline & 1 & 1 & 1 \end{array}$$

$$\int \left(x + 1 + \frac{1}{x-1} \right) dx$$

37.) $\int \tan\left(\frac{2}{t}\right) \cdot \frac{1}{t^2}\, dt$

$u = 2t^{-1}$

$du = \frac{-2}{t^2}$

$-\frac{1}{2}\int \tan u\, du$

$-\frac{1}{2}\left(-\ln\left|\cos\frac{2}{t}\right| + C\right.$

39.) $\displaystyle\int \frac{6}{\sqrt{10x-x^2}}\,dx$

$-(x^2-10x+25)+25$

$\displaystyle\int \frac{6}{\sqrt{25-(x-5)^2}}\,dx$

31. $$\int \frac{\ln x^2}{x}\,dx$$

$$2\int \frac{\ln x}{x}\,dx$$

$$\frac{\cancel{2}(\ln x)^2}{\cancel{2}} + C$$

7.2 Integration By Part (IBP)

Integration By Parts is an integration technique necessary when an integrand contains a product of algebraic and transcendental functions.

ex: Which integrals are candidates for integration by parts?

- $\int 3x\sin\left(x^2+1\right)dx$ No
- $\int x\ln x\,dx$ Yes
- $\int e^x\cos x\,dx$ Yes

In general: $\int u(x)v'(x)\,dx =$ $uv - \int v\,du$

The Product Rule Revisted:

$$\frac{d}{dx}\left[u(x)v(x)\right] = \qquad u\,dv + v\,du$$

Integrate both sides

$$uv = \underbrace{\int u\,dv} + \int v\,du$$

Rearrange

$$\int u\,dv = uv - \int v\,du$$

Guidelines For Picking u and dv

GUIDELINES FOR INTEGRATION BY PARTS

1. Try letting dv be the most complicated portion of the integrand that fits a basic integration rule. Then u will be the remaining factor(s) of the integrand.
2. Try letting u be the portion of the integrand whose derivative is a function simpler than u. Then dv will be the remaining factor(s) of the integrand.

Note that dv always includes the dx of the original integrand.

Easier - Remember the acronym DETAIL.

D - pick dv 1st

E - exponential

T - trigonometric

A - algebraic

I - inverse trigonometric

L - logarithmic

ex: Identify u and dv.

a) $\int x \ln x \, dx$ $u = \ln x$ $dv = x$

b) $\int e^x \cos x \, dx$ $u = \cos x$ $dv = e^x$

c) $\int 2^x x^2 \, dx$ $u = x^2$ $dv = 2^x$

ex: Identify u and dv.

d) $\int \ln x\, dx$ $u = \ln x$ $dv = 1$

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e) $\int \frac{xe^{2x}}{(2x+1)^2}\, dx$ $u = xe^{2x}$ $dv = \frac{1}{(2x+1)^2}$

ex: Evaluate.

a) $\int xe^x \, dx$

$u = x \qquad dv = e^x$

$du = 1dx \qquad v = e^x$

$uv - \int v\,du$

$xe^x - \int e^x dx$

$xe^x - e^x + C$

Check:

$xe^x + \cancel{e^x} - \cancel{e^x}$

ex: Evaluate.

b) $\int \arcsin x \, dx$

$u = \arcsin x \qquad dv = 1$

$du = \frac{1}{\sqrt{1-x^2}} \qquad v = x$

$x \arcsin x - \int \frac{x}{\sqrt{1-x^2}} dx$

$x \arcsin x + \sqrt{1-x^2} + C$

ex: Evaluate.

c) $\int \ln x \, dx$

$u = \ln x$ $\quad$ $dv = 1$

$du = \frac{1}{x}$ $\quad$ $v = x$

$x \ln x - \int 1 \, dx$

$x \ln x - x + C$

ex: Evaluate.

d) $\int 3x^2 \cos(2x)\,dx$

$u = 3x^2$ $\quad dv = \cos 2x$

$du = 6x$ $\quad v = \frac{1}{2}\sin 2x$

$$\frac{3}{2}x^2 \sin 2x - \underbrace{\int 3x \sin 2x\,dx}$$

$u = 3x$ $\quad dv = \sin 2x$

$du = 3$ $\quad v = -\frac{1}{2}\cos 2x$

$$\frac{3}{2}x^2 \sin 2x - \left(-\frac{3}{2}x\cos 2x + \int \frac{3}{2}\cos 2x\,dx\right)$$

$$\frac{3}{2}x^2 \sin 2x + \frac{3}{2}x\cos 2x - \frac{3}{4}\sin 2x + C$$

ex: Evaluate.

e) $\int e^x \sin x \, dx$

$u = \sin x \qquad dv = e^x$

$du = \cos x \qquad v = e^x$

$$\int e^x \sin x \, dx = e^x \sin x - \underbrace{\int e^x \cos x \, dx}$$

$u = \cos x \qquad dv = e^x$

$du = -\sin x \qquad v = e^x$

$$\int e^x \sin x \, dx = e^x \sin x - \left(e^x \cos x + \int e^x \sin x \, dx\right)$$

$$\int e^x \sin x \, dx = e^x \sin x - e^x \cos x - \int e^x \sin dx$$

$$\frac{\cancel{2}\int e^x \sin x \, dx}{\cancel{2}} = \frac{e^x \sin x - e^x \cos x}{2} + C$$

Integration By Parts - Tabular Method

*Tabular Method is convenient in problems involving repeated applications of IBP.

Tabular method can ONLY be used when...

1. The integrand has a polynomial present.
2. The non-polynomial factor of the integrand is easy to integrate.

ex: Which integrals are candidates for tabular method?

- $\int x^3 e^x \, dx$ ✓
- $\int x \arcsin x \, dx$ X
- $\int 5x^2 \sin 4x \, dx$ ✓

ex: Evaluate.

a) $\int xe^x\,dx$

$u = x \qquad dv = e^x$

$$\begin{array}{ccc} x & & e^x \\ 1 & + & e^x \\ 0 & - & e^x \end{array}$$

$xe^x - e^x + C$

ex: Evaluate.

b) $\int 3x^2 \cos(2x)\,dx$

$u = 3x^2 \quad + \quad dv = \cos 2x$

$6x \quad - \quad \frac{1}{2}\sin 2x$

$6 \quad + \quad -\frac{1}{4}\cos 2x$

$0 \quad -\frac{1}{8}\sin 2x$

$\frac{3}{2}x^2\sin 2x + \frac{3}{2}x\cos 2x + -\frac{3}{4}\sin 2x + C$

ex: Evaluate.

c) $\int x\sqrt{x-5}\,dx$

$u = x \qquad dv = \sqrt{x-5}$

$du = 1 \qquad v = \frac{2(x-5)^{3/2}}{3}$

$$\frac{2x(x-5)^{3/2}}{3} - \int \frac{2}{3}(x-5)^{3/2}\,dx$$

$$\frac{2x(x-5)^{3/2}}{3} - \frac{4}{15}(x-5)^{5/2} + C$$

ex:

If $\int f(x)\sin x\,dx = -f(x)\cos x + \int 3x^2 \cos x\,dx$, then $f(x)$ could be

(A) $3x^2$ (B) x^3 (C) $-x^3$ (D) $\sin x$ (E) $\cos x$

$u = x^3 \qquad dv = \sin x$

$du = 3x^2 \qquad v = -\cos x$

$-x^3 \cos x + \int 3x^2 \cos x\, dx$