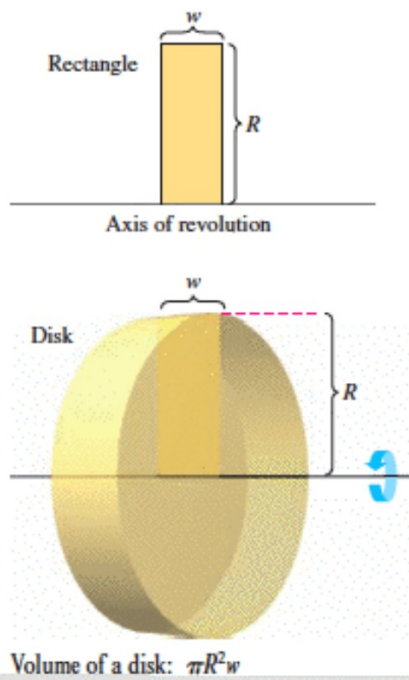


## 7.2 Volume: The Disk Method

- Find the volume of a solid of revolution using the disk method.
- Find the volume of a solid of revolution using the washer method.



$$V = \pi r^2 h$$

$$V = \pi \int_a^b r^2 dx$$

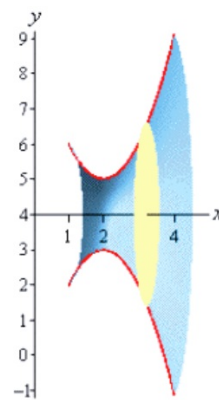
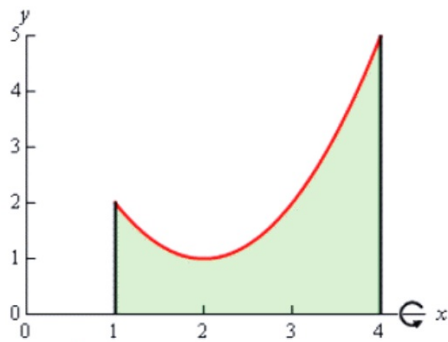
$$r^2 = (\text{top} - \text{bottom})^2$$
$$= (f(x) - \text{axis})^2$$

$$(3-7)^2 = (7-3)^2$$

$$(\text{Axis} - f(x))^2 = (f(x) - \text{axis})^2$$

## Volume by Revolution (Disk Method)

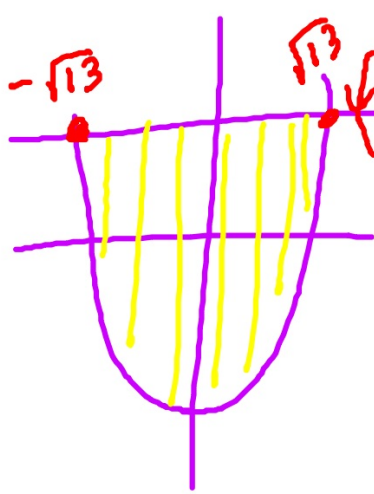
**Example 1** Determine the volume of the solid obtained by rotating the region bounded by  $y = x^2 - 4x + 5$ ,  $x = 1$ ,  $x = 4$ , and the  $x$ -axis about the  $x$ -axis.



$y = x^2 - 4x + 5$   
 $x = 1, x = 4$ ,  
rotate  
about the  
 $x$ -axis

$$\pi \int_1^4 (x^2 - 4x + 5)^2 dx \approx 49.009$$

#2 Find the volume of the solid that is formed by  $y = x^2 - 9$ ,  $y = 4$ ; rotated about  $y = 4$ .

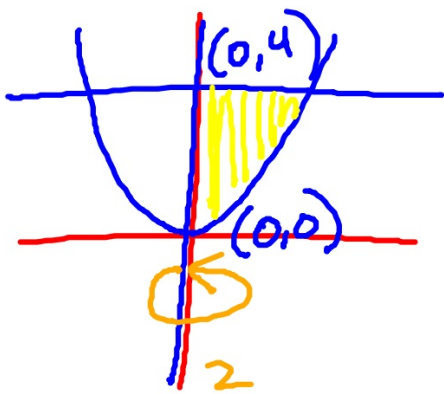


$$2\pi \int_0^{\sqrt{13}} (x^2 - 9 - 4)^2 dx$$

$$2\pi \int_0^{\sqrt{13}} (x^2 - 13)^2 dx$$

$$2041.912$$

#3 Region:  $y = x^2$ ,  $x = 0$ ,  $y = 4$  (1st quad)  
rotated about y-axis

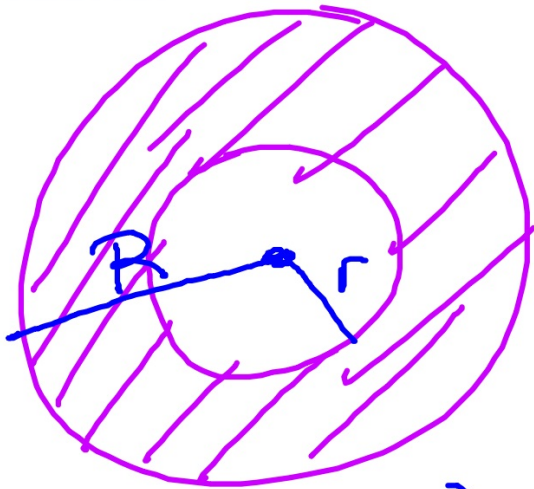


$$y = x$$
$$\pm\sqrt{y} = x$$
$$\sqrt{y} = x$$

$$\pi \int_0^4 (\sqrt{y} - 0)^2 dy$$

$$\pi \int_0^4 y dy = \frac{\pi y^2}{2} \Big|_0^4$$
$$= 8\pi$$

Washer



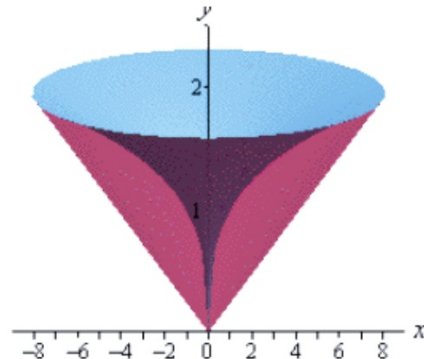
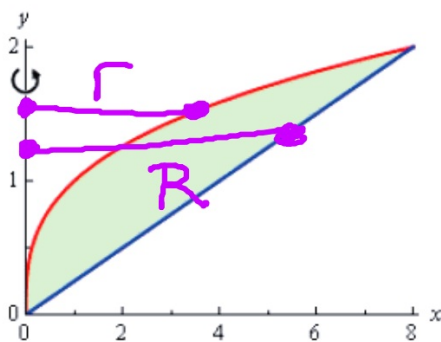
$$\pi R^2 - \pi r^2$$

## Volume by Revolution (Washer Method)

$$V = \pi \int_a^b [R(x)]^2 - [r(x)]^2 dx$$

$$V = \pi \int_a^b [(outer - axis)^2 - (inner - axis)^2] dx$$

**Example 2** Determine the volume of the solid obtained by rotating the portion of the region bounded by  $y = \sqrt[3]{x}$  and  $y = \frac{x}{4}$  that lies in the first quadrant about the  $y$ -axis.



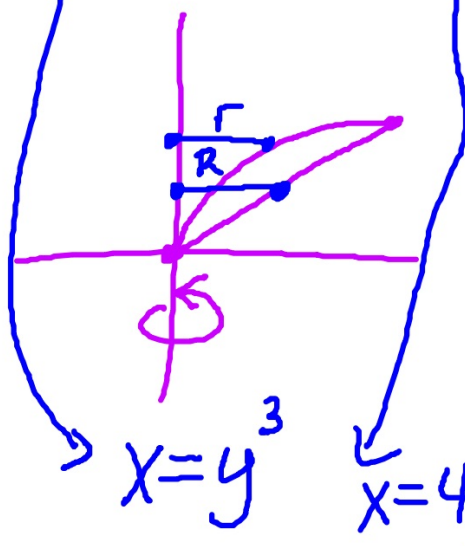
#4

$$y = \sqrt[3]{x}$$

$$y = \frac{x}{4}$$

rotate : y-axis

1st Quad.



Washer (dy)

$$\pi \int_0^2 \left( (4y-0)^2 - (y^3-0)^2 \right) dy$$

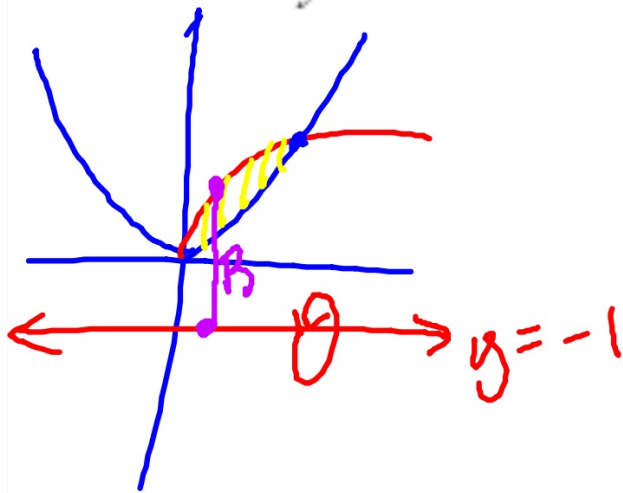
$$76.595$$

$$y^3 = 4y \Rightarrow y^3 - 4y = 0$$

$$y(y^2 - 4) = 0$$



#5 Find the volume of the solid obtained by rotating the region bounded by the curve  $y = x^2$  and  $y = \sqrt{x}$  about the line  $y = -1$ .



Washer

$$\pi \int_0^1 \left( (\sqrt{x} - (-1))^2 - (x^2 - (-1))^2 \right) dx$$

3.037

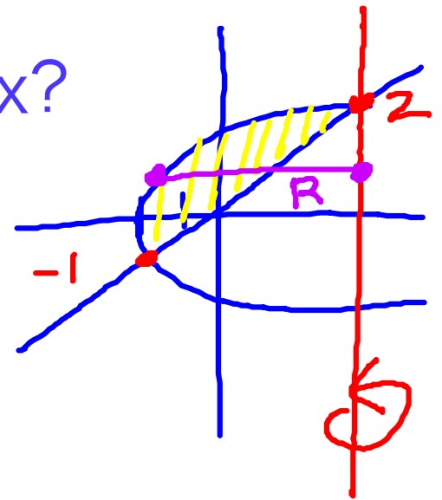
#6

Find the volume of the solid obtained by rotating the region bounded by  $x = y^2 - 2$  and  $y = x$  about the line  $x=4$

Disk or washer? dy or dx?

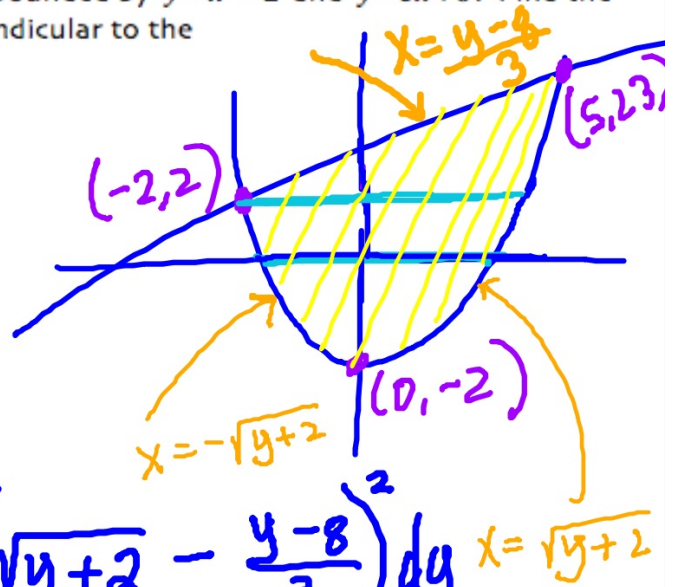
$$\pi \int_{-1}^2 \left( (y^2 - 2 - 4)^2 - (y - 4)^2 \right) dy$$

124.407



3. (Calculator) The base of a solid is the region bounded by  $y = x^2 - 2$  and  $y = 3x + 8$ . Find the volume of each solid whose cross sections perpendicular to the

- a.  $x$ -axis are semicircles
- b.  $y$ -axis are squares**
- c.  $y$ -axis are equilateral triangles
- d.  $x$ -axis are rectangles with height of 4



$$\int_a^b s^2 dy$$

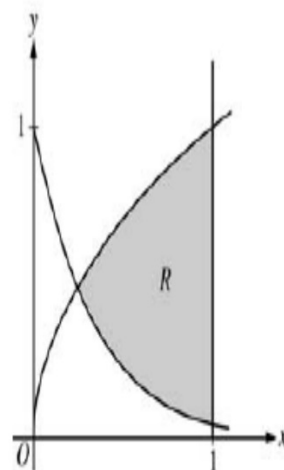
$$\int_{-2}^2 (\sqrt{y+2} - (-\sqrt{y+2}))^2 dy + \int_2^{23} (\sqrt{y+2} - \frac{y-8}{3})^2 dy$$

$$\int_{-2}^2 (2\sqrt{y+2})^2 dy + \int_2^{23} (\sqrt{y+2} - \frac{y-8}{3})^2 dy$$

## AP Practice Question

Let  $R$  be the shaded region bounded by the graphs of  $y = \sqrt{x}$  and  $y = e^{-3x}$  and the vertical line  $x = 1$ , as shown in the figure above.

- Find the area of  $R$ .
- Find the volume of the solid generated when  $R$  is revolved about the horizontal line  $y = 1$ .
- The region  $R$  is the base of a solid. For this solid, each cross section perpendicular to the  $x$ -axis is a rectangle whose height is 5 times the length of its base in region  $R$ . Find the volume of this solid.



## Answers to AP Question

Point of intersection

$$e^{-3x} = \sqrt{x} \text{ at } (T, S) = (0.238734, 0.488604)$$

$$\begin{aligned} \text{(a) Area} &= \int_T^1 (\sqrt{x} - e^{-3x}) dx \\ &= 0.442 \text{ or } 0.443 \end{aligned}$$

$$\begin{aligned} \text{(b) Volume} &= \pi \int_T^1 ((1 - e^{-3x})^2 - (1 - \sqrt{x})^2) dx \\ &= 0.453\pi \text{ or } 1.423 \text{ or } 1.424 \end{aligned}$$

$$\begin{aligned} \text{(c) Length} &= \sqrt{x} - e^{-3x} \\ \text{Height} &= 5(\sqrt{x} - e^{-3x}) \end{aligned}$$

$$\text{Volume} = \int_T^1 5(\sqrt{x} - e^{-3x})^2 dx = 1.554$$

1: Correct limits in an integral in  
(a), (b), or (c)

2:  $\left\{ \begin{array}{l} 1 : \text{integrand} \\ 1 : \text{answer} \end{array} \right.$

3:  $\left\{ \begin{array}{l} 2 : \text{integrand} \\ < -1 > \text{ reversal} \\ < -1 > \text{ error with constant} \\ < -1 > \text{ omits 1 in one radius} \\ < -2 > \text{ other errors} \\ 1 : \text{answer} \end{array} \right.$

3:  $\left\{ \begin{array}{l} 2 : \text{integrand} \\ < -1 > \text{ incorrect but has} \\ \quad \sqrt{x} - e^{-3x} \\ \quad \text{as a factor} \\ 1 : \text{answer} \end{array} \right.$