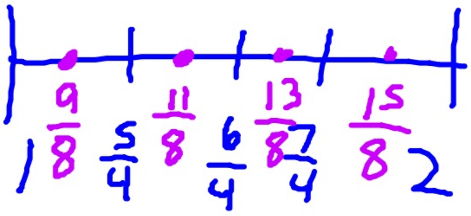
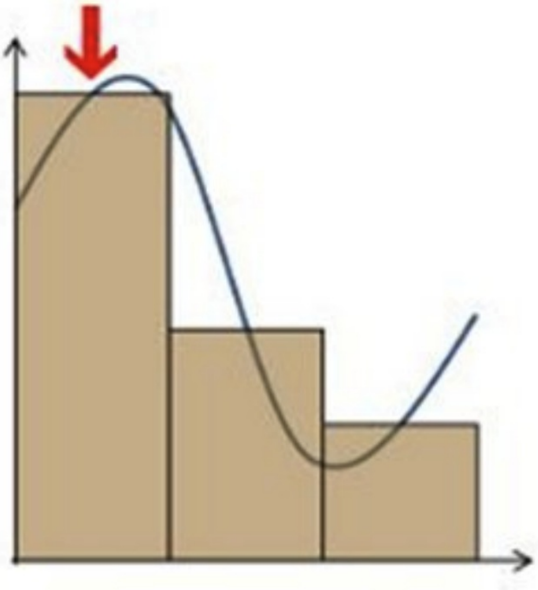
3c.) $\int_1^2 \frac{1}{x^2}\,dx \approx \frac{1}{4}\left(\frac{1}{\left(\frac{9}{8}\right)^2} + \dots\dots\right)$



$\frac{1}{4}\left(\frac{64}{81} + \frac{64}{121} + \dots\right)$

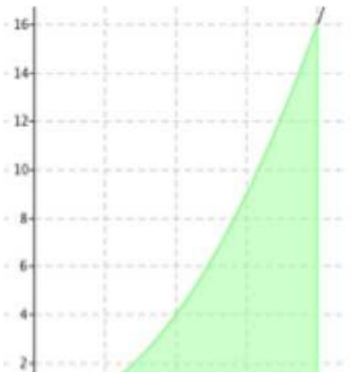


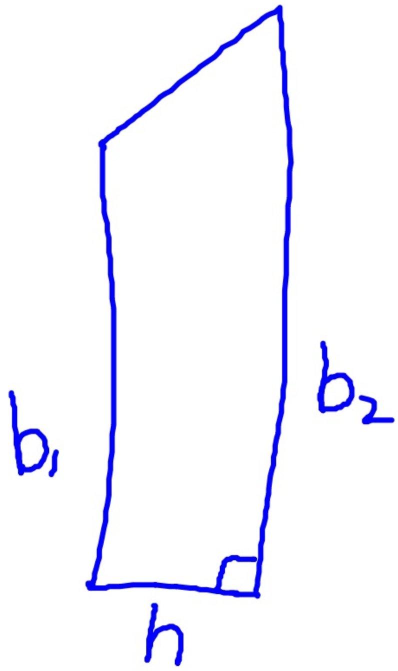
The middle of the base of the rectangle

At the arrow, the function is concave down. Since the rectangle is including more area, this will be an over-approximation.

Riemann Approximations - cont.

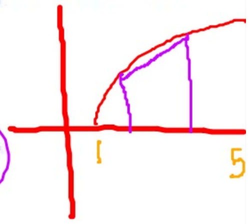
ex: Approximate the integral $\int_0^4 x^2 dx$ using a trapezoidal approximation with two equal subdivisions. Then determine if the approximation is an over or under estimate.





$$A = \frac{1}{2} h \left(b_1 + b_2\right)$$

ex: Approximate the integral $\int_1^5 \ln x\,dx$ $= 4.047$ using a trapezoidal approximation with four equal subdivisions. Then determine if the approximation is an over or under estimate.



$$\int_1^5 \ln x\,dx \approx \frac{1}{2}(1)\left[0+\ln 2+\ln 2+\ln 3+\ln 3+\ln 4+\ln 4+\ln 5\right]$$

$$\frac{1}{2}\left[2\ln 2+2\ln 3+2\ln 4+\ln 5\right]$$

3.983

ex: Approximate the integral $\int_{2}^{14} \frac{1}{x}\,dx$ using a trapezoidal approximation with three equal subdivisions. Then determine if the approximation is an over or under estimate.

$$\approx \frac{1}{2}(4)\left[\frac{1}{2} + 2\left(\frac{1}{6}\right) + 2\left(\frac{1}{10}\right) + \frac{1}{14}\right]$$

$$2.210$$

THEOREM 4.9 The Trapezoidal Rule

Let f be continuous on $[a, b]$. The Trapezoidal Rule for approximating $\int_a^b f(x)\,dx$ is

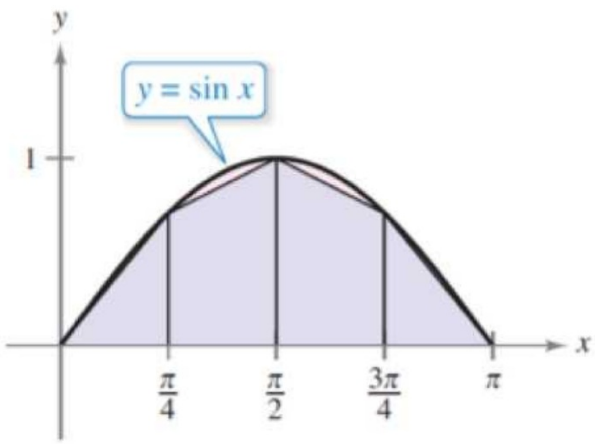
$$\int_a^b f(x)\,dx \approx \frac{b-a}{2n}[f(x_0) + 2f(x_1) + 2f(x_2) + \cdot \cdot \cdot + 2f(x_{n-1}) + f(x_n)].$$

Moreover, as $n \to \infty$, the right-hand side approaches $\int_a^b f(x)\,dx$.

Remark

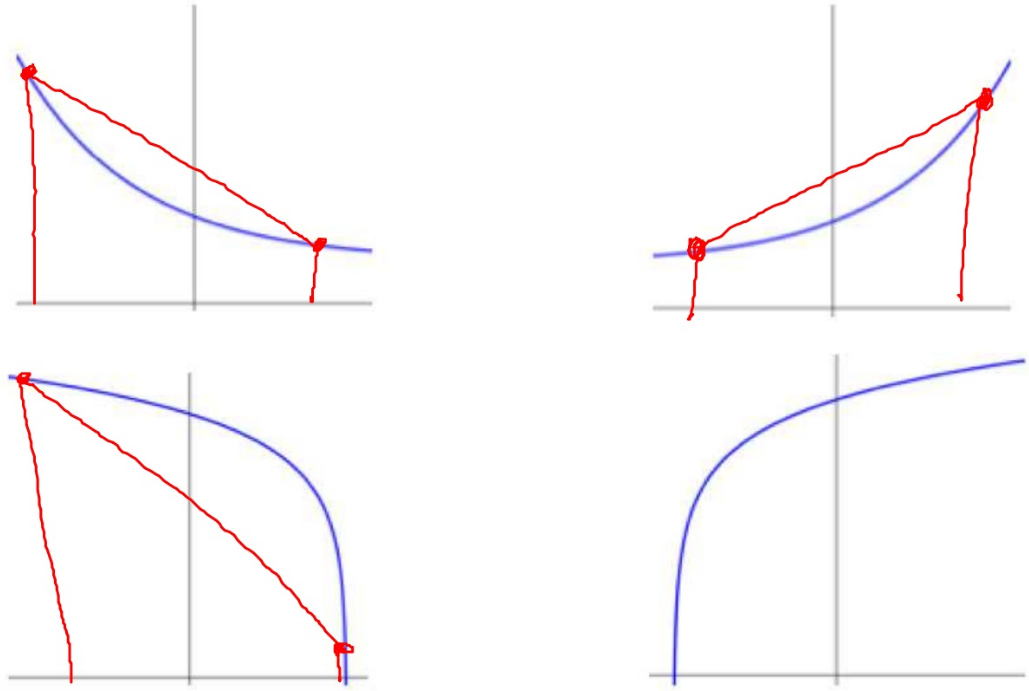
Observe that the coefficients in the Trapezoidal Rule have the following pattern.

1 2 2 2 . . . 2 2 1



- Over and Under Estimates

ex: Determine if Trapezoid Approximation would yield an over or under approximations.



CCU: over

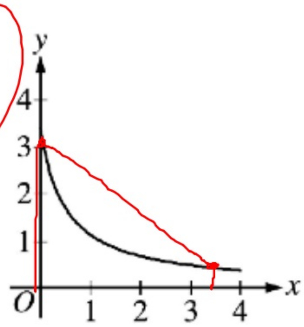
CCD: under

When estimating an integral value using a Trapezoidal Approximation, the approximation will be an over or underestimate depending on whether the curve is

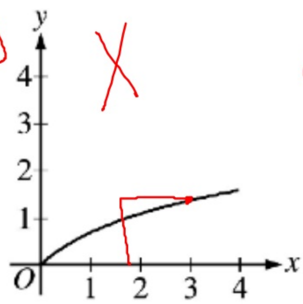
concave up or concave down.

If a trapezoidal sum over approximates $\int_0^4 f(x)dx$, and a right Riemann sum under approximates $\int_0^4 f(x)dx$, which of the following could be the graph of $y = f(x)$?

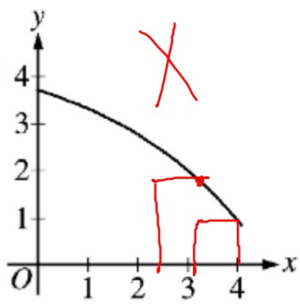
A.)



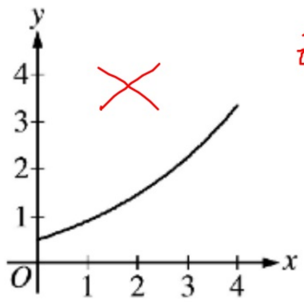
B.)



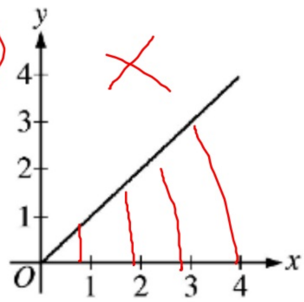
C.)



D)



E)



- Riemann Approximations and Tabular Data

ex: Estimate the value of the integral using the indicated method and n subdivisions indicated by the data.

x	2	4	6	8	10
$f(x)$	17	1	-2	8	7

a) $\int_2^{10} f(x)dx$ Left Riemann, n=4

$$\approx 2\left(\underline{f(2)} + \underline{f(4)} + \underline{f(6)} + \underline{f(8)}\right)$$

$$2(17 + 1 + -2 + 8) = 48$$

x	2	4	6	8	10
$f(x)$	17	1	-2	8	7

b) $\int_{2}^{10} f(x)dx$ Right Riemann, n=4

$2[f(10)+f(8)+f(6)+f(4)]$

28

x	2	4	6	8	10
$f(x)$	17	1	-2	8	7

c) $\int_2^{10} f(x)dx$ Midpoint Riemann, n=2

$4 \cdot f(4) + 4 \cdot f(8)$

$4 \cdot 1 + 4 \cdot 8$

36

x	2	4	6	8	10
$f(x)$	17	1	-2	8	7

d) $\int_{2}^{10} f(x)dx$ Trapezoid Approximation, n=4

$$\frac{1}{2}\cdot 2\left[17+1+1+-2+-2+8+8+7\right]$$

38

ex: Estimate the value of the integral using the indicated method and n subdivisions indicated by the data.

x	1	3	9	12	21
$f(x)$	2	-10	11	5	6

a) $\int_1^{21} f(x)dx$ Right Riemann, n=4

$9 \cdot f(21) + 3 \cdot f(12) + 6 \cdot f(9) + 2 \cdot f(3)$

$9 \cdot 6 + 3 \cdot 5 + 6 \cdot 11 + 2 \cdot (-10)$

115

x	1	3	9	12	21
$f(x)$	2	-10	11	5	6

c) $\int_1^9 |f(x)|dx$ Left Riemann, n=2

$2(2) + 6(10) = 64$

FR 20

A test plane flies in a straight line with positive velocity $v(t)$, in miles per minute at time t minutes, where v is a differentiable function of t. Selected values of $v(t)$ for $0 \le t \le 40$ are shown in the table above.

t (min)	0	5	10	15	20	25	30	35	40
$v(t)$ (mpm)	7.0	9.2	9.5	7.0	4.5	2.4	2.4	4.3	7.3

(a) Use a midpoint Riemann sum with four subintervals of equal length and values from the table to approximate $\int_0^{40} v(t)\,dt$. Show the computations that lead to your answer. Using correct units, explain the meaning of $\int_0^{40} v(t)\,dt$ in terms of the plane's flight.

(b) Based on the values in the table, what is the smallest number of instances at which the acceleration of the plane could equal zero on the open interval $0 < t < 40$? Justify your answer.

a) 229 miles; The plane traveled 229 miles in 40min.

b) Twice; Since v(0) = v(15) and v(t) is diff. and therefore continous, by Rolle's Theorem there must...