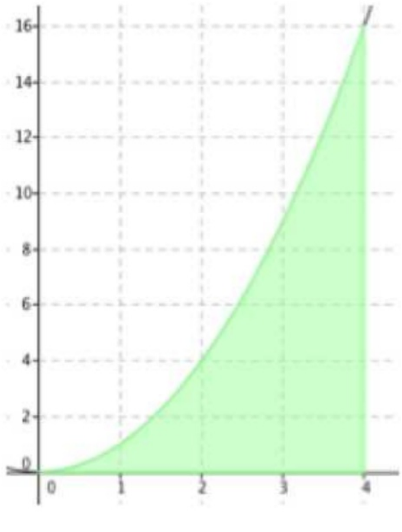
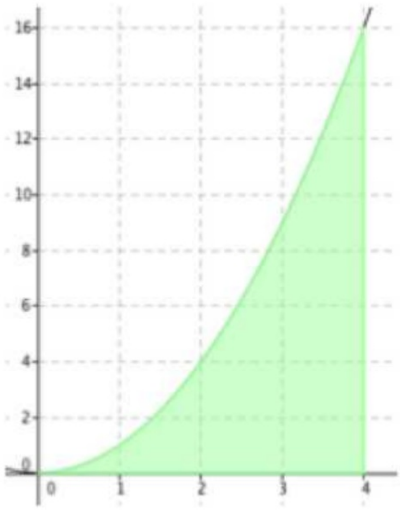
Riemann Approximations

ex: Evaluate: $\int_0^4 x^2 dx$



Since we can't find the exact area using "shapes", we will approximate the area using a Riemann Approximation.



Approximation Techniques:

1. Left Riemann
2. Right Riemann
3. Midpoint Riemann
4. Trapezoidal

If there is a constant width, the width can be calculated by:

Width = (b - a)/n

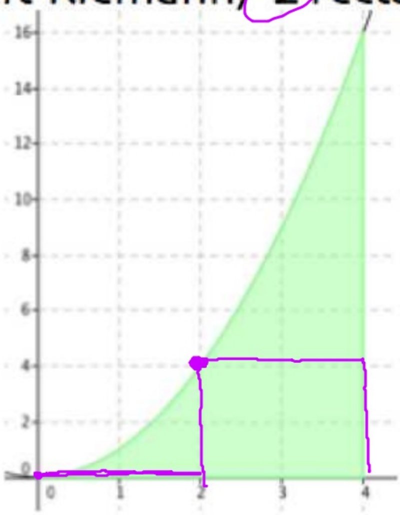
n: # of rectangles

$[0, 4]$ $n = 2$

$$\frac{4-0}{2} = 2 = w$$

ex: Approximate the integral $\int_0^4 x^2\,dx$ using the indicated technique. Then determine if the approximation is an over or under estimate.

a) Left Riemann, 2 rectangles

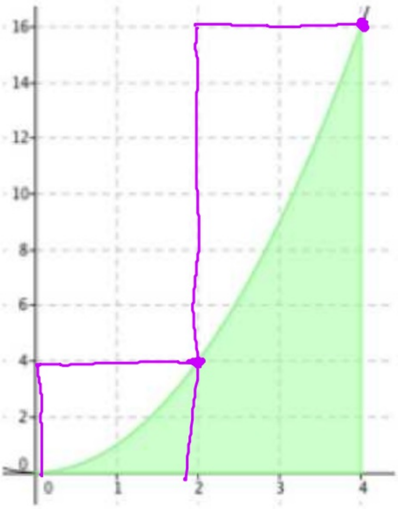


$$\int_0^4 x^2\,dx \approx 2\cdot 0 + 2\cdot 4$$

$$\approx 8$$

Under

b) Right Riemann, 2 rectangles



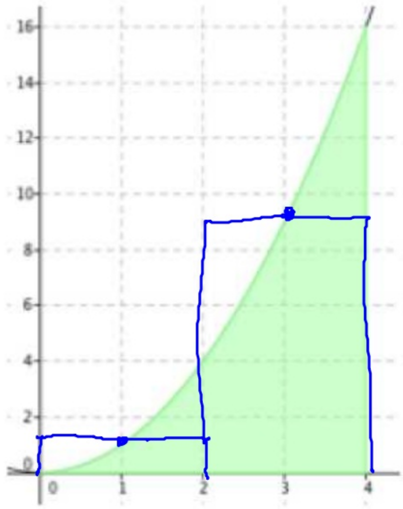
$$\int_0^4 x^2\,dx \approx 2\cdot 16 + 2\cdot 4$$

$$\approx 32 + 8$$

$$\approx 40$$

over

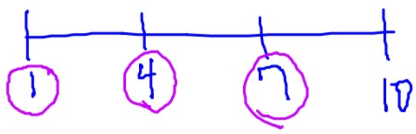
c) Midpoint Riemann, 2 rectangles



$$\int_0^4 x^2\,dx \approx 2(1 + 9)$$
$$\approx 20$$

ex: Approximate the integral $\int_{1}^{10} \sqrt{x}\,dx$ using the indicated technique. Then determine if the approximation is an over or under estimate.

$W = \frac{10-1}{3} = 3$



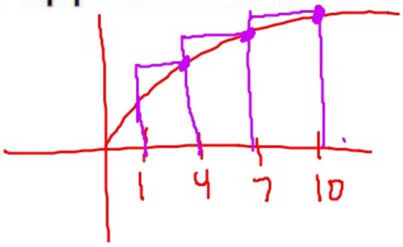
a) Left Riemann, 3 rectangles

$$\int_{1}^{10} \sqrt{x}\,dx \approx 3\left(\underline{\sqrt{1}} + \underline{\sqrt{4}} + \underline{\sqrt{7}}\right)$$

$$3(1 + 2 + 2.646)$$

$$3(5.646) = 16.937$$

ex: Approximate the integral $\int_1^{10} \sqrt{x}\,dx$ using the indicated technique. Then determine if the approximation is an over or under estimate.



b) Right Riemann, 3 rectangles

$$\int_1^{10} \sqrt{x}\,dx \approx 3\left(\sqrt{10}+\sqrt{7}+\sqrt{4}\right)$$

$$\approx 23.424$$

ex: Approximate the integral $\int_{1}^{10} \sqrt{x}dx$ using the indicated technique. Then determine if the approximation is an over or under estimate.

c) Midpoint Riemann, 3 rectangles

$$\int_{1}^{10} \sqrt{x}\,dx \approx 3\left(\sqrt{2.5}+\sqrt{5.5}+\sqrt{8.5}\right)$$

$$20.525$$

ex: Approximate the integral $\int_{2}^{22} \frac{1}{x}\,dx$ using the indicated technique. Then determine if the approximation is an over or under estimate.

a) Left Riemann, 5 rectangles

$$4\left(\frac{1}{2}+\frac{1}{6}+\frac{1}{10}+\frac{1}{14}+\frac{1}{18}\right)$$

3.575

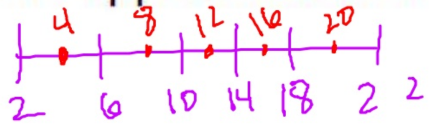
ex: Approximate the integral $\int_{2}^{22} \frac{1}{x} dx$ using the indicated technique. Then determine if the approximation is an over or under estimate.

b) Right Riemann, 5 rectangles

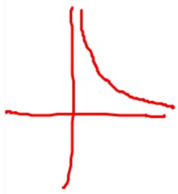
$$4\left(\frac{1}{22}+\frac{1}{18}+\frac{1}{14}+\frac{1}{10}+\frac{1}{6}\right)$$

1.756

ex: Approximate the integral $\int_{2}^{22} \frac{1}{x}\,dx$ using the indicated technique. Then determine if the approximation is an over or under estimate.



c) Midpoint Riemann, 5 rectangles

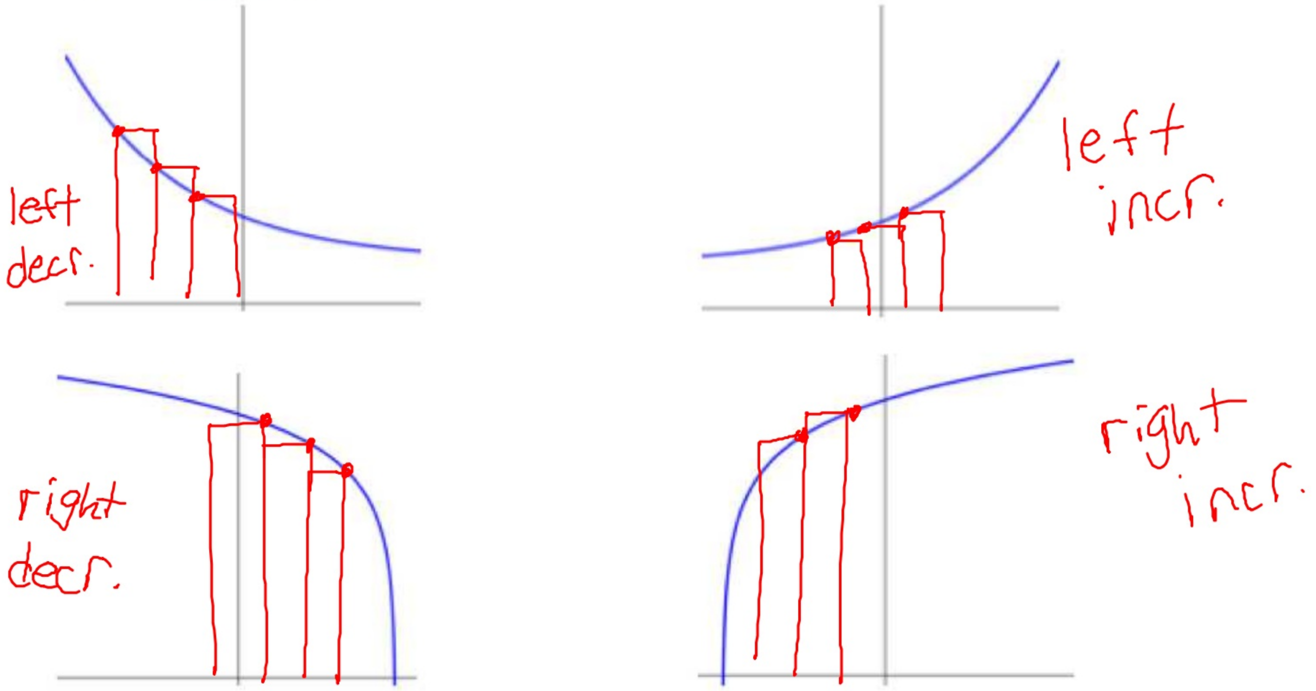


$$4\left(\frac{1}{4}+\frac{1}{8}+\frac{1}{12}+\frac{1}{16}+\frac{1}{20}\right)$$

2.283

- Over and Under Estimates

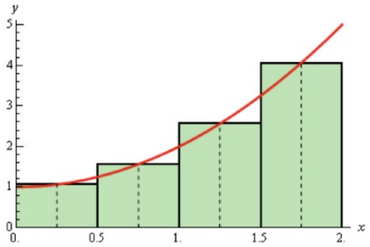
ex: Determine if the Left and Right estimates are over or under approximations.

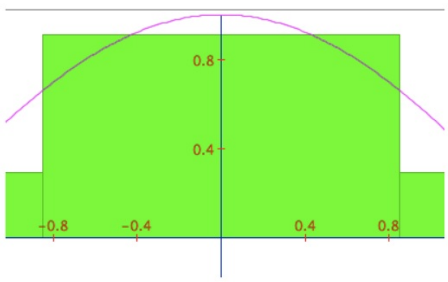


When estimating an integral value using a left or right Riemann Approximation, the approximation will be an over or underestimate depending on whether the curve is

increasing or decreasing ____.

For Midpoints: Will the estimate be an under or over approximation for Concave Up/Concave Down functions





Concave up
Under approx.
more area under
vs. over

Concave down
Over approx.
More area over
vs. under