Rates of change and Rectilinear Motion

- Application of the Derivative: ______________________________

Slope, rate of change, tangent line

- Rate of Change(definition): change in y ÷ change in x.

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Examples:

- ______________________
- ______________________
- ______________________
- ______________________
- ______________________

When you are asked to find the rate of change, you are finding the slope (derivative).

 ex:

The temperature, in degrees Fahrenheit (°F), of water in a pond is modeled by the function H given by $\boxed{H(t) = 55 - 9\cos\left(\frac{2\pi}{365}(t+10)\right)}$, where t is the number of days since January 1 ($t = 0$). What is the instantaneous rate of change of the temperature of the water at time $t = 90$ days?

(A) 0.114°F/day

(B) 0.153°F/day

(C) 50.252°F/day

(D) 56.350°F/day

- Rectilinear Motion Problems – When we talk about these types of problems, we often talk about three types of functions

1. position:
   Notation: $s(t)$, $x(t)$, $f(t)$
   Application(s): position at $t = 0, 1, 2 \ldots$ / when does the object hit the ground.
2. velocity:
   Notation: $s'(t)$, $v(t)$
   - (rate of change of position)

   Application(s): when is the particle moving to the right, when does the particle change direction,
3. acceleration:
   Notation: $s''(t)$, $a(t)$
   - rate of change of velocity

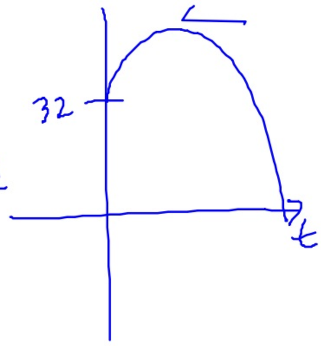
   Application(s): find acceleration at $t = 4$, what times is the velocity decreasing.

Ex 3: At time $t=0$ seconds a diver jumps from a platform diving board that is 32 feet above the water. The position of the diver is given by $s(t) = -16t^2 + 16t + 32$.

a) When does the diver hit the water?

$$0 = -16t^2 + 16t + 32$$
$$0 = -16(t^2 - t - 2)$$
$$0 = -16(t-2)\cancel{(t+1)}$$

$t = 2$ sec



b) What is the diver's velocity at impact?

$$s'(t) = -32t + 16$$
$$s'(2) = -32(2) + 16$$
$$= -48 \text{ ft/sec}$$

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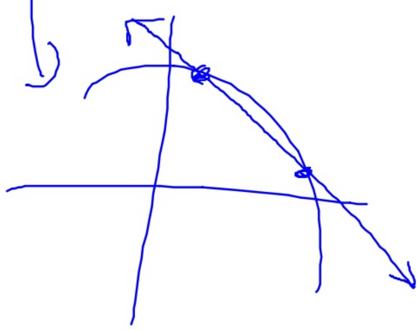
- Average Velocity vs. Instantaneous Velocity

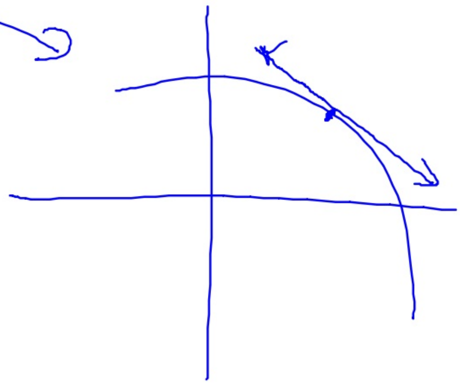
  - Average Velocity: slope of a secant line

    Formula: $\frac{f(b)-f(a)}{b-a}$

  - Instantaneous Velocity: slope at a point

    Formula: $v(a)$ or $s'(a)$





- Speed: $|v(t)|$

- Rest: when $v(t) = 0$

- Left and Right Motion:
  - Left: $v(t) < 0$
  - Right: $v(t) > 0$
  - Changes Direction: $v(a) = 0$ and $v(t)$ changes signs at $t = a$.

Ex 4: A billiard ball is dropped from a height of 100 ft, its height s at time t is given by the position function $s(t) = -16t^2 + 100$, where s is measured in feet and t is measured in seconds.

a) Find the average velocity over time interval [1, 2].

$$\frac{s(2)-s(1)}{2-1} = \frac{36-84}{1} = -48 \text{ ft/sec}$$

b) Find the Instantaneous velocity at the endpoints of the interval.

$s'(t) = -32t$

$s'(1) = -32$ ft/sec

$s'(2) = -64$ ft/sec

c) Find the speed at the endpoints of the interval.

speed at $t=1$
32 ft/sec

speed at $t=2$
64 ft/sec

Ex 5: A particle starts at time t=0 and moves along the x-axis so that its position at any time $t \geq 0$ is given by $x(t) = (t-1)^3(2t-3)$.

a) Find the velocity of the particle at any time $t \geq 0$. Simplify.

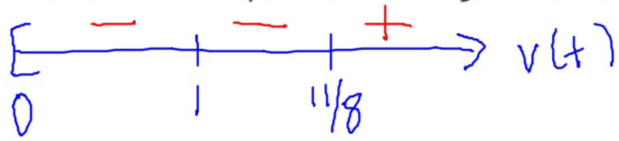
$$v(t) = (t-1)^2(8t-11)$$

b) Determine the values of t for which the particle is at rest.

$$0 = (t-1)^2(8t-11)$$

$$t = 1, \ 11/8$$

c) Determine the values of t for which the particle is moving to the left. JYA.



The particle is moving to the left on (0,1) U (1, 11/8) because v(t) <0 on these intervals.

d) Determine the values of t for which the particle is moving to the right. Justify your answer.

The particle is moving to the right (11/8, inf) because $v(t) > 0$ on this interval.

e) Determine the values of t for which the particle changes direction. Justify your answer.

The particle is changing direction at $t = 11/8$ because $v(11/8) = 0$ and $v(t)$ changes signs at this time.

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A particle moves along the x-axis so that at time t its position is given by $x(t)=\sin\left(\pi t^2\right)$ for $-1\le t\le 1$.

(a) Find the velocity at time t. $v(t) = 2\pi t\cos\left(\pi t^2\right)$

(b) Find the acceleration at time t. $a(t) = 2\pi\left[\cos\left(\pi t^2\right) - 2\pi t^2 \sin\left(\pi t^2\right)\right]$

(c) For what values of t does the particle change direction?

(d) Find all values of t for which the particle is moving to the left.

c.) $0 = 2\pi t\cos(\pi t^2)$ $\qquad [-1, 1]$

$$0 = \underbrace{t}\,\underbrace{\cos(\pi t^2)}$$

$t = 0 \qquad \cos(\pi t^2) = 0$

$$\pi t^2 = -\frac{3\pi}{2},\ -\frac{\pi}{2},\ \frac{\pi}{2},\ \frac{3\pi}{2}$$

$$\sqrt{t^2} = \sqrt{-\frac{3}{2}},\ \sqrt{-\frac{1}{2}},\ \sqrt{\frac{1}{2}},\ \sqrt{\frac{3}{2}}$$

$$t = \pm\sqrt{\frac{1}{2}},\ \cancel{\pm\sqrt{\frac{3}{2}}} \leftarrow$$ bigger than 1

-1		$-\sqrt{\frac{1}{2}}$		0		$\sqrt{\frac{1}{2}}$		1
	$+$		$-$		$+$		$-$	

d.) left?

The particle is moving to the left on 2 intervals....
(you write these intervals), because $v(t) < 0$
on these intervals.

$$y = \underbrace{2x\arccos x}_{} - 2\sqrt{1-x^2}$$

$$y' = \cancel{2x\cdot\frac{-1}{\sqrt{1-x^2}}} + \arccos x\cdot 2 + \cancel{2}\cdot\cancel{\frac{1}{2}}(1-x^2)^{-1/2}(+2x)$$

$$\cancel{\frac{2x}{\sqrt{1-x^2}}}$$

$$y' = 2\arccos x$$

35.) $y = \sin(\arccos t)$

$$y' = \underline{\cos(\arccos t)} \cdot \frac{-1}{\sqrt{1-t^2}}$$

$$y' = t \cdot \frac{-1}{\sqrt{1-t^2}}$$

$$y' = \frac{-t}{\sqrt{1-t^2}}$$

54.) $y = \operatorname{arcsec}(4x)$

$$y' = \frac{4}{|4x|\sqrt{16x^2-1}}$$

$$y' = \frac{1}{|x|\sqrt{16x^2-1}}$$

$$y'\left(\frac{\sqrt{2}}{4}\right) = \frac{1}{\frac{\sqrt{2}}{4}\sqrt{1}} = \frac{4}{\sqrt{2}}$$