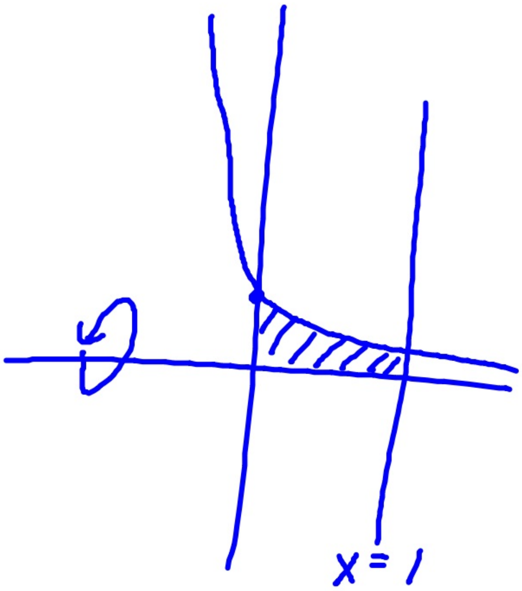
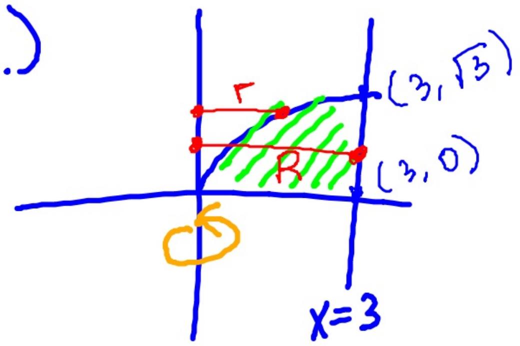


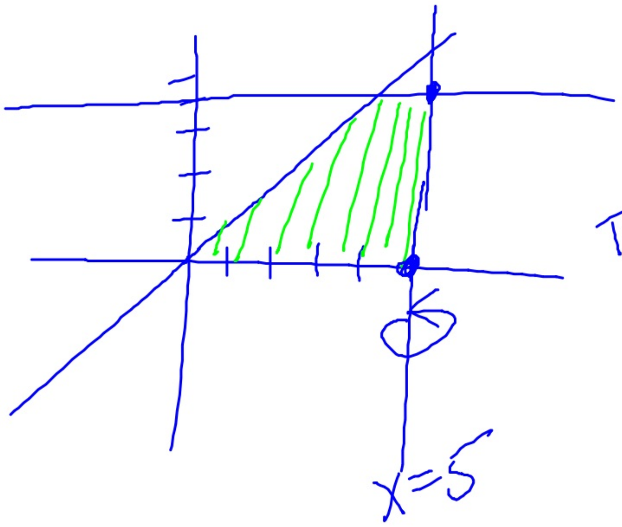
$$\pi \int_0^1 (e^{-x})^2 \, dx = \pi \int_0^1 e^{-2x} \, dx$$

$$-\frac{1}{2}\pi \left(e^{-2} - e^{0} \right)$$

13b.)



$$\pi \int_0^{\sqrt{3}} \left(\left(\underset{\text{outer}}{3} - \underset{\text{axis}}{0} \right)^2 - \left(\underset{\text{inner}}{y^2} - \underset{\text{axis}}{0} \right)^2 \right) dy$$



disk

$$\pi \int_0^4 (y-5)^2\, dy$$

$$\left.\frac{\pi (y-5)^3}{3}\right]_0^4$$

22c.)

22.

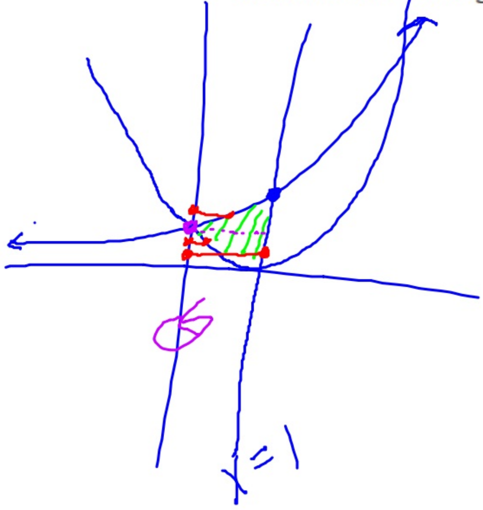
Let R be the region enclosed by the graphs of $y = e^x$, $y = (x-1)^2$, and the line $x = 1$.

$x = \ln y$ $\quad$ $x = \sqrt{y} + 1$

(a) Find the area of R.

(b) Find the volume of the solid generated when R is revolved about the x-axis.

(c) Set up, but do not integrate, an integral expression in terms of a single variable for the volume of the solid generated when R is revolved about the y-axis.



Washer

$$\pi \int_0^1 \left((1-0)^2 - (\sqrt{y}+1-0)^2\right) dy +$$

$$\pi \int_1^e \left((1-0)^2 - (\ln y - 0)^2\right) dy$$

17. CALCULATOR

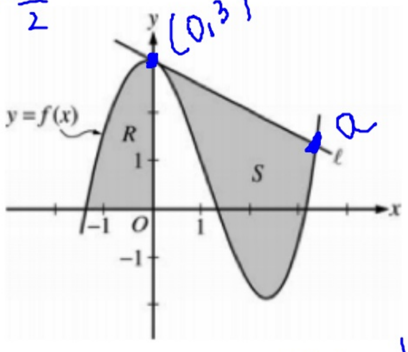
$f'(0) = -\frac{1}{2}$

Let f be the function given by $f(x) = \frac{x^3}{4} - \frac{x^2}{3} - \frac{x}{2} + 3\cos x$. Let R be the shaded region in the second quadrant bounded by the graph of f, and let S be the shaded region bounded by the graph of f and line ℓ, the line tangent to the graph of f at $x = 0$, as shown above.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is rotated about the horizontal line $y = -2$.

(c) Write, but do not evaluate, an integral expression that can be used to find the area of S.



$$\int_0^a \left(\left(-\frac{1}{2}x + 3\right) - f(x)\right) dx$$

$y = mx + b$

$y = -\frac{1}{2}x + 3$

19b

21.

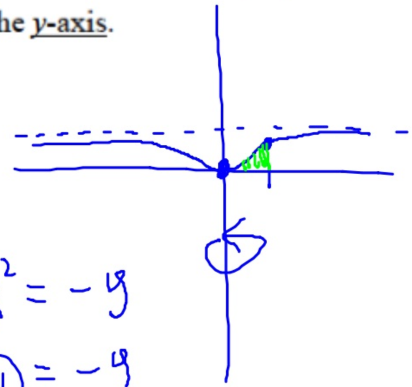
Let R be the region enclosed by the graph of $y = \dfrac{x^2}{x^2+1}$, the line $x = 1$, and the x-axis.

$(1, \frac{1}{2})$

$(2, \frac{4}{5})$

(a) Find the area of R .

(b) Find the volume of the solid generated when R is rotated about the y-axis.



$$\pi \int_0^{1/2} \left((1-0)^2 - \left(\sqrt{\frac{-y}{y-1}} - 0 \right)^2 \right) dy$$

$$(x^2+1)y = x^2$$

$$x^2 y + y = x^2$$

$$x^2 y - x^2 = -y$$

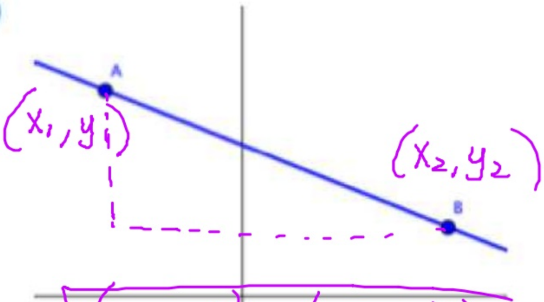
$$x^2 (y-1) = -y$$

$$x = \pm \sqrt{\frac{-y}{y-1}}$$

6.4 Arc Length

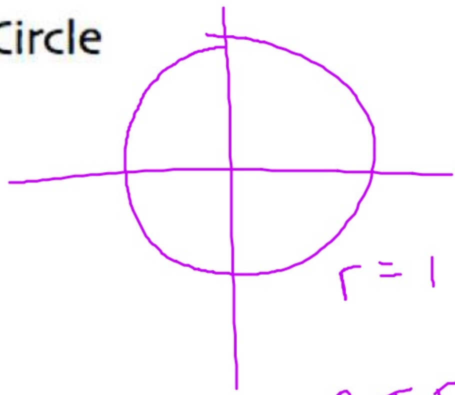
ex: Find the length.

a)



$$d = \sqrt{(y_1 - y_2)^2 + (x_1 - x_2)^2}$$

b) Unit Circle



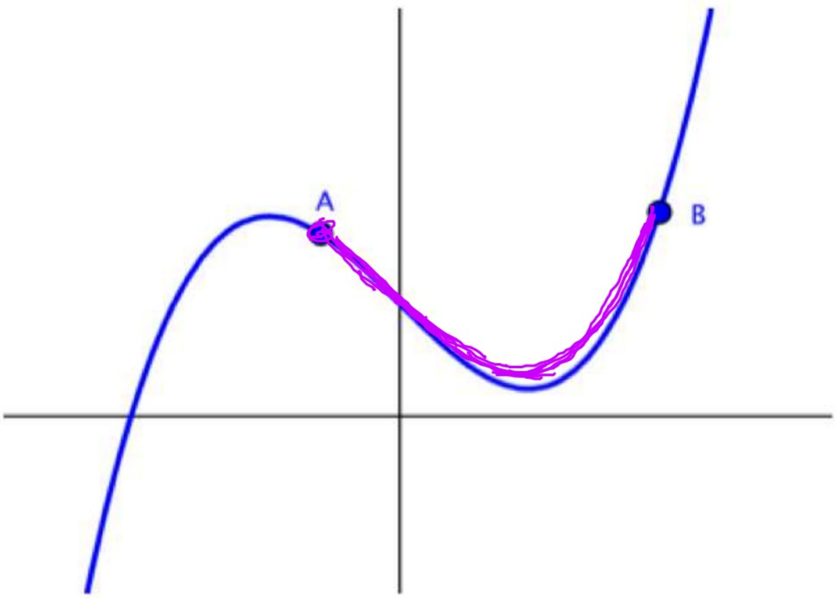
$r = 1$

$C = 2\pi r$

$C = 2\pi$

ex: Find the length.

c)



$$\begin{aligned}
\left|P_{i-1}\ P_i\right| &= \sqrt{\left(x_i - x_{i-1}\right)^2 + \left(y_i - y_{i-1}\right)^2} \\
&= \sqrt{\frac{\Delta x^2}{\Delta x^2} + \left[f'\left(x_i^*\right)\right]^2 \frac{\Delta x^2}{\Delta x^2}} \\
&= \sqrt{1 + \left[f'\left(x_i^*\right)\right]^2}\ \ \Delta x
\end{aligned}$$

By the **Mean Value Theorem** we know that on the interval $\left[x_{i-1}, x_i\right]$ there is a point x_i^* so that,

$$\begin{aligned}
f\left(x_i\right) - f\left(x_{i-1}\right) &= f'\left(x_i^*\right)\left(x_i - x_{i-1}\right) \\
\Delta y_i &= f'\left(x_i^*\right)\Delta x
\end{aligned}$$

$$\begin{aligned}
L &= \lim_{n\to\infty} \sum_{i=1}^{n} \left|P_{i-1}\ P_i\right| \\
&= \lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{1 + \left[f'\left(x_i^*\right)\right]^2}\ \ \Delta x
\end{aligned}$$

$$\boxed{L = \int_a^b \sqrt{1 + \left[f'\left(x\right)\right]^2}\, dx}$$

Arc Length

Let the function $y = f(x)$ represent a smooth curve on the interval $[a, b]$. The **arc length** of f between a and b is

$$L = \int_a^b \sqrt{1 + (f'(x))^2}\, dx$$

ex: Find the arc length on the indicated interval.

$$f(x) = \sin x, \quad [0, 2\pi]$$

$$L = \int_0^{2\pi} \sqrt{1 + \cos^2 x}\, dx = 7.640$$

ex: Find ~~a~~ the curve that passes through the point (1,1) whose length on the interval [1, 4] is given by

$$\int_1^4 \sqrt{1+\frac{1}{4x^2}}\,dx$$

$f'(x) = \frac{1}{2x} = \frac{1}{2} \cdot \frac{1}{x}$

$f(x) = \frac{1}{2}\ln x + C$

$1 = C$

$\boxed{f(x) = \frac{1}{2}\ln x + 1}$

Arc Length WKST

1.

Find the arc length of the graph of the function over the indicated interval.

a) $y = \dfrac{x^5}{10} + \dfrac{1}{6x^3}$, $[2,5]$

b) $y = \ln(\sin x)$, $\left[\dfrac{\pi}{4}, \dfrac{3\pi}{4}\right]$

c) $y = \dfrac{1}{2}\left(e^x + e^{-x}\right)$, $[0,2]$

*See printout.

Arc Length WKST

2.

The length of the curve $y = x^3$ from $x = 0$ to $x = 2$ is given by

~~(A)~~ $\int_0^2 \sqrt{1+x^6}\,dx$ (B) $\int_0^2 \sqrt{1+3x^2}\,dx$ ~~(C)~~ $\pi\int_0^2 \sqrt{1+9x^4}\,dx$

~~(D)~~ $2\pi\int_0^2 \sqrt{1+9x^4}\,dx$ (E) $\int_0^2 \sqrt{1+9x^4}\,dx$

Arc Length WKST

3.

The length of a curve from $x = 1$ to $x = 4$ is given by $\int_1^4 \sqrt{1+9x^4}\,dx$. If the curve contains the point $(1,6)$, which of the following could be an equation for this curve?

(A) $y = 3 + 3x^2$ (B) $y = 5 + x^3$ (C) $y = 6 + x^3$

(D) $y = 6 - x^3$ (E) $y = \frac{16}{5} + x + \frac{9}{5}x^5$

$f'(x) = 3x^2$

$f(x) = x^3 + C$

$6 = 1 + C$

$5 = C$

Arc Length WKST

4.

(Calculator Permitted) Which of the following gives the best approximation of the length of the arc of $y = \cos(2x)$ from $x = 0$ to $x = \dfrac{\pi}{4}$?

(A) 0.785 (B) 0.955 (C) 1.0 (D) 1.318 (E) 1.977

Arc Length WKST

5.

Find the length of the curve described by $y = \frac{2}{3}x^{3/2}$ from $x = 0$ to $x = 8$.

(A) $\frac{26}{3}$ (B) $\frac{52}{3}$ (C) $\frac{512\sqrt{2}}{15}$ (D) $\frac{512\sqrt{2}}{15} + 8$ (E) 96

Arc Length WKST

6.

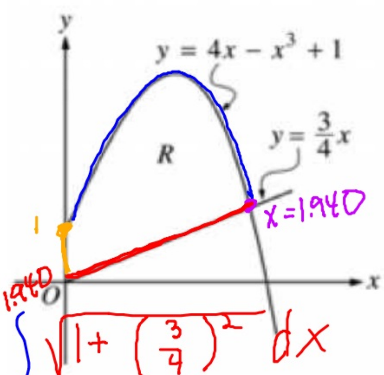
$y = 4x - x^3 + 1$

Let R be the region in the first quadrant bounded by the y-axis and the graphs of $y = 4x - x^3 + 1$ and $y = \frac{3}{4}x$.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is revolved about the x-axis.

(c) Write an expression involving one or more integrals that gives the perimeter of R. Do not evaluate.

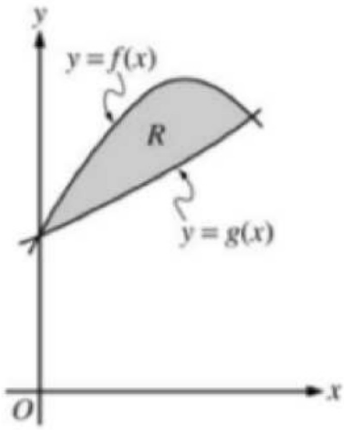


$$1 + \int_0^{1.940} \sqrt{1+(4-3x^2)^2}\,dx + \int_0^{1.940} \sqrt{1+\left(\frac{3}{4}\right)^2}\,dx$$

FR 18

$g(x) = e^{x/2}$

Let f and g be the functions given by $f(x) = 1 + \sin(2x)$ and $g(x) = e^{x/2}$. Let R be the shaded region in the first quadrant enclosed by the graphs of f and g as shown in the figure above.



(a) Find the area of R.

(b) Find the volume of the solid generated when R is revolved about the x-axis.

(c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are semicircles with diameters extending from $y = f(x)$ to $y = g(x)$. Find the volume of this solid.

.078

FR 15

Let R be the region in the first quadrant bounded by the graphs of $y = \sqrt{x}$ and $y = \frac{x}{3}$.

(a) Find the area of R.

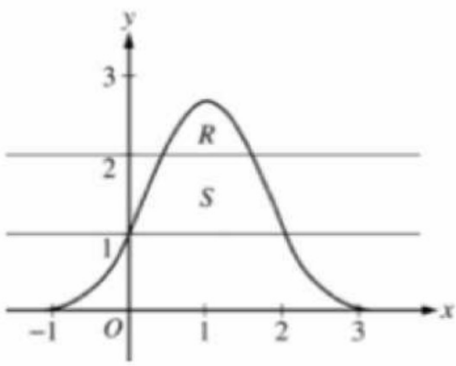
(b) Find the volume of the solid generated when R is rotated about the vertical line $x = -1$.

(c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the y-axis are squares. Find the volume of this solid.

$\int_0^3 (y^2 - 3y)^2 \, dy = 8.1$

FR 16

Let R be the region bounded by the graph of $y = e^{2x-x^2}$ and the horizontal line $y = 2$, and let S be the region bounded by the graph of $y = e^{2x-x^2}$ and the horizontal lines $y = 1$ and $y = 2$, as shown above.



(a) Find the area of R.

(b) Find the area of S.

(c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 1$.

FR 4

Consider the differential equation $\frac{dy}{dx} = \frac{3x^2}{e^{2y}}$.

(a) Find a solution $y = f(x)$ to the differential equation satisfying $f(0) = \frac{1}{2}$.

(b) Find the domain and range of the function f found in part (a).

If $\boxed{\frac{dy}{dt} = -2y}$ and if $y = 1$ when $t = 0$, what is the value of t for which $y = \frac{1}{2}$?

(0, 1) (—, $\frac{1}{2}$)

(A) $-\frac{1}{2}\ln 2$ (B) $-\frac{1}{4}$ (C) $\frac{1}{2}\ln 2$ (D) $\frac{\sqrt{2}}{2}$ (E) $\ln 2$

(Calculator permitted) Population y grows according to the equation $\frac{dy}{dt} = ky$, where k is a constant and t is measured in years. If the population doubles every 10 years, then the value of k is

(A) 0.069 (B) 0.200 (C) 0.301 (D) 3.322 (E) 5.000