6.2: Volume by cross section

You will be given:

- base (determined by region enclosed by functions)
- geometric shape
- perpendicular to x-axis or y-axis

Geometric Shapes

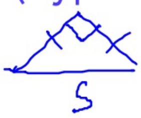
Square s^2

Rectangle $s \cdot h$

Equilateral triangle $\frac{\sqrt{3}}{4} s^2$

Right Isosceles Triangle(leg on base) $\frac{1}{2} s^2$

Right Isosceles Triangle (hyp. on base) $\frac{s^2}{4}$



Semi-circles $\frac{\pi}{8} s^2$

Perpendicular to the x-axis

$$V = \int_a^b (\text{Area})\,dx$$

in terms of x

Perpedicular to the y-axis

$$V = \int_a^b (\text{Area})\,dy$$

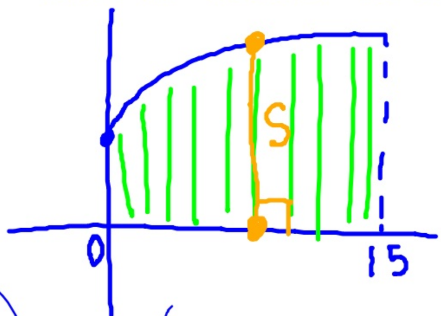
in terms of y

1 $f(x) = 2\sqrt{x} + 5$ $[0, 15]$

Cross section perpendicular to the x-axis are squares. (1ST Quadrant)

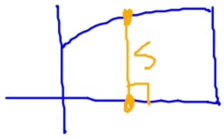
$$\int_0^{15} S^2\,dx = \int_0^{15} (2\sqrt{x}+5)^2\,dx$$

1599.597



$S = (2\sqrt{x} + 5 - 0)$

② $f(x) = 2\sqrt{x} + 5 \quad [0, 15]$



Cross section perpendicular to the x-axis are semi-circles.

$$V = \int_0^{15} \frac{\pi}{8} s^2 \, dx = \frac{\pi}{8} \int_0^{15} \left(2\sqrt{x} + 5\right)^2 dx$$

$$= 628.160$$

③ $f(x) = 2\sqrt{x} + 5 \quad [0, 15]$

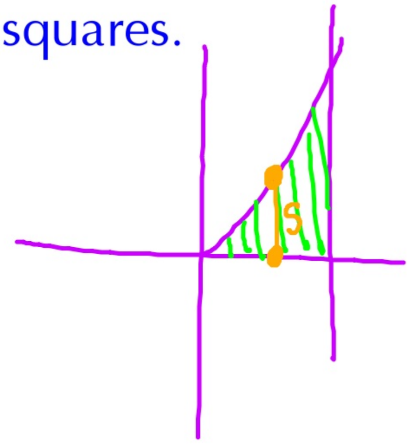
Cross section perpendicular to the x-axis are rectangles of height 4.

$$V = \int_0^{15} 4s\,dx = \int_0^{15} 4(2\sqrt{x} + 5)\,dx$$

609.839

4) Base enclosed by $y = x^2$, $y = 0$, and $x = 2$. Cross sections perpendicular to the x-axis are squares.

$$\int_0^2 s^2\,dx = \int_0^2 x^4\,dx = 6.4$$

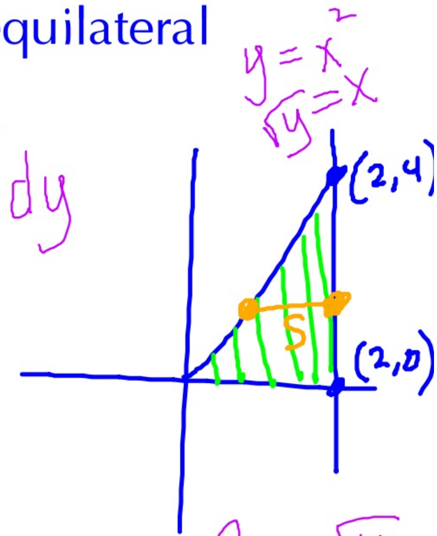


perp
S =

5) Base enclosed by $y = x^2$, $y = 0$, and $x = 2$. Cross sections perpendicular to the y-axis are equilateral triangles.

$$\int_0^4 \frac{\sqrt{3}}{4} s^2 \, dy = \frac{\sqrt{3}}{4}\int_0^4 (2-\sqrt{y})^2 \, dy$$

$$= 1.155$$

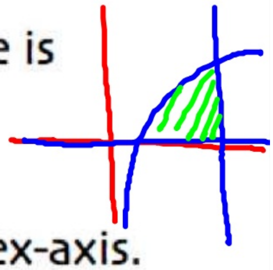


$S = \underset{\substack{\uparrow \\ \text{right}}}{2} - \underset{\substack{\uparrow \\ \text{left}}}{\sqrt{y}}$

perpendicular to y-axis:

S = right - left

ex: Find the volume of the solid whose base is bounded by $y = \ln x$, $x = e$, and the $x-\text{axis}$ with...

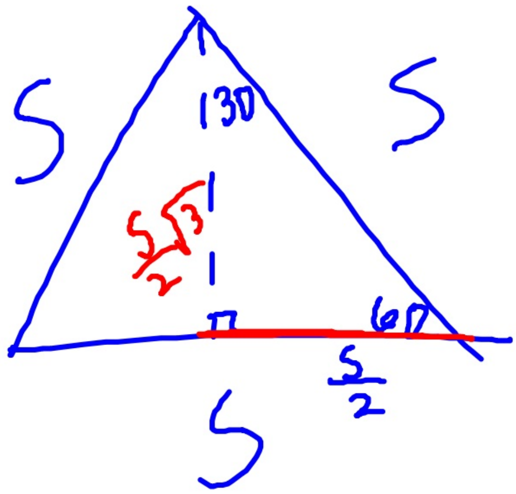


a) square cross sections taken perpendicular to thex-axis.

$$\int_1^e (\ln x)^2 dx = .718$$

b) equilateral triangular cross sections taken perpendicular to the y-axis.

$$\frac{\sqrt{3}}{4}\int_0^1 (e-e^y)^2 dy = .538$$



$$\frac{1}{2}(s)\left(\frac{s\sqrt{3}}{2}\right)$$

$$\frac{\sqrt{3}}{4}s^2$$