$$|y| = 7$$

$$y = 7, \; y = -7$$

$$|y-2| = 2e^{x^5/5}$$

$$y-2 = 2e^{x^5/5}$$

$$y = 2e^{x^5/5}+2$$

$$y-2 = -2e^{x^5/5}$$

$$y = -2e^{x^5/5}+2$$

$(0,0)$

25.) $(0,2)$

$$\frac{dy}{dx} = \frac{x}{4y}$$

$$\int y\,dy = \int \frac{1}{4} x\,dx$$

$$\frac{y^2}{2} = \frac{1}{4}\frac{x^2}{2} + C$$

$$\frac{y^2}{2} = \frac{x^2}{8} + C$$

$$2 = C$$

$$2\left(\frac{y^2}{2} = \frac{x^2}{8} + 2\right)$$

$$y^2 = \frac{1}{4}x^2 + 4$$

$$y = \sqrt{\frac{1}{4}x^2 + 4}$$

17.) $\frac{dy}{dx} = -y(x+1)$ $(-2,1)$

$\int \frac{dy}{y} = \int -(x+1)\,dx$

$0 = -2+2 = c$

$0 = c$

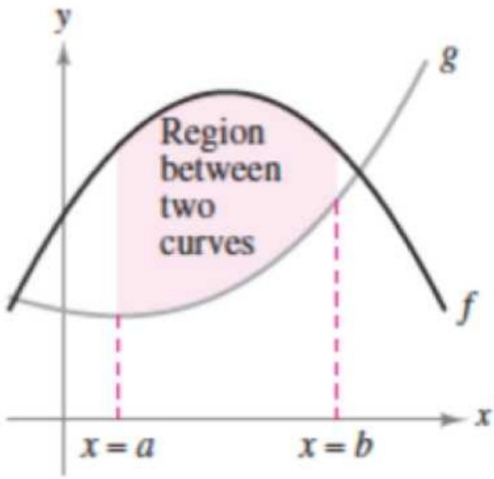
$e^{\ln|y|} = e^{-\frac{x^2}{2} - x + c}$

$0 = c$

$|y| = e^{-x^2/2 - x}$

$y = e^{-\frac{x^2}{2} - x}$

6.1 Area Between Two Curves

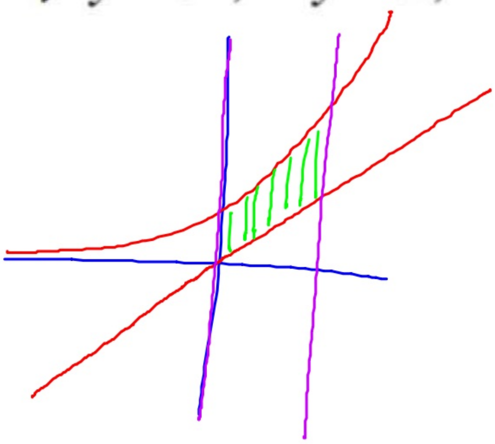


top − bottom

Area = $\int_a^b (f(x) - g(x))\,dx$

ex: Find the area enclosed by the given curves.

a) $y = e^x, \quad y = x, \quad x = 0, \quad x = 1$

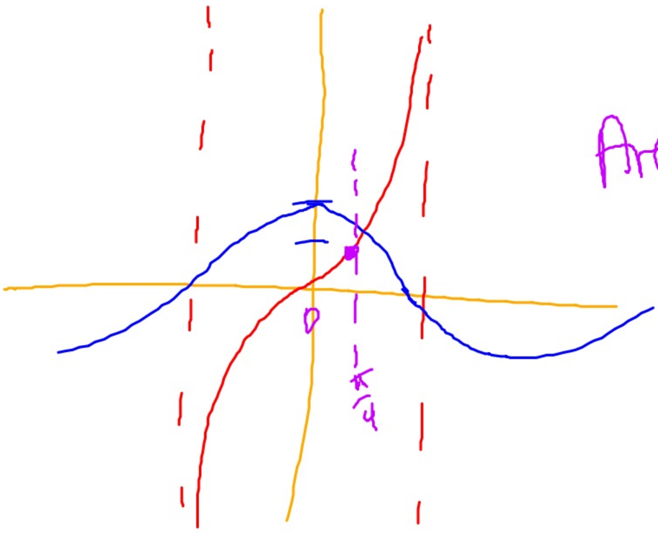


$$\text{Area} = \int_0^1 (e^x - x)\,dx$$

$$= 1.218$$

ex: Find the area enclosed by the given curves.

b) $y = \tan x, \quad y = 2\cos x, \quad 0 \le x \le \frac{\pi}{4}$

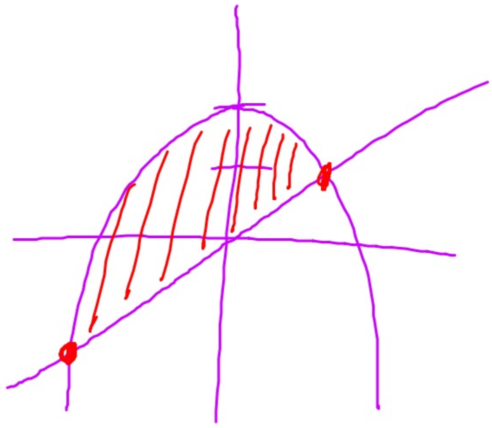


$$\text{Area} = \int_0^{\pi/4} (2\cos x - \tan x)\,dx$$

$$= 1.068$$

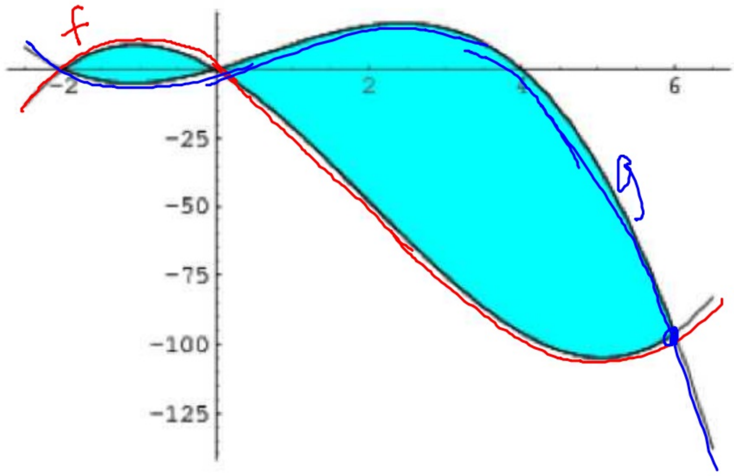
ex: Find the area enclosed by the given curves.

c) $f(x) = 2 - x^2, \quad g(x) = x$



$$\int_{-2}^{1} (2 - x^2 - x)\,dx$$

4.5



(by hand) Area = $\int_{-2}^{0} (f(x)-g(x))\,dx + \int_{0}^{6} (g(x)-f(x))\,dx$

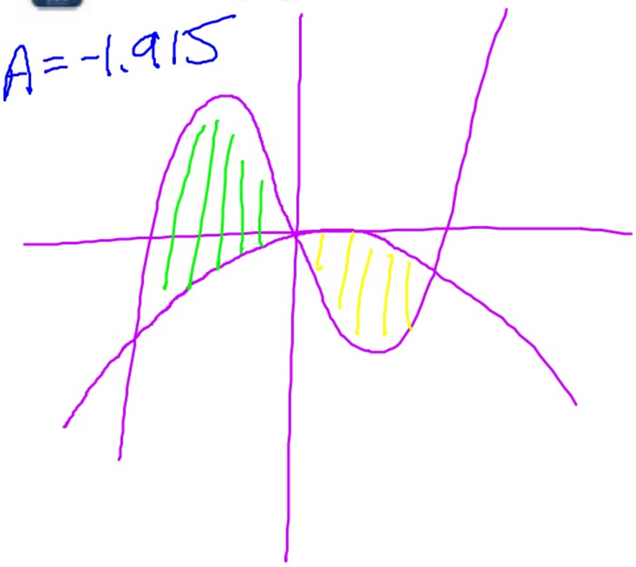
OR

(calculator) Area = $\int_{-2}^{6} |f(x)-g(x)|\,dx$

ex: Find the area enclosed by the given curves.

d) $f(x) = 3x^3 - x^2 - 10x, \quad g(x) = -x^2 + x$

$A = -1.915$



$$\int_{-1.915}^{0} (f(x) - g(x))\,dx + \int_{0}^{1.915} (g(x) - f(x))\,dx$$

20.167

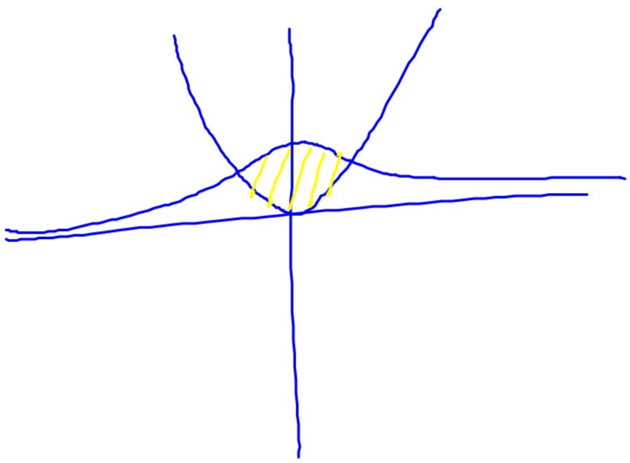
$$\int_{-1.915}^{1.915} |(f(x) - g(x)|\,dx$$

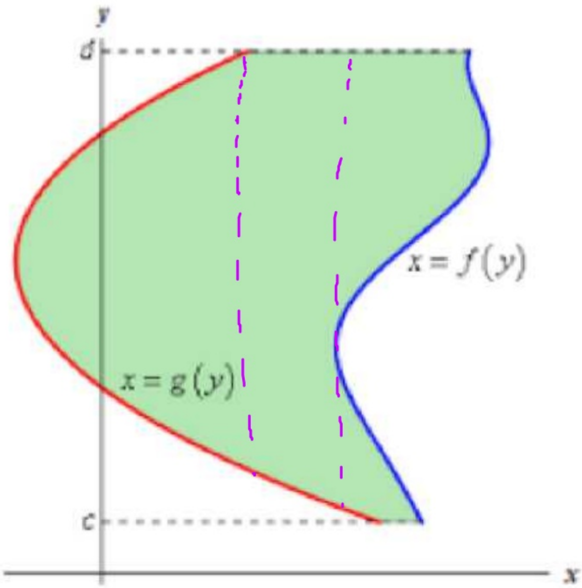
ex: Find the area enclosed by the given curves.

e) $y = \dfrac{1}{1+x^2}, \quad y = \dfrac{x^2}{2}$

$$2\int_0^1 \left(\frac{1}{1+x^2} - \frac{x^2}{2}\right)dx$$

1.237



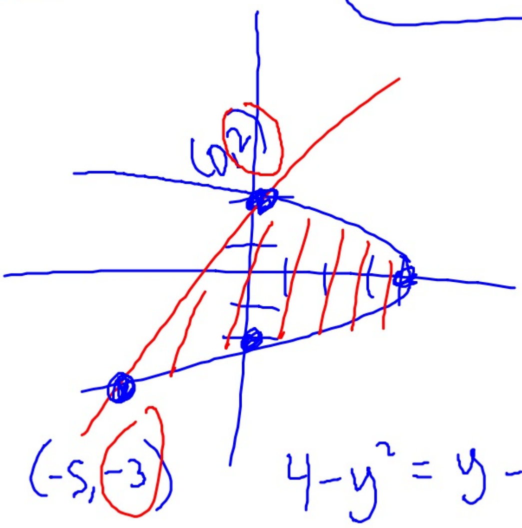


Right - left

Area = $\int_c^d (f(y) - g(y))\,dy$

ex: Find the area enclosed by the given curves.

f) $x = 4 - y^2, \quad x = y - 2 \longrightarrow y = x + 2$



x	y
0	-2
4	0
0	2

$$\int_{-3}^{2} (4 - y^2 - y + 2)\,dy$$

$$\int_{-3}^{2} (6 - y^2 - y)\,dy$$

$$20.833$$

$$4 - y^2 = y - 2$$

$$0 = y^2 + y - 6$$

$$(y+3)(y-2)$$

ex:

a) Find the area bounded by

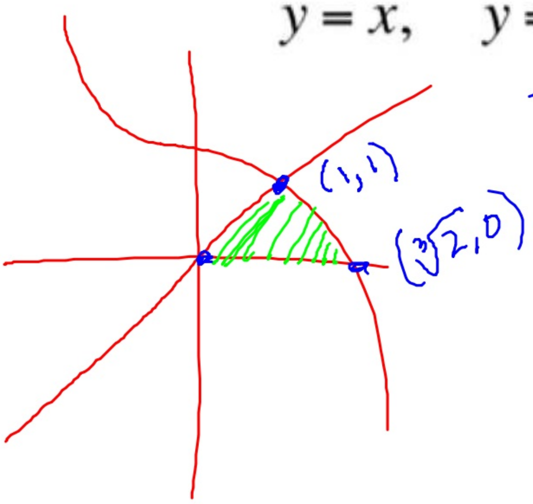
$$y = x, \quad y = -x^3 + 2, \quad \text{y-axis}$$

ex:

b) Find the area bounded by

$$x = \sqrt[3]{2-y}$$

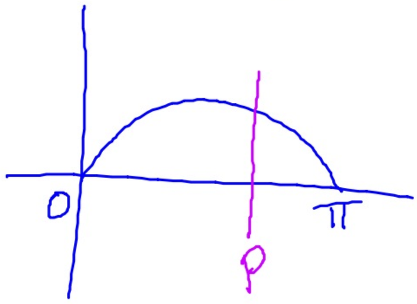
$$y = x, \quad y = -x^3 + 2, \quad \text{x-axis}$$



Top − Bottom: $\displaystyle\int_0^1 x\,dx + \int_1^{\sqrt[3]{2}} (-x^3+2)\,dx$

Right − left $\displaystyle\int_0^1 \left(\sqrt[3]{2-y} - y\right) dy$

ex: The line x=p divides the area bounded by $y = \sin x$ on $0 \le x \le \pi$ into 2 regions such that the area from $0 \le x \le p$ exceeds the area from $p \le x \le \pi$ by 1 square unit. Find p.



$$\int_0^p \sin x\,dx = \int_p^\pi \sin x\,dx + 1$$

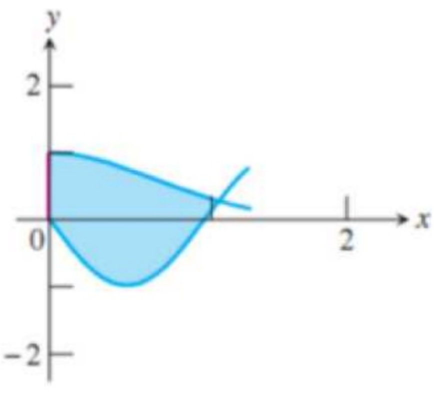
$$p = \frac{2\pi}{3}$$

5.1, 5.3, 6.1 Extra Practice

2.

(Calculator permitted) Let R be the shaded region enclosed by the graphs of $y = e^{-x^2}$, $y = -\sin(3x)$, and the y-axis as shown at right. Which of the following gives the approximate area of the region R?

(A) 1.139 (B) 1.445 (C) 1.869 (D) 2.114 (E) 2.340



5.1, 5.3, 6.1 Extra Practice

3.

If $\frac{dy}{dt} = -2y$ and if $y = 1$ when $t = 0$, what is the value of t for which $y = \frac{1}{2}$?

(A) $-\frac{1}{2}\ln 2$ (B) $-\frac{1}{4}$ (C) $\frac{1}{2}\ln 2$ (D) $\frac{\sqrt{2}}{2}$ (E) $\ln 2$

5.1, 5.3, 6.1 Extra Practice

4.

(Calculator permitted) Population y grows according to the equation $\frac{dy}{dt} = ky$, where k is a constant and t is measured in years. If the population doubles every 10 years, then the value of k is

(A) 0.069 (B) 0.200 (C) 0.301 (D) 3.322 (E) 5.000

5.1, 5.3, 6.1 Extra Practice

5.

Let f and g be the functions given by $f(x) = e^x$ and $g(x) = \frac{1}{x}$. Which of the following gives the area of the region enclosed by the graphs of f and g between $x = 1$ and $x = 2$?

(A) $e^2 - e - \ln 2$ (B) $\ln 2 - e^2 + e$ (C) $e^2 - \frac{1}{2}$ (D) $e^2 - e - \frac{1}{2}$ (E) $\frac{1}{e} - \ln 2$

5.1, 5.3, 6.1 Extra Practice

6.

Let R be the region in the first quadrant bounded by the x-axis, the graph of $x = y^2 + 2$, and the line $x = 4$. Which of the following integrals gives the area of R?

(A) $\int_0^{\sqrt{2}} \left[4 - \left(y^2 + 2\right)\right] dy$ (B) $\int_0^{\sqrt{2}} \left[\left(y^2 + 2\right) - 4\right] dy$ (C) $\int_{-\sqrt{2}}^{\sqrt{2}} \left[4 - \left(y^2 + 2\right)\right] dy$

(D) $\int_{-\sqrt{2}}^{\sqrt{2}} \left[\left(y^2 + 2\right) - 4\right] dy$ (E) $\int_2^4 \left[4 - \left(y^2 + 2\right)\right] dy$

5.1, 5.3, 6.1 Extra Practice

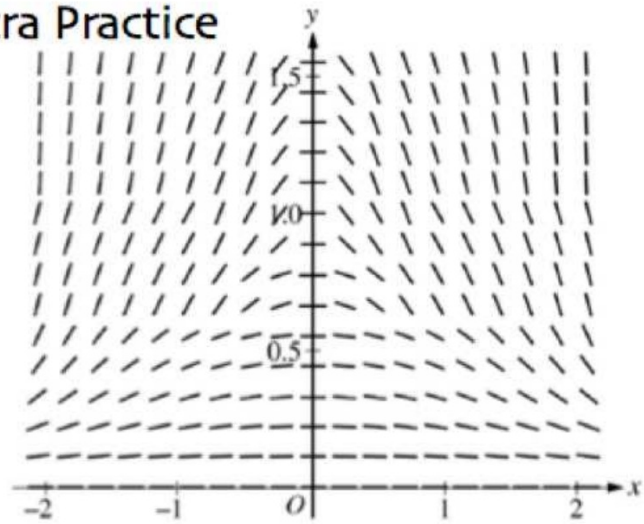
7.

Which of the following gives the area of the region between the graphs of $y = x^2$ and $y = -x$ from $x = 0$ to $x = 3$.

(A) 2 (B) $\frac{9}{2}$ (C) $\frac{13}{2}$ (D) 13 (E) $\frac{27}{2}$

5.1, 5.3, 6.1 Extra Practice

8.



The slope field for a certain differential equation is shown above. Which of the following could be a solution to the differential equation with the initial condition $y(0) = 1$?

(A) $y = \cos x$

(B) $y = 1 - x^2$

(C) $y = e^x$

(D) $y = \sqrt{1 - x^2}$

(E) $y = \dfrac{1}{1 + x^2}$

5.1, 5.3, 6.1 Extra Practice

9.

Which of the following is the solution to the differential equation $\frac{dy}{dx} = e^{y+x}$ with the initial condition $y(0) = -\ln 4$?

(A) $y = -x - \ln 4$

(B) $y = x - \ln 4$

(C) $y = -\ln\left(-e^{x} + 5\right)$

(D) $y = -\ln\left(e^{x} + 3\right)$

(E) $y = \ln\left(e^{x} + 3\right)$

5.1, 5.3, 6.1 Extra Practice

11.

What is the area of the region in the first quadrant bounded by the graph of $y = e^{x/2}$ and the line $x = 2$?

(A) $2e - 2$ (B) $2e$ (C) $\frac{e}{2} - 1$ (D) $\frac{e-1}{2}$ (E) $e - 1$