85.) $\int_1^x \frac{3}{t}\,dt = \int_{1/4}^x \frac{1}{t}\,dt$

$3\ln|t|+c\Big|_1^x = \ln|t|+c\Big|_{1/4}^x$

$3\ln x = \ln x - \ln\frac{1}{4}$

$3\ln x - 1\ln x =$

$2\ln x = \ln 4$

$e^{\ln x^2} = e^{\ln 4}$

$x^2 = 4$

$x = 2$

35.) $\int(\cos^3\theta - 1)\,d\theta$

$\int \cos 3\theta\,d\theta - \int 1\,d\theta$

$\frac{1}{3}\sin 3\theta - \theta + C$

15.)
17.) $\int \frac{x^3 - 3x^2 + 5}{x-3}\,dx$

$$\begin{array}{r|rrrr} 3 & 1 & -3 & 0 & 5 \\ & & 3 & 0 & 0 \\ \hline & 1 & 0 & 0 & 5 \end{array}$$

$\int \left(x^2 + \frac{5}{x-3}\right)dx$

$\frac{1}{3}x^3 + 5\ln|x-3| + C$

57.) $\int_1^2 \frac{1-\cos\theta}{\theta-\sin\theta}\,d\theta = \int \frac{1}{u}\,du$

$u = \theta - \sin\theta$

$du = 1-\cos\theta\,d\theta$

$\ln|\theta-\sin\theta|\Big|_1^2$

$\ln|2-\sin 2| - \ln|1-\sin 1|$

4.7 Inverse Trigonometry: Integration

Review:

$$\frac{d}{dx}\left[\sin^{-1}x\right] = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\left[\csc^{-1}x\right] = \frac{-1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx}\left[\cos^{-1}x\right] = \frac{-1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\left[\sec^{-1}x\right] = \frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx}\left[\tan^{-1}x\right] = \frac{1}{1+x^2} \qquad \frac{d}{dx}\left[\cot^{-1}x\right] = \frac{-1}{1+x^2}$$

THEOREM 4.20 Integrals Involving Inverse Trigonometric Functions

Let u be a differentiable function of x, and let $a > 0$.

1. $\displaystyle\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin\frac{u}{a} + C$

2. $\displaystyle\int \frac{du}{a^2 + u^2} = \frac{1}{a}\arctan\frac{u}{a} + C$

3. $\displaystyle\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + C$

ex: Integrate.

a) $\int \frac{dx}{9+x^2} = \frac{1}{3}\arctan\frac{x}{3} + C$

$\int \frac{x}{9+x^2}dx$

$u = x$

$du = 1dx$

$a = 3$

check

$\frac{1}{3} \cdot \frac{\frac{1}{3}}{1+\frac{x^2}{9}}$ ✓

$\frac{1}{9\left(1+\frac{x^2}{9}\right)} = \frac{1}{9+x^2}$

ex: Integrate.

b) $\int \frac{dx}{x\sqrt{x^2-9}} = \frac{1}{3}\operatorname{arcsec}\frac{|x|}{3} + C$

$u = x$

$du = 1\,dx$

$a = 3$

ex: Integrate.

c) $\int \frac{\sin x dx}{\sqrt{25-\cos^2 x}}$ $= -\arcsin\frac{\cos x}{5} + C$

$u = \cos x$

$du = -\sin x dx$

$a = 5$

ex: Integrate.

d) $\int \frac{7dx}{16+3x^2} = 7\int \frac{dx}{16+3x^2} = \frac{7}{\sqrt{3}\,4}\arctan\frac{\sqrt{3}\,x}{4} + C$

$u = \sqrt{3}\,x$

$du = \sqrt{3}\,dx$

$a = 4$

ex: Integrate.

e) $\int \frac{x\,dx}{\sqrt{25-x^2}}$

1) Arcsin
2) u-sub
3) natural log u-sub

$\int \frac{1}{u}\,du$

$u = 25 - x^2$

$du = -2x\,dx$

$\int \frac{x\,dx}{\sqrt{25-x^4}}$

$-\frac{1}{2}\int u^{-1/2}\,du$

$-\frac{1}{2}\cdot\frac{u^{1/2}}{1/2} + C$

$-\sqrt{25-x^2} + C$

ex: Integrate.

g) $\int \frac{e^{2x}dx}{1+e^{2x}}$

$u = 1+e^{2x}$

$du = 2e^{2x}dx$

$\frac{1}{2}\ln|1+e^{2x}|+C$

Compare with:

$\frac{1}{2}\int \frac{2\cdot e^{2x}}{1+e^{4x}}dx$

$u = e^{2x}$

$du = 2e^{2x}dx$ ✱

$a = 1$

$\frac{1}{2}\arctan e^{2x} + C$

ex: Integrate.

h) $\int \frac{\sin x dx}{1+\cos^2 x}$

ex: Integrate.

k) $\int \frac{\arccos x\,dx}{\sqrt{1-x^2}} = \int \arccos x \cdot \frac{1}{\sqrt{1-x^2}}\,dx$

$u = \arccos x$

$du = \frac{-1}{\sqrt{1-x^2}}\,dx$

$-\int u\,du = \frac{-u^2}{2} + C$

$= -\frac{1}{2}(\arccos x)^2 + C$

ex: Integrate.

★ 1) $\int \frac{dx}{x^2-2x+2} = \int \frac{dx}{(x-1)^2+1} = \arctan(x-1)+C$

Complete the square

$x^2-2x+1+2-1$

$(x-1)^2+1$

$u = x-1$

$du = 1dx$

$a = 1$

ex: Integrate.

n) $\int \frac{dx}{\sqrt{-x^2-4x}}$ $=$ $\int \frac{dx}{\sqrt{4-(x+2)^2}}$

CTS

$-(x^2+4x+4)+4$

$-(x+2)^2+4$

$u = x+2$

$du = 1\,dx$

$a = 2$

$\arcsin \frac{x+2}{2} + C$

ex: Integrate.

★ p) $\int \frac{x+5}{\sqrt{9-x^2}}dx = \int \frac{x}{\sqrt{9-x^2}}dx + \int \frac{5}{\sqrt{9-x^2}}dx$

$= -\sqrt{9-x^2} + 5\arcsin\frac{x}{3} + C$

ex: Integrate.

★ q) $\int \frac{x+1}{x^2+9}dx$ = $\int \frac{x}{x^2+9}dx + \int \frac{1}{x^2+9}dx$

$\frac{1}{2}\ln|x^2+9| + \frac{1}{3}\arctan\frac{x}{3} + C$

ex: Integrate.

r) $\int \frac{2x+5}{x^2+2x+2}dx = \int \frac{2x+2}{x^2+2x+2}dx + \int \frac{5-2}{x^2+2x+2}dx$

$= \ln|x^2+2x+2| + 3\int \frac{1}{(x+1)^2+1}dx$

$= \ln|x^2+2x+2| + 3\arctan(x+1) + C$

ex: Integrate.

★ t) $\int \frac{x^3}{1+x^2}dx$ $= \int \left(x - \frac{x}{x^2+1}\right)dx$

$$\begin{array}{r} x - \frac{x}{x^2+1} \\ x^2+1 \overline{)\, x^3} \\ -x^3 - x \\ \hline -x \end{array}$$

$\frac{x^2}{2} - \frac{1}{2}\ln|x^2+1| + C$