63.) $\int x\sqrt{x+6}\,dx = \int (u-6)\sqrt{u}\,du$

$u = x+6$

$du = 1\,dx$

$u-6 = x$

$\int \left(u^{3/2} - 6u^{1/2}\right) du$

$\frac{2}{5}u^{5/2} - \frac{2\cdot 6u^{3/2}}{3} + C$

$\frac{2}{5}(x+6)^{5/2} - 4(x+6)^{3/2} + C$

73.) $2\int_1^2 x^2 \sqrt{x^3+1}\, dx$

$u = x^3 + 1$

$du = 3x^2 dx$

$\frac{2}{3}\int_2^9 u^{1/2} du = \frac{2}{3} \cdot \frac{u^{3/2}}{3/2} + c \Big|_2^9$

$\frac{4}{9} u^{3/2} + c \Big|_2^9$

$\frac{4}{9} \cdot 27 - \frac{4}{9}\sqrt{8}$

$$u = e^{3/x}$$

$$du = -\frac{1}{x^2}dx$$

$$\int e^{3/x} \cdot \frac{1}{x^2}dx$$

$$-\int e^u du$$

118) $\dfrac{1}{\frac{\pi}{4}-0}\displaystyle\int_0^{\pi/4} \sec^2 x\,(1+2\tan x)^3\,dx$

$\dfrac{1}{2}\cdot\dfrac{4}{\pi}\displaystyle\int_1^3 u^3\,du = \dfrac{2}{\pi}\cdot\dfrac{u^4}{4}+c\,\Big|_1^3$

$= \dfrac{u^4}{2\pi}+c\,\Big|_1^3$

$= \dfrac{81}{2\pi} - \dfrac{1}{2\pi}\quad \dfrac{80}{2\pi}$

4.6 The Natural Logarithmic Function: Integration

Review:

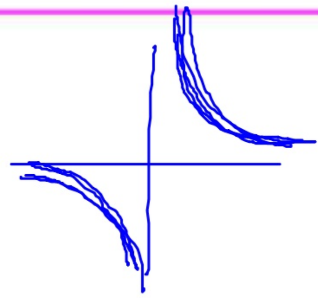
$$\frac{d}{dx}[\ln x] = \frac{1}{x}$$

$$\frac{d}{dx}[\ln|x|] = \frac{1}{x}$$

$$\frac{d}{dx}[\ln u] = \frac{1}{u} \cdot u'$$

THEOREM 4.19 Log Rule for Integration

Let u be a differentiable function of x.



1. $\displaystyle\int \frac{1}{x}\,dx = \ln|x| + C$

2. $\displaystyle\int \frac{1}{u}\,du = \ln|u| + C$

When integrating with a power of -1, you will need "ln"

Other powers will need power rule

ex: Integrate.

a) $\int \frac{6}{x}dx = 6\int \frac{1}{x}dx = 6\ln|x| + C$

Check:

$6 \cdot \frac{1}{x}$ ✓

ex: Integrate.

b) $\int \frac{1}{3x+5}\,dx$ $= \int \frac{1}{u} \cdot \frac{du}{3} = \frac{1}{3}\int \frac{1}{u}\,du$

$u = 3x+5$

$du = 3\,dx$

$\frac{du}{3} = dx$

$\frac{1}{3}\ln|3x+5| + C$

Check:

$\frac{1}{3} \cdot \frac{1}{3x+5} \cdot 3$ ✓

ex: Integrate.

c) $\int \frac{2x}{x^2+6}dx$ $= \ln|x^2+6| + C$

or

$\ln(x^2+6) + C$

Which ones will have "ln" with the answer?

On your own...

ex: Integrate.

d) $\int \frac{\sec^2 x}{\tan x} dx$

$\ln|\tan x| + C$

e) $\int \frac{\sec^2 x}{\tan^2 x} dx$

f) $\int \tan x \, dx$

g) $\int \frac{x}{\sqrt{9-x^2}} dx$

h) $\int \frac{1}{x \ln x} dx$

i) $\int \frac{x^3 + 4x^2 - 2x + 1}{x^2} dx$

ex: Integrate.

d) $\int \frac{\sec^2 x}{\tan x} dx$

ex: Integrate.

e) $\int \frac{\sec^2 x}{\tan^2 x} dx$

$$\int \frac{\frac{1}{\cos^2 x}}{\frac{\sin^2 x}{\cos^2 x}} dx$$

$$\int \csc^2 x \, dx$$

$$-\cot x + C$$

$u = \tan x$

$du = \sec^2 x \, dx$

$$\int \frac{1}{u^2} du$$

$$\int u^{-2} du$$

$$-u^{-1} + C$$

$$-\frac{1}{\tan x} + C$$

ex: Integrate.

f) $\int \tan x \, dx$

$$\int \frac{\sin x}{\cos x} dx = -\ln|\cos x| + C$$

ex: Integrate.

g) $\int \frac{x}{\sqrt{9-x^2}}dx \quad = \quad -\frac{1}{2}\int u^{-1/2}\,du$

$u = 9 - x^2$

$du = -2x\,dx$

$-\frac{1}{2} \cdot \frac{u^{1/2}}{1/2} + C$

$-\sqrt{9-x^2} + C$

ex: Integrate.

h) $\int \frac{1}{x \ln x} dx = \int \frac{1}{x} \cdot \frac{1}{\ln x} dx$

$u = \ln x$

$du = \frac{1}{x} dx$

$\int \frac{1}{u} du$

$\ln|u| + C$

$\ln|\ln x| + C$

ex: Integrate.

i) $\int \frac{x^3+4x^2-2x+1}{x^2}\,dx$

$$\int \left(x + 4 - \frac{2}{x} + x^{-2}\right) dx$$

$$\frac{x^2}{2} + 4x - 2\ln|x| + \frac{x^{-1}}{-1} + C$$

ex: Integrate.

☆ j) $\int \frac{x^2 + x + 1}{x^2 + 1} dx = \int \left(1 + \frac{x}{x^2+1}\right) dx$

$$
\begin{array}{r|l}
 & 1 + \frac{x}{x^2+1} \\
x^2 + 1 & x^2 + x + 1 \\
 & -x^2 \qquad -1 \\
\hline
 & x + 0
\end{array}
$$

$x + \frac{1}{2}\ln|x^2+1| + C$

$$\int \frac{x^2 + 1 + x}{x^2 + 1} dx = \int \frac{x^2+1}{x^2+1} dx + \int \frac{x}{x^2+1} dx$$

ex: Integrate.

k) $\int \frac{2x}{(x+1)^2}\,dx$

ex: Integrate.

When the degree of the numerator is greater than or equal to the degree of the denominator, use division

1) $\int_0^1 \frac{x-1}{x+1}dx \Longrightarrow \int_0^1 \left(1 - \frac{2}{x+1}\right)dx$

$$\begin{array}{r|rr} -1 & 1 & -1 \\ & & -1 \\ \hline & 1 & -2 \end{array}$$

$$x - 2\ln|x+1| + c \Big|_0^1$$

$$(1-2\ln 2) - (\cancel{0 - 2\ln 1})$$

$1 - 2\ln 2$ or $1 - \ln 4$

Trigonometric Antiderivatives

$$\int \sin x\, dx = -\cos x + C$$

$$\int \cos x\, dx = \sin x + C$$

$$\int \tan x\, dx = -\ln|\cos x| + C$$

$$\int \csc x\, dx = -\ln|\csc x + \cot x| + C$$

$$\int \sec x\, dx = \ln|\sec x + \tan x| + C$$

$$\int \cot x\, dx = \ln|\sin x| + C$$

ex: Integrate.

a) $\int \tan 5x\, dx = \frac{1}{5}\int \tan u\, du$

$u = 5x$

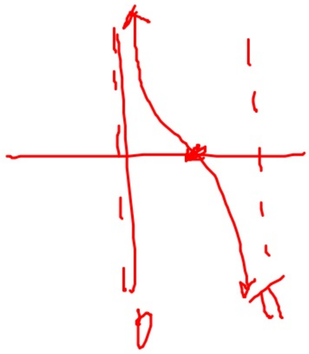
$du = 5dx$

$\frac{1}{5}\left(-\ln|\cos u|\right) + C$

$-\frac{1}{5}\ln|\cos 5x| + C$

ex: Integrate.

b) $\int_{\pi/4}^{\pi/2} \sqrt{\csc^2 x - 1}\,dx \quad = \int_{\pi/4}^{\pi/2} \sqrt{\cot^2 x}\,dx$



$\int_{\pi/4}^{\pi/2} |\cot x|\,dx$

$= \ln|\sin x| + C \Big|_{\pi/4}^{\pi/2}$

$-1\ln 3$

$\ln 3^{-1}$

$(\ 0\ ) - (\ \ln \frac{\sqrt{2}}{2}\ )$

$\ln \frac{2}{\sqrt{2}} = \ln\sqrt{2}$

ex:

Evaluate $\int_0^{\frac{\pi}{4}} \frac{2e^{\tan x}+5}{\cos^2 x}\,dx$

(A) $2e+3$ (B) $2e$ (C) $2e-3$ (D) e (E) $e+5$

ex:

Evaluate $\displaystyle\int_{e}^{e^4} \frac{5}{x\sqrt{\ln x}}\,dx$

(A) 6 (B) 7 (C) 8 (D) 9 (E) 10

FR 7

A particle starts at the point $(5, 0)$ at $t = 0$ and moves along the x-axis in such a way that at time $t > 0$ its velocity $v(t)$ is given by $v(t) = \frac{t}{1+t^2}$.

(a) Determine the maximum velocity attained by the particle. Justify your answer.

(b) Determine the position of the particle at $t = 6$.

(c) Find the limiting value of the velocity as t increases without bound.

FR 21

t (sec)	0	15	25	30	35	50	60
$v(t)$ (ft/sec)	−20	−30	−20	−14	−10	0	10
$a(t)$ $\left(\text{ft}/\text{sec}^2\right)$	1	5	2	1	2	4	2

A car travels on a straight track. During the time interval $0 \le t \le 60$ seconds, the car's velocity v, measured in feet per second, and acceleration a, measured in feet per second per second, are continuous functions. The table above shows selected values of these functions.

(a) Using appropriate units, explain the meaning of $\int_{30}^{60} |v(t)|\, dt$ in terms of the car's motion. Approximate $\int_{30}^{60} |v(t)|\, dt$ using a trapezoidal approximation with the three subintervals determined by the table.

(b) Using appropriate units, explain the meaning of $\int_{0}^{30} a(t)\, dt$ in terms of the car's motion. Find the exact value of $\int_{0}^{30} a(t)\, dt$.