47.) $\int e^{\sin \pi x} \cos \pi x \, dx$

$u = \sin \pi x$

$du = \pi \cos \pi \, dx$

$\frac{1}{\pi} \int e^{u} \, du$

$\frac{1}{\pi} e^{\sin \pi x} + C$

51.) $\int 3^{x/2}\,dx = \int 3^u \cdot 2\,du$

$u = \frac{1}{2}x$

$du = \frac{1}{2}dx$

$2du = dx$

$= 2\int 3^u\,du$

$2 \cdot \frac{1}{\ln 3} \cdot 3^u + C$

$\frac{2}{\ln 3} \cdot 3^{x/2} + C$

39.) $\int \frac{\csc^2 x}{\cot^3 x}\,dx = -\int \frac{1}{u^3}\,du$

$u = \cot x$

$du = -\csc^2 x\,dx$

$-\csc^2 x\,du = dx$

$-\int u^{-3}\,du$

$-\frac{u^{-2}}{-2} + C$

$\frac{1}{2}(\cot x)^{-2} + C$

2 3.) $\int \left(1+\frac{1}{t}\right)^3 \left(\frac{1}{t^2}\right) dt = \int u^3 \left(\frac{1}{t^2}\right)\left(-t^2 du\right)$

$u = 1 + \frac{1}{t}$

$du = \left(0 - \frac{1}{t^2}\right) dt$

$-t^2 du = dt$

$-\int u^3 du$

$-\frac{1}{4}\left(1+\frac{1}{t}\right)^4 + C$

37.) $\int \sin 2x \cos 2x \, dx = \frac{1}{2}\int u^1 \, du$

$u = \sin 2x$

$du = 2\cos 2x \, dx$

$\frac{1}{2} \cdot \frac{u^2}{2} + C$

$\frac{1}{4}(\sin^2 2x) + C$

4.5 U-Substitution - cont.

ex: Evaluate.

a) $\int_0^1 x\left(x^2+1\right)^3 dx = \frac{1}{2}\int u^3\,du = \frac{1}{2}\cdot\frac{u^4}{4}$

$u = x^2+1$

$du = 2x\,dx$

$\frac{1}{8}(x^2+1)^4 + C\Big|_0^1$

$\frac{1}{8}\left(2^4 - 1^4\right)$

$\frac{15}{8}$

ALTERNATE WAY

ex: Evaluate.

a) $\int_0^1 x\left(x^2+1\right)^3 dx = \frac{1}{2}\int_1^2 u^3 du = \frac{1}{2}\cdot\frac{u^4}{4}\Big|_1^2$

$\frac{1}{8}\left(2^4-1^4\right)$

$\frac{15}{8}$

$u = x^2+1$

$du = 2x\,dx$

lower	upper
$x=0$	$x=1$
$u=x^2+1$	$u=x^2+1$
$u=1$	$u=2$

ex: Evaluate.

b) $\int_0^{\pi/2} \cos\left(\frac{2x}{3}\right)dx = \frac{3}{2}\int \cos u \, du = \frac{3}{2}\sin\left(\frac{2x}{3}\right)+C\Big|_0^{\pi/2}$

$u = \frac{2x}{3}$

$du = \frac{2}{3}dx$

$\frac{3}{2}\left(\frac{\sqrt{3}}{2} - 0\right)$

$\frac{3\sqrt{3}}{4}$

ALTERNATE WAY

ex: Evaluate.

b) $\int_{0}^{\pi/2} \cos\left(\frac{2x}{3}\right)dx = \frac{3}{2}\int_{0}^{\pi/3} \cos u\,du = \frac{3}{2}\sin u \Big|_{0}^{\pi/3}$

$\frac{3}{2}\left(\frac{\sqrt{3}}{2} - 0\right)$

ex: Evaluate.

c) $\displaystyle\int_0^1 \frac{e^{\tan^{-1} x}}{1+x^2}\,dx = \int_0^1 e^{\tan^{-1} x}\left(\frac{1}{1+x^2}\right)dx$

$u = \tan^{-1} x$

$du = \frac{1}{x^2+1}\,dx$

lower	upper
$x=0$	$x=1$
$u=\tan^{-1} x$	$u=\tan^{-1} x$
$u=0$	$u=\frac{\pi}{4}$

$$\int_0^{\pi/4} e^u\,du = e^u \Big|_0^{\pi/4}$$

$$= e^{\pi/4} - 1$$

ex: Rewrite the definite integral in terms of u.

a) Let $u = 3x - 2$; $\int_0^1 (3x-2)^3 \, dx$

$du = 3dx$

lower	upper
$x = 0$	$x = 1$
$u = 3x - 2$	$u = 3x - 2$
$u = -2$	$u = 1$

$\frac{1}{3} \int_{-2}^{1} u^3 \, du$

ex: Rewrite the definite integral in terms of u.

b) Let $u = 2x + 3$; $\int_{-1}^{1} \frac{1}{2x+3} dx$

$\frac{1}{2}\int_{1}^{5} \frac{1}{u} du$

ex: Rewrite the definite integral in terms of u.

c) Let $u = \sqrt{x+1}$; $\int_0^3 \frac{dx}{x\sqrt{x+1}}$ $= \int_1^2 \frac{2u\,du}{(u^2-1)u}$

$u^2 = x+1$

$u^2 - 1 = x$

$2u\,du = dx$

$\int_1^2 \frac{2}{u^2-1}\,du$

ex: Rewrite the definite integral in terms of u.

4 d) Let $u = 4 - x^2$; $\int_0^{\sqrt{3}} \frac{x\,dx}{\sqrt{4-x^2}}$ $= -\frac{1}{2}\int_4^1 \frac{1}{\sqrt{u}}\,du$

$u = 4 - x^2$

$du = -2x\,dx$

$\frac{1}{2}\int_1^4 \frac{1}{\sqrt{u}}\,du$

ex:

Using the substitution $u = \sqrt{x}$, $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$ is equal to which of the following?

(A) $\cancel{2\int_1^{16} e^u\,du}$ (B) $\cancel{2\int_1^{4} e^u\,du}$ (C) $2\int_1^{2} e^u\,du$ (D) $\frac{1}{2}\int_1^{2} e^u\,du$ (E) $\cancel{\int_1^{4} e^u\,du}$

$u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}\,dx$

$2\sqrt{x}\,du = dx$

FR 18

The function f is defined by $f(x) = \sqrt{25 - x^2}$ for $-5 \le x \le 5$.

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(a) Find $f'(x)$. $f'(x) = \frac{1}{2}(25 - x^2)^{-1/2}(-2x) = \frac{-x}{\sqrt{25 - x^2}}$

(b) Write an equation for the line tangent to the graph of f at $x = -3$. $(-3, 4)$ $m = 3/4$

$y - 4 = \frac{3}{4}(x + 3)$

(c) Let g be the function defined by $g(x) = \begin{cases} f(x) & \text{for } -5 \le x \le -3 \\ x + 7 & \text{for } -3 < x \le 5. \end{cases}$

Is g continuous at $x = -3$? Use the definition of continuity to explain your answer.

Yes $\lim_{x \to -3^-} g(x) = \lim_{x \to -3^+} g(x) = g(-3)$

$4 = 4 = 4$

(d) Find the value of $\int_0^5 x\sqrt{25 - x^2}\, dx$.

$u = 25 - x^2$

$du = -2x\, dx$

$-\frac{1}{2}\int_{25}^{0} \sqrt{u}\, du = -\frac{1}{2} \cdot \frac{u^{3/2}}{3/2} = -\frac{1}{3}\left(u^{3/2}\right)\Big|_{25}^{0}$

$-\frac{1}{3}(0 - 125) = \frac{125}{3}$

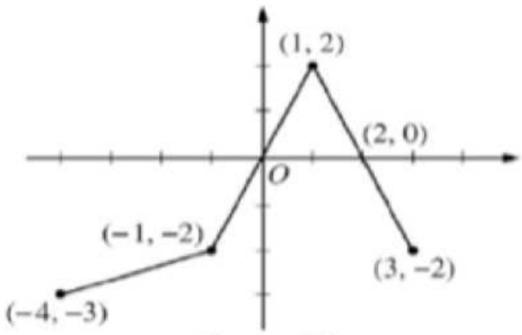
FR 15

The graph of the function f above consists of three line segments.

(a) Let g be the function given by $g(x) = \int_{-4}^{x} f(t)\,dt$.

For each of $g(-1)$, $g'(-1)$, and $g''(-1)$, find the value or state that it does not exist.

(b) For the function g defined in part (a), find the x-coordinate of each point of inflection of the graph of g on the open interval $-4 < x < 3$. Explain your reasoning.



Graph of f

(c) Let h be the function given by $h(x) = \int_{x}^{3} f(t)\,dt$. Find all values of x in the closed interval $-4 \le x \le 3$ for which $h(x) = 0$.

(d) For the function h defined in part (c), find all intervals on which h is decreasing. Explain your reasoning.

21. Calculator

The temperature, in degrees Celsius (°C), of the water in a pond is a differentiable function W of time t. The table above shows the water temperature as recorded every 3 days over a 15-day period.

t (days)	$W(t)$ (°C)
0	20
3	31
6	28
9	24
12	22
15	21

(a) Use data from the table to find an approximation for $W'(12)$. Show the computations that lead to your answer. Indicate units of measure.

(b) Approximate the average temperature, in degrees Celsius, of the water over the time interval $0 \le t \le 15$ days by using a trapezoidal approximation with subintervals of length $\Delta t = 3$ days.

(c) A student proposes the function P, given by $P(t) = 20 + 10te^{(-t/3)}$, as a model for the temperature of the water in the pond at time t, where t is measured in days and $P(t)$ is measured in degrees Celsius. Find $P'(12)$. Using appropriate units, explain the meaning of your answer in terms of water temperature.

(d) Use the function P defined in part (c) to find the average value, in degrees Celsius, of $P(t)$ over the time interval $0 \le t \le 15$ days.

FR 10

The acceleration of a particle moving along a straight line is given by $a = 10e^{2t}$.

(a) Write an expression for the velocity v, in terms of time t, if $v = 5$ when $t = 0$.

(b) During the time that the velocity increases from 5 to 15, how far does the particle travel?

(c) Write an expression for the position s, in terms of time t, of the particle if $s = 0$ when $t = 0$.