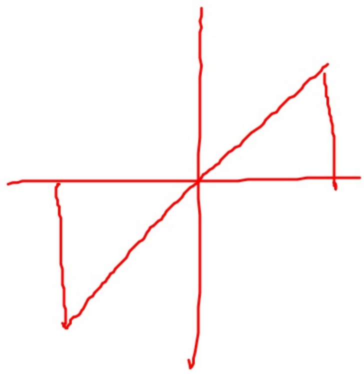
d.)

x	$F(x)$
-2	0
0	-2
4	
6	$\pi + 4$



$$F(x) = \int_2^x f(t)\,dt$$

$$F(6) = \int_2^6 f(t)\,dt$$

$v(t) = t^2 + 1$

Average velocity $[1, 3]$

$$\frac{1}{3-1} \int_1^3 (t^2 + 1)\, dt$$

4.5 U-Substitution

U-Substitution is an integration technique used when an integrand involves a composite function.

$$\boxed{f(g(x))}$$

Review:

$$\frac{d}{dx}\left[f(g(x))\right] = \underbrace{f'(g(x))}\cdot\underbrace{g'(x)}$$

THEOREM 4.15 Antidifferentiation of a Composite Function

Let g be a function whose range is an interval I, and let f be a function that is continuous on I. If g is differentiable on its domain and F is an antiderivative of f on I, then

$$\int f'(g(x))g'(x)dx = f(g(x)) + C$$

Letting $u = g(x)$ gives $du = g'(x)\,dx$ and

$$\int f'(f(u))du = f(u) + C$$

ex: Identify g(x) and g'(x).

a) $\int 2x\left(x^2+1\right)^4 dx$ $\quad g(x)=x^2+1 \quad g'(x)=2x$

b) $\int 5\sin(5x)\,dx$ $\quad g(x)=5x \quad g'(x)=5$

c) $\int \frac{\sec^2 x}{\tan x}dx$ $\quad g(x)=\tan x \quad g'(x)=\sec^2 x$

$\int \sec^2 x\,dx = \tan x + C$

ex: Evaluate.

b) $\int 4\cos(4x-3)\,dx = \int \cos u\,du$

$u = 4x-3$

$du = 4dx$

$= \sin u + C$

$\int x^2 dx = \frac{x^3}{3} + C$

$= \sin(4x-3) + C$

ex: Evaluate.

d) $\int \tan^2 x \sec^2 x\, dx = \int (\tan x)^2 \sec^2 x\, dx$

$u = \tan x$

$du = \sec^2 x\, dx$

$\int u^2\, du = \frac{1}{3}u^3 + C$

$= \frac{1}{3}(\tan x)^3 + C$

$\frac{1}{3}\tan^3 x + C$

ex: Evaluate.

e) $\int x(3x^2-5)^{12}\,dx = \int x u^{12} \cdot \frac{du}{6x} = \frac{1}{6}\int u^{12}\,du$

$u = 3x^2 - 5$

$du = 6x\,dx$

$\frac{du}{6x} = dx$

$\frac{1}{6} \cdot \frac{u^{13}}{13} + C$

$\frac{(3x^2-5)^{13}}{78} + C$

ex: Evaluate.

f) $\int 5\sin(7x)\,dx = \int 5\sin u \frac{du}{7} = \frac{5}{7}\int \sin u\,du$

$u = 7x$

$du = 7dx$

$\frac{du}{7} = dx$

$-\frac{5}{7}\cos(7x) + C$

ex: Evaluate.

g) $\int (x+1)e^{x^2+2x}\,dx$ $= \int \cancel{(x+1)}e^{u}\frac{du}{2\cancel{(x+1)}}$

$u = x^2 + 2x$

$du = (2x+2)dx$

$du = 2(x+1)dx$

$\frac{du}{2(x+1)} = dx$

$\frac{1}{2}\int e^{u}\,du$

$\frac{1}{2}e^{u} + C$

$\frac{1}{2}e^{x^2+2x} + C$

ex: Evaluate.

h) $\int \left(x^2 - 3 + \sec^2(5x)\right) dx = \int x^2 dx - \int 3 dx + \int \sec^2(5x) dx$

$\frac{1}{3}x^3 - 3x + \frac{1}{5}\tan(5x) + C$

ex: Evaluate.

i) $\int \frac{e^{2x} + 2e^{x} + 5}{e^{x}} dx = \int (e^{x} + 2 + 5e^{-x}) dx$

$u = -x$
$du = -1dx$
$\frac{du}{-1} = dx$

$y = e^{-x}$

$= e^{x} + 2x + 5\int e^{u} \frac{du}{-1}$

$y' = -e^{-x}$

$e^{x} + 2x - 5e^{u} + C$

$\int e^{-x} dx = -e^{-x} + C$

$e^{x} + 2x - 5e^{-x} + C$

ex: Evaluate.

k) $\int (\cos 2x - \sin 3x)\, dx$

$\frac{1}{2}\sin(2x) + \frac{1}{3}\cos(3x) + C$

ex: Evaluate.

1) $\int \frac{x}{\sqrt[3]{1-2x^2}}\,dx = -\frac{1}{4}\int u^{-1/3}\,du = -\frac{1}{4}\cdot\frac{u^{2/3}}{2/3} + C$

$= -\frac{3}{8}(1-2x^2)^{2/3} + C$

$u = 1-2x^2$

$du = -4x\,dx$

$\frac{du}{-4x} = dx$

ex: Evaluate.

m) $\int \cos^4(x)\sin(x)\,dx$ $= \int (\cos x)^4 \sin x \, dx$

$$= \frac{-\cos^5 x}{5} + C$$

ex: Evaluate.

o) $\int \sin^2(x)\,dx$

$\cos 2x = 1 - 2\sin^2 x$

$\frac{\cos 2x - 1}{-2} = \sin^2 x$

$-\frac{1}{2}\int (\cos(2x) - 1)\,dx$

$-\frac{1}{2}(\cos 2x - 1)$

$-\frac{1}{2}\left(\frac{\sin 2x}{2} - x\right) + C$

$-\frac{1}{4}\sin 2x + \frac{1}{2}x + C$

ex: Evaluate.

q) $\int x\sqrt{x+1}\,dx$ $= \int (u-1)\sqrt{u}\,du$

$u = x+1$

$du = 1\,dx$

$u-1 = x$

$\int \left(u^{3/2} - u^{1/2}\right) du$

$\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2} + C$

$\frac{2}{5}(x+1)^{5/2} - \frac{2}{3}(x+1)^{3/2} + C$

ex:

$\int e^x \cos\left(e^x + 1\right) dx =$

(A) $\sin\left(e^x + 1\right) + C$

(B) $e^x \sin\left(e^x + 1\right) + C$

(C) $e^x \sin\left(e^x + x\right) + C$

(D) $\frac{1}{2}\cos^2\left(e^x + 1\right) + C$

FR 1

A particle moves along the x-axis in such a way that its acceleration at time t for $t \geq 0$ is given by $a(t) = 4\cos(2t)$. At time $t = 0$, the velocity of the particle is $v(0) = 1$ and its position is $x(0) = 0$.

(a) Write an equation for the velocity $v(t)$ of the particle. $v(t) = 2\sin 2t + 1$

(b) Write an equation for the position $x(t)$ of the particle. $x(t) = -\cos 2t + t + C$

(c) For what values of t, $0 \leq t \leq \pi$, is the particle at rest?

$0 = -1 + 0 + C$

$1 = C$

$x(t) = -\cos 2t + t + 1$