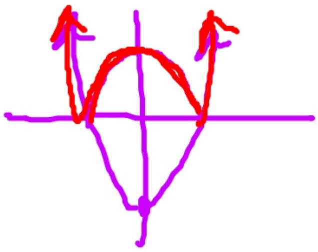
25.) $f(x) = |x^2 - 9|$



$$f(x) = \begin{cases} -x^2 + 9 & x \le 3 \\ x^2 - 9 & x > 3 \end{cases}$$

$$\int_0^4 f(x)\,dx = \int_0^3 (-x^2+9)\,dx + \int_3^4 (x^2-9)\,dx$$

e.) $\displaystyle\int_0^6 (2 + f(x))\,dx$

$$\int_0^6 2\,dx + \int_0^6 f(x)\,dx$$

$$12 + 3.5$$

19) $\frac{1}{3}\int_0^1 (x-\sqrt{x})\,dx = \frac{1}{3}\left(\frac{x^2}{2} - \frac{2x^{3/2}}{3}\right)\Big|_0^1$

$= \frac{1}{3}\left(\frac{1}{2} - \frac{2}{3}\right)$

4.4 The FUNdamental Theorem Of Calculus - cont.

ex:

Let h be the function defined by $h(x)=\frac{1}{\sqrt{x^5+1}}$. If g is an antiderivative of h and $g(2)=3$, what is the value of $g(4)$?

(A) –0.020

(B) 0.152

(C) 3.031

(D) 3.152

$$g(4) = 3 + \int_2^4 h(x)\,dx$$

For HW # 55 - 59 odd, just find average value.

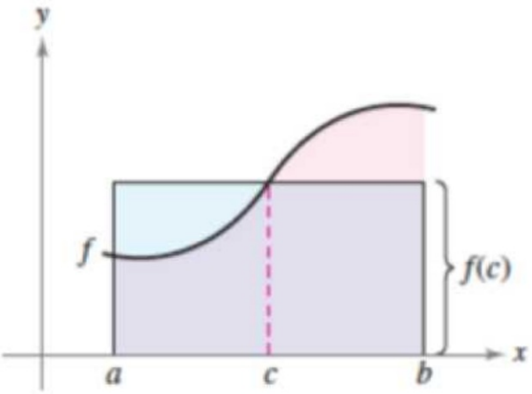
4.4 The FUNdamental Theorem Of Calculus - cont.

- Mean Value Theorem For Integrals

THEOREM 4.12 Mean Value Theorem for Integrals

If f is continuous on the closed interval $[a, b]$, then there exists a number c in the closed interval $[a, b]$ such that

$$\int_a^b f(x)\,dx = f(c)(b - a).$$



The Mean Value Theorem for Integrals states that somewhere "between" the inscribed and circumscribed rectangles there is a rectangle whose area is precisely equal to the area of the region under the curve.

- Average Value

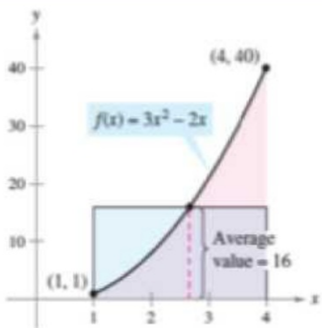
The value f(c) given in the Mean Value Theorem for Integrals is called the <u>average value</u> of f on the interval [a, b].

Definition of the Average Value of a Function on an Interval

If f is integrable on the closed interval $[a, b]$, then the **average value** of f on the interval is

$$\boxed{\frac{1}{b-a}\int_a^b f(x)\,dx.}$$

See Figure 4.36.



Average Value is not to be confused with Average Rate of Change!!!!

Average Value of f(x) on [a, b]:

$$\frac{1}{b-a}\int_a^b f(x)\,dx$$

Averate Rate of Change of f(x) on [a,b]:

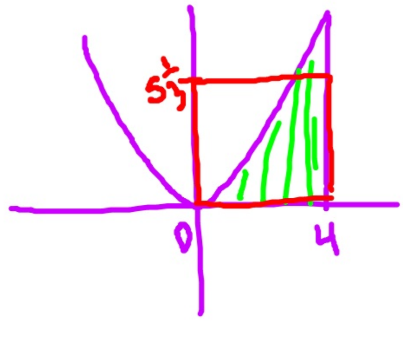
$$\frac{f(b)-f(a)}{b-a}$$

$$\frac{1}{b-a}\int_a^b f'(x)\,dx = \frac{f(b)-f(a)}{b-a}$$

ex: Find the average value of f(x) on the given interval

a) $f(x) = x^2, \quad [0,4]$

$$\frac{1}{4-0}\int_0^4 x^2\,dx = \frac{1}{4}\cdot\frac{x^3}{3}\Bigg|_0^4 = \frac{1}{4}\cdot\frac{64}{3} = \frac{16}{3}$$



ex: Find the average value of f(x) on the given interval

b) $f(x)=\begin{cases} x^2-3, & x<3 \\ x+3, & x\geq 3 \end{cases}$ $[0,5]$

$$\frac{1}{3-0}\int_0^3 (x^2-3)\,dx + \frac{1}{5-3}\int_3^5 (x+3)\,dx$$

- Accumulation Functions

The Definite Integral as a Number

Constant

$$\int_a^b f(x)\,dx$$

Constant

f is a function of x.

The Definite Integral as a Function of x

F is a function of x.

$$F(x) = \int_a^x f(t)\,dt$$

Constant

f is a function of t.

In general, an accumulation function comes in the form:

$$F(x) = \int_a^{g(x)} f(t)\,dt$$

- The 2nd FUNdamental Theorem of Calculus

THEOREM 4.13 **The Second Fundamental Theorem of Calculus**

If f is continuous on an open interval I containing a, then, for every x in the interval,

$$\frac{d}{dx}\left[\int_a^{g(x)} f(t)\,dt\right] = f(g(x)) \cdot g'(x)$$

*The 2nd FUNdamental Theorem of Calculus is used to DIFFERENTIATE an accumulation function.

$$F(x) = \int_1^x 2t\,dt$$

$$F(x) = t^2 \Big|_1^x$$

$$F(x) = x^2 - 1$$

$$F'(x) = 2x$$

$$F(x) = \int_1^x \ln t\,dt$$

$$F'(x) = \ln x$$

$$F(x) = \int_2^{x^2} 3t^2\,dt$$

$$F(x) = t^3 \Big|_2^{x^2}$$

$$F(x) = x^6 - 8$$

$$F'(x) = 6x^5$$

$$6x^5 = 3x^4 \cdot 2x$$

$$F(x) = \int_3^{\sin x} \ln t\,dt$$

$$F'(x) = (\ln(\sin x))(\cos x)$$

ex: Find the derivative.

a) $y = \int_{7}^{3x^2} \tan t\,dt$; $y' = \tan(3x^2)\cdot 6x$

b) $y = \int_{10}^{\tan x} \ln t\,dt$ $y' = \ln(\tan x)\cdot \sec^2 x$

ex: Find the derivative.

c) $y = \int_{2}^{\sqrt{x}} \left(t^4 - t^2 + 3\right) dt$

$\sqrt{(-3)^2}$ $\sqrt{3^2}$

d) $y = \int_{x^2}^{5} \sqrt{t}\, dt$

$y = -\int_{5}^{x^2} \sqrt{t}\, dt$; $y' = -\sqrt{x^2} \cdot 2x = -|x| \cdot 2x$

ex: If $f(x)=\int_{1}^{x}\frac{t^4+1}{t}dt$ find $f''(2)$.

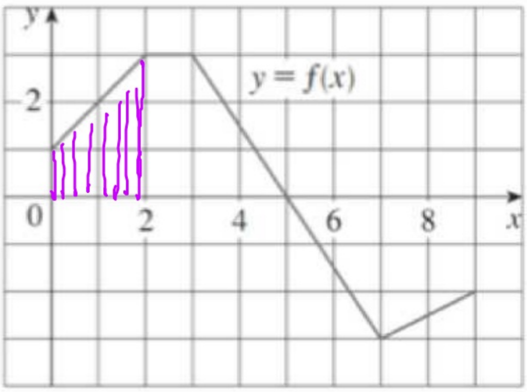
$$f'(x)=\frac{x^4+1}{x}=x^3+x^{-1}$$

$$f''(x)=3x^2-\frac{1}{x^2}$$

$$f''(2)=12-\frac{1}{4}$$
$$=11\frac{3}{4}$$

4.4 Notes WKST

The graph of $f(x)$ is shown below. If $\boxed{g(x)=\int_2^x f(t)\,dt}$, evaluate the following or explain why they do not exist.



a) $g(0)$

$$g(x)=\int_2^x f(t)\,dt$$

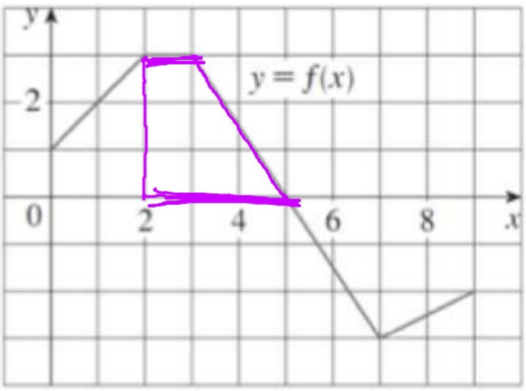
$$g(0)=\int_2^0 f(t)\,dt$$

$$=-\int_0^2 f(t)\,dt$$

$$=-4$$

*See printout.

The graph of $f(x)$ is shown below. If $g(x)=\int_2^x f(t)\,dt$, evaluate the following or explain why they do not exist.



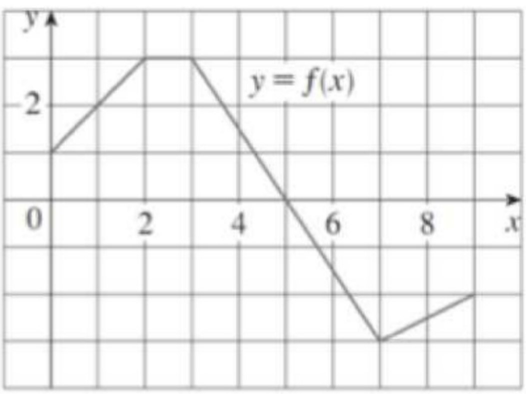
b) $g(5) = \int_2^5 f(t)\,dt$

$\frac{1}{2}(3)(3+1)$

6

The graph of $f(x)$ is shown below. If $g(x) = \int_2^x f(t)\,dt$, evaluate the following or explain why they do not exist.

c) $g'(5)$

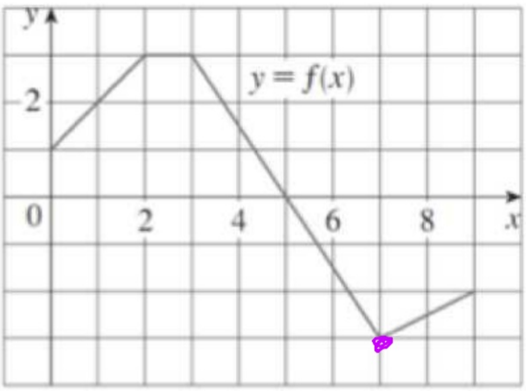


$g'(x) = f(x)$

$g'(5) = f(5)$

$= 0$

The graph of $f(x)$ is shown below. If $g(x) = \int_2^x f(t)\,dt$, evaluate the following or explain why they do not exist.



d) $g'(7)$

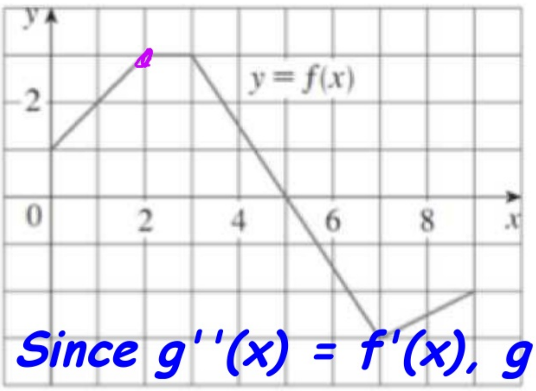
$g'(x) = f(x)$

$g'(7) = f(7)$

$= -3$

The graph of $f(x)$ is shown below. If $g(x) = \int_2^x f(t)\,dt$, evaluate the following or explain why they do not exist.

e) $g''(2)$



$g'(x) = f(x)$

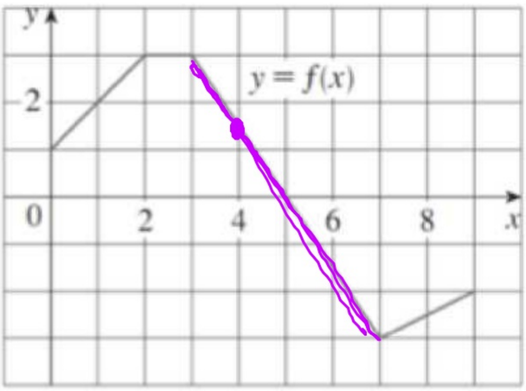
$g''(x) = f'(x)$

$g''(2) = f'(2)$

Since g''(x) = f'(x), g''(2) does not exist (f(x) is not differentiable at x = 2)

The graph of $f(x)$ is shown below. If $g(x)=\int_2^x f(t)dt$, evaluate the following or explain why they do not exist.

f) $g''(4)$

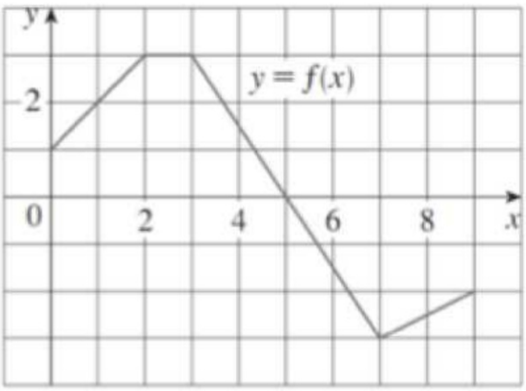


$g''(x) = f'(x)$

$g''(4) = f'(4)$

$= \frac{-3}{2}$

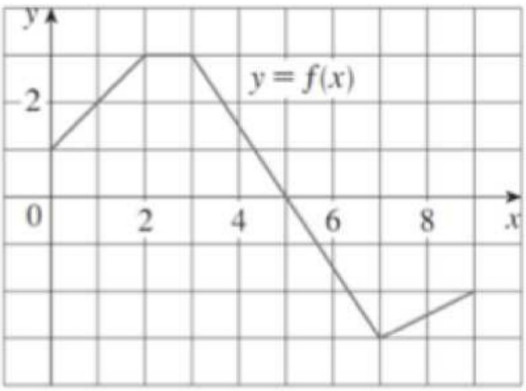
The graph of $f(x)$ is shown below. If $g(x)=\int_2^x f(t)dt$, evaluate the following or explain why they do not exist.



g) On what interval does $g(x)$ increase? Justify your answer

Since g'(x) = f(x), g(x) is increasing on (0,5) because g' (or f) is > 0 on this interval

The graph of $f(x)$ is shown below. If $g(x)=\int_2^x f(t)dt$, evaluate the following or explain why they do not exist.



h) At what x-value(s) does $g(x)$ have a point of inflection? Justify your answer.

Since $g'(x) = f(x)$, there is a POI at = 7 because $f(x)$ has a relative extrema at $x = 7$

ex:

If $f(x)=\int_1^{x^3}\frac{1}{1+\ln t}\,dt$ for $x\geq 1$, then $f'(2)=$

$f'(x)=\frac{1}{1+\ln x^3}\cdot 3x^2$

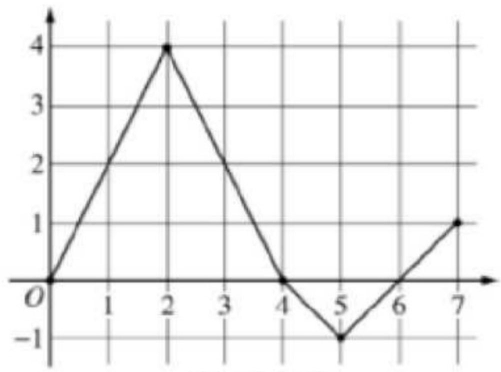
(A) $\frac{1}{1+\ln 2}$

(B) $\frac{12}{1+\ln 2}$

(C) $\frac{1}{1+\ln 8}$

(D) $\frac{12}{1+\ln 8}$ *

FR 16



Graph of f

Let f be a function defined on the closed interval $[0, 7]$. The graph of f, consisting of four line segments, is shown above. Let g be the function given by $g(x) = \int_2^x f(t)\, dt$.

(a) Find $g(3)$, $g'(3)$, and $g''(3)$.

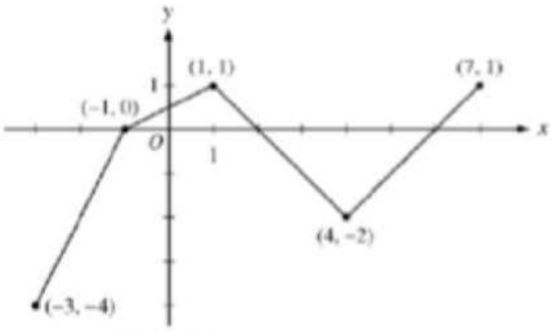
(b) Find the average rate of change of g on the interval $0 \le x \le 3$.

(c) For how many values c, where $0 < c < 3$, is $g'(c)$ equal to the average rate found in part (b) ? Explain your reasoning.

(d) Find the x-coordinate of each point of inflection of the graph of g on the interval $0 < x < 7$. Justify your answer.

FR 13

Let g be a continuous function with $g(2) = 5$. The graph of the piecewise-linear function g', the derivative of g, is shown above for $-3 \le x \le 7$.



Graph of g'

(a) Find the x-coordinate of all points of inflection of the graph of $y = g(x)$ for $-3 < x < 7$. Justify your answer.

(b) Find the absolute maximum value of g on the interval $-3 \le x \le 7$. Justify your answer.

(c) Find the average rate of change of $g(x)$ on the interval $-3 \le x \le 7$.

(d) Find the average rate of change of $g'(x)$ on the interval $-3 \le x \le 7$. Does the Mean Value Theorem applied on the interval $-3 \le x \le 7$ guarantee a value of c, for $-3 < c < 7$, such that $g''(c)$ is equal to this average rate of change? Why or why not?

FR 20

A test plane flies in a straight line with positive velocity $v(t)$, in miles per minute at time t minutes, where v is a differentiable function of t. Selected values of $v(t)$ for $0 \le t \le 40$ are shown in the table above.

t (min)	0	5	10	15	20	25	30	35	40
$v(t)$ (mpm)	7.0	9.2	9.5	7.0	4.5	2.4	2.4	4.3	7.3

(c) The function f, defined by $f(t) = 6 + \cos\left(\frac{t}{10}\right) + 3\sin\left(\frac{7t}{40}\right)$, is used to model the velocity of the plane, in miles per minute, for $0 \le t \le 40$. According to this model, what is the acceleration of the plane at $t = 23$? Indicates units of measure.

(d) According to the model f, given in part (c), what is the average velocity of the plane, in miles per minute, over the time interval $0 \le t \le 40$?