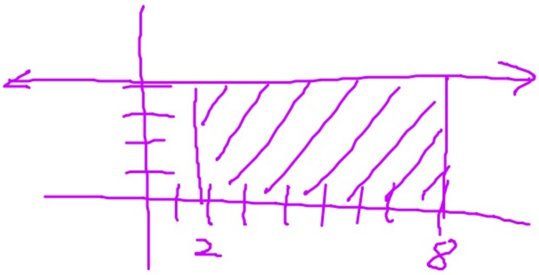
4.4 The FUNdamental Theorem Of Calculus

THEOREM 4.11 The Fundamental Theorem of Calculus

If a function f is continuous on the closed interval $[a, b]$ and F is an antiderivative of f on the interval $[a, b]$, then

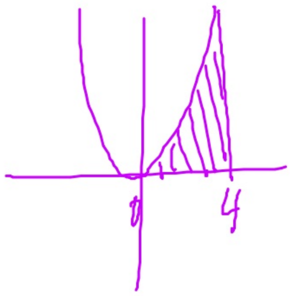
$$\int_a^b f(x)\,dx = F(b) - F(a)$$



$f(x) = 4 \quad [2, 8]$

ex: Evaluate.

a) $\int_0^4 x^2 dx = \frac{1}{3}x^3 + C \Big]_0^4$



$= \left(\frac{1}{3}\cdot 4^3 + C\right) - \left(\frac{1}{3}(0)^3 + C\right)$

$\frac{64}{3}$

ex: Evaluate.

b) $\int_1^4 \frac{3}{\sqrt{x}}dx = 3\int_1^4 x^{-1/2}dx = \frac{3 \cdot x^{1/2}}{1/2} + C$

$= 6x^{1/2} + C \Big]_1^4$

$= (6 \cdot \sqrt{4} + C) - (6\sqrt{1} + C)$

6

ex: Evaluate.

$$\frac{\sin^2 x+\cos^2 x}{\sin^2 x}=\frac{1}{\sin^2 x}$$

$$1+\cot^2 x=\csc^2 x$$

c) $\displaystyle\int_{\pi/4}^{\pi/3}(\underbrace{\cot^2 x+1}_{\csc^2 x})dx$

$$= -\cot x + C \Big]_{\pi/4}^{\pi/3}$$

$$-\left[\cot\frac{\pi}{3}-\cot\frac{\pi}{4}\right]=-\left[\frac{\sqrt{3}}{3}-1\right]=-\frac{\sqrt{3}}{3}+1$$

$$=\frac{-\sqrt{3}+3}{3}$$

ex: Evaluate.

d) $\int_0^2 f(x)dx$ if $f(x)=\begin{cases} x^4, & x<1 \\ x^5, & x\geq 1 \end{cases}$

$$\int_0^1 x^4 dx + \int_1^2 x^5 dx = \frac{x^5}{5}\Big]_0^1 + \frac{x^6}{6}\Big]_1^2$$

$$= \left(\frac{1}{5} - 0\right) + \left(\frac{64}{6} - \frac{1}{6}\right)$$

$$\frac{1}{5} + \frac{64}{6} - \frac{1}{6}$$

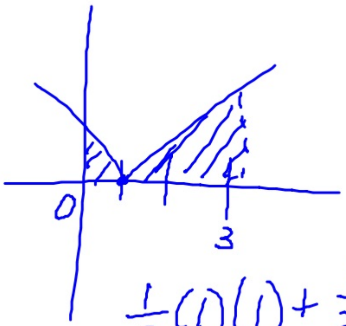
$$\frac{1}{5} + \frac{63}{6}$$

$$\frac{6 + 315}{30}$$

$$\frac{321}{30}$$

ex: Evaluate.

e) $\int_0^3 |x-1|dx = \int_0^1 (-x+1)dx + \int_1^3 (x-1)dx$



$\frac{1}{2}(1)(1) + \frac{1}{2}(2)(2)$

$\frac{1}{2} + \frac{4}{2} = \frac{5}{2}$

ex: Evaluate.

f) $\int_{0}^{3} \sqrt{9-x^2}\,dx$

Chocolate-Studded Dream Cookies

"The next part of this recipe will involve some calculus."

*See printout.

- Average Value

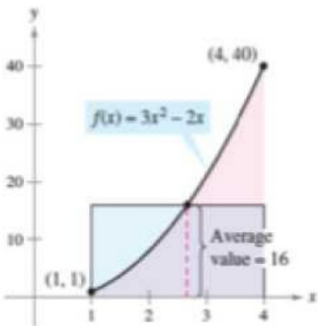
The value f(c) given in the Mean Value Theorem for Integrals is called the <u>average value</u> of f on the interval [a, b].

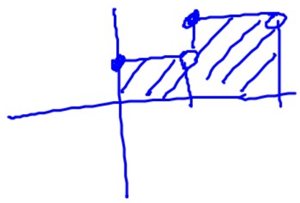
Definition of the Average Value of a Function on an Interval

If f is integrable on the closed interval $[a, b]$, then the **average value** of f on the interval is

$$\frac{1}{b-a}\int_a^b f(x)\,dx.$$

See Figure 4.36.





Average Value is not to be confused with Average Rate of Change!!!!

Average Value of f(x) on [a, b]:

$$\frac{1}{b-a}\int_a^b f(x)\,dx$$

Average Rate of Change of f(x) on [a,b]:

$$\frac{f(b)-f(a)}{b-a}$$

ex: Find the average value of f(x) on the given interval

a) $f(x) = x^2, \quad [0,4]$

$$\frac{1}{4-0}\int_0^4 x^2\,dx = \frac{1}{4}\cdot\frac{x^3}{3}\Bigg]_0^4$$

$$= \frac{16}{3}$$