27.) $\int (\tan^2 y + 1)\,dy = \int \sec^2 y\,dy$

$= \tan y + C$

31.) $\int \left(x - \frac{5}{x}\right)dx = \frac{1}{2}x^2 - 5\ln|x| + C$

$y = \ln x$

$y' = \frac{1}{x}$

$\int$

4.3 Definite Integration - Geometric Interpretation

- Definite Integral

$$\boxed{\int_a^b f(x)dx}$$

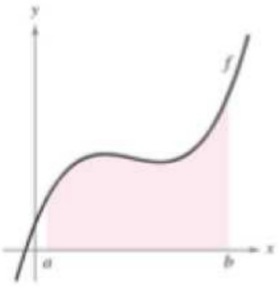
where

a - lower limit
b - upper limit

Definition - If f(x) is continuous and nonnegative on [a,b] then

$$\int_a^b f(x)dx$$

represents the area bounded by f(x), the x-axis, and the lines x=a and x=b.



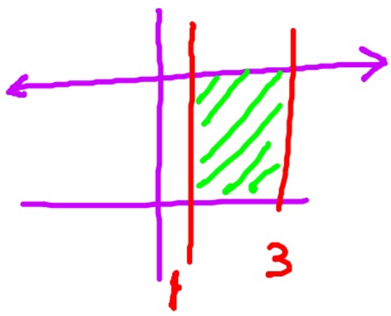
You can use a definite integral to find the area of the region bounded by the graph of f, the x-axis, $x = a$, and $x = b$.

This area is often referred to as "the area under the curve."

ex: Evaluate.

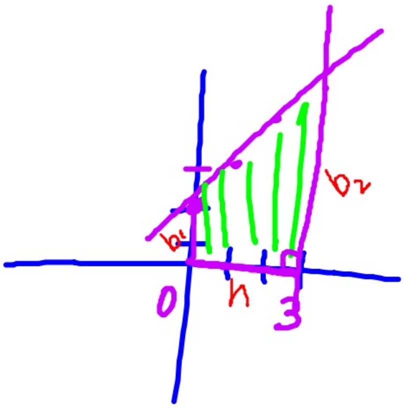
a) $\int_{1}^{3} 4dx$ $= 8$

$f(x) = 4$



ex: Evaluate.

b) $\int_0^3 (x+2)\,dx$



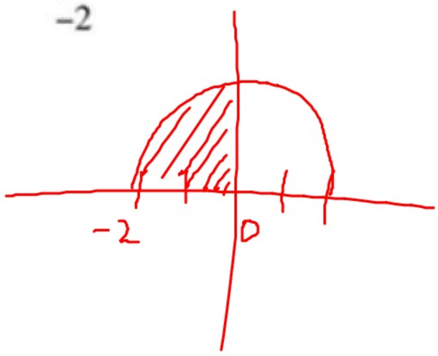
$\frac{1}{2}h(b_1+b_2)$

$\frac{1}{2}(3)(2+5)$

$\frac{21}{2}$

ex: Evaluate.

c) $\int_{-2}^{0} \sqrt{4-x^2}\,dx = \pi$



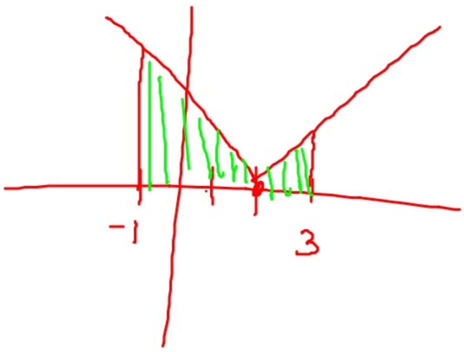
$\frac{1}{4}\pi r^2$

$\frac{1}{4}\pi(2)^2$

π

ex: Evaluate.

d) $\int_{-1}^{3} |x-2|\,dx$



$\frac{1}{2}bh$

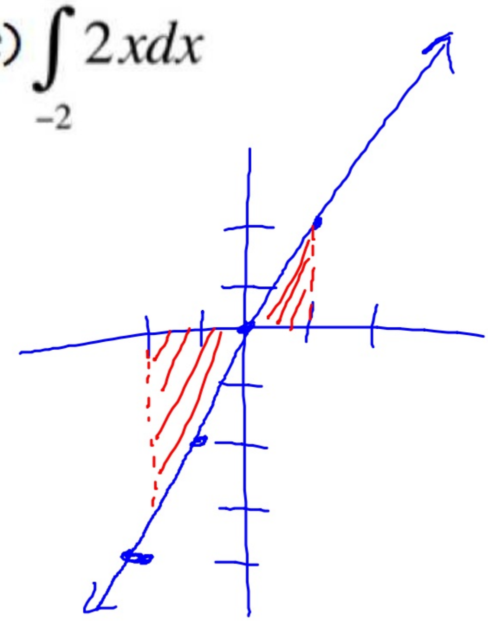
$\frac{1}{2}(3)(3) + \frac{1}{2}(1)(1)$

$\frac{9}{2} + \frac{1}{2}$

5

ex: Evaluate.

e) $\int_{-2}^{1} 2x\,dx$



$-\frac{1}{2}bh + \frac{1}{2}bh$

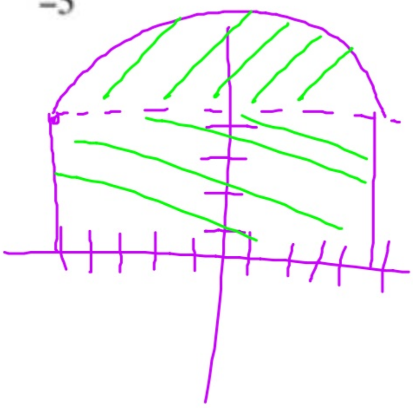
$-\frac{1}{2}(2)(4) + \frac{1}{2}(1)(2)$

$-4 + 1$

-3

ex: Evaluate.

f) $\int_{-5}^{5}\left(\sqrt{25-x^2}+4\right)dx$



$\frac{1}{2}\pi r^2 + bh$

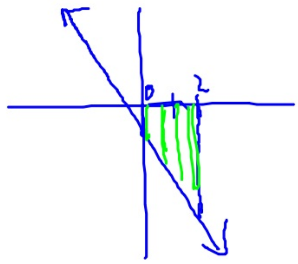
$\frac{1}{2}\pi(5)^2 + (10)(4)$

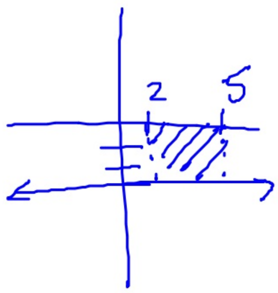
$\frac{25\pi}{2} + 40$

ex: Evaluate.

j) $\int_0^5 f(x)dx$ if $f(x)=\begin{cases}-x-1, & x\le 2\\ -3, & x>2\end{cases}$

$$\int_0^2 (-x-1)\,dx + \int_2^5 -3\,dx$$





$$-4 \quad - \quad 9$$

$$-13$$

Definite Integral Properties

If f(x) is continuous on [a,b] then...

1. $\int_a^a f(x)dx =$ 0

2. $\int_b^a f(x)dx =$ $-\int_a^b f(x)dx$

3. $\int_a^b kf(x)dx =$ $k\int_a^b f(x)dx$

Definite Integral Properties - cont.

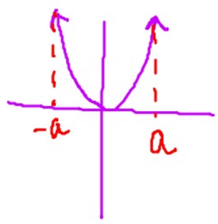
4. $$\int_a^b \left(f(x) \pm g(x)\right)dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$$

5. $$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

$a \le c \le b$

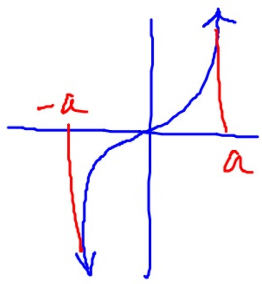
Definite Integral Properties - cont.

* 6. $\int_a^b \left(f(x)+c\right)dx = \int_a^b f(x)\,dx + \int_a^b c\,dx$



Definite Integral Properties - cont.

8. If f(x) is even, $\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)\,dx$



9. If f(x) is odd, $\int_{-a}^{a} f(x)dx = 0$

ex: If f(x) is continuous and

$$\int_0^1 f(x)dx = -4 \qquad \int_0^3 f(x)dx = 6 \qquad \int_3^5 f(x)dx = -7$$

use the properties of integrals to evaluate.

a) $\int_3^0 f(x)dx = -\int_0^3 f(x)dx = -6$

ex: If f(x) is continuous and

$$\int_0^1 f(x)dx = -4 \qquad \int_0^3 f(x)dx = 6 \qquad \int_3^5 f(x)dx = -7$$

use the properties of integrals to evaluate.

b) $\int_3^5 (f(x)+6)dx =$ $\int_3^5 f(x)dx + \int_3^5 6dx$

$-7 + 12 = 5$

ex: If f(x) is continuous and

$$\int_0^1 f(x)dx = -4 \qquad \boxed{\int_0^3 f(x)dx = 6} \qquad \boxed{\int_3^5 f(x)dx = -7}$$

use the properties of integrals to evaluate.

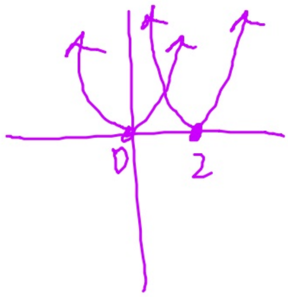
c) $\int_0^5 f(x)dx =$ $6 + -7 = -1$

ex: If f(x) is continuous and

$$\int_0^1 f(x)dx = -4 \qquad \int_0^3 f(x)dx = 6 \qquad \int_3^5 f(x)dx = -7$$

use the properties of integrals to evaluate.

d) $\int_5^7 f(x-2)dx =$ -7



ex: If f(x) is continuous and

$$\int_0^1 f(x)dx = -4 \qquad \int_0^3 f(x)dx = 6 \qquad \int_3^5 f(x)dx = -7$$

use the properties of integrals to evaluate.

e) $\int_0^1 3f(x)dx =$ -12

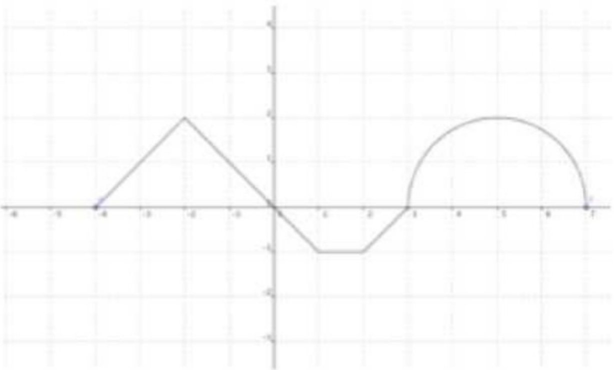
ex: Evaluate.

$$\int_{-13}^{13} \sin x dx = 0$$

ex: If f(x) is even and $\int_0^{20} f(x)dx = -7$ then

$$\int_{-20}^{20} f(x)dx = -14$$

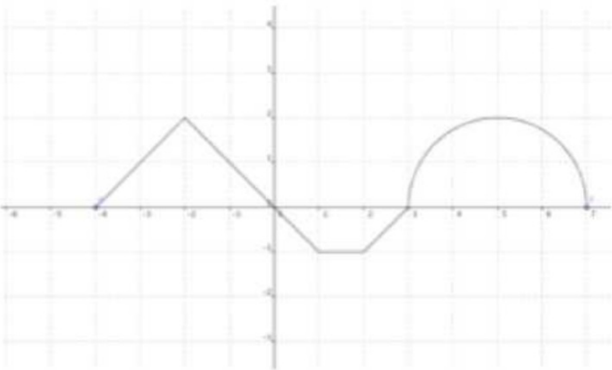
ex: The graph below represents the function, $f(x)$.



Evaluate.

1. $\int_3^7 f(x)dx$
2. $\int_{-4}^0 f(x)dx$
3. $\int_{-4}^7 f(x)dx$
4. $\int_3^0 f(x)dx$

*See printout.



5. $\int_{-4}^{0}[f(x)+1]dx$

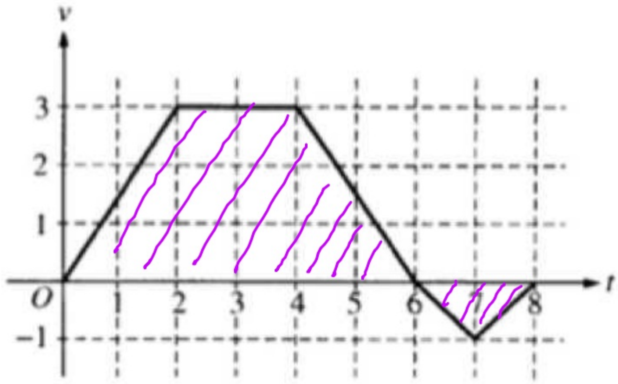
6. $\int_{3}^{7}-f(x)dx$

7. $\int_{-4}^{7}|f(x)|dx$

8. $\int_{3}^{-4}5f(x)dx$

4.1-4.3 Extra Practice WKST

7. & 8.



Area ⏢ + Area △
12 + 1
13

A bug begins to crawl up a vertical wire at time $t = 0$. The velocity v of the bug at time t, $0 \le t \le 8$, is given by the function whose graph is shown above.

At what value of t does the bug change direction?

(A) 2 (B) 4 (C) 6 (D) 7 (E) 8

What is the total distance the bug traveled from $t = 0$ to $t = 8$?

(A) 14 (B) 13 (C) 11 (D) 8 (E) 6

*See printout.

4.1-4.3 Extra Practice WKST

9.

If $\int_2^5 f(x)dx = 18$, then $\int_2^5 (f(x)+4)dx =$

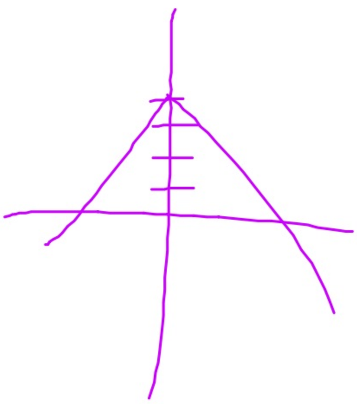
(A) 20 (B) 22 (C) 23 (D) 25 (E) 30

4.1-4.3 Extra Practice WKST

10.

$$\int_{-4}^{4} (4-|x|)\,dx =$$

(A) 0 (B) 4 (C) 8 (D) 16 (E) 32



$$\int_{-4}^{4} 4dx - \int_{-4}^{4} |x|dx$$

$$32 - 16$$

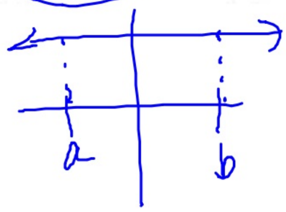
4.1-4.3 Extra Practice WKST

11.

If $\int_a^b f(x)\,dx = a+2b$, then $\int_a^b \left(f(x)+3\right)dx =$

(A) $a+2b+3$ (B) $3b-3a$ (C) $4a-b$ (D) $5b-2a$ (E) $5b-3a$

$$\int_a^b f(x)\,dx + \int_a^b 3\,dx$$



$$a+2b+(b-a)3$$

4.1-4.3 Extra Practice WKST

12.

Given that $\int_4^9 \sqrt{x}\,dx = \frac{38}{3}$, using your knowledge of transformations, what is

(a) $\int_9^4 \sqrt{t}\,dt$

(b) $\int_4^9 \left(\sqrt{x}+3\right)dx$

(c) $\int_9^{14} \sqrt{x-5}\,dx$

(d) $\int_4^4 \sqrt{x}\,dx$

4.1-4.3 Extra Practice WKST

13. $f(x) = \begin{cases} x & \text{for } x < 2 \\ 3 & \text{for } x \geq 2 \end{cases}$

If f is the function defined above, then $\int_{-1}^{4} f(x)\,dx$ is

(A) $\frac{9}{2}$

(B) $\frac{15}{2}$

(C) $\frac{17}{2}$

(D) undefined

4.1-4.3 Extra Practice WKST

14.

A race car is traveling on a straight track at a velocity of 80 meters per second when the brakes are applied at time $t = 0$ seconds. From time $t = 0$ to the moment the race car stops, the acceleration of the race car is given by $a(t) = -6t^2 - t$ meters per second per second. During this time period, how far does the race car travel?

(A) 188.229 m

(B) 198.766 m

(C) 260.042 m

(D) 267.089 m