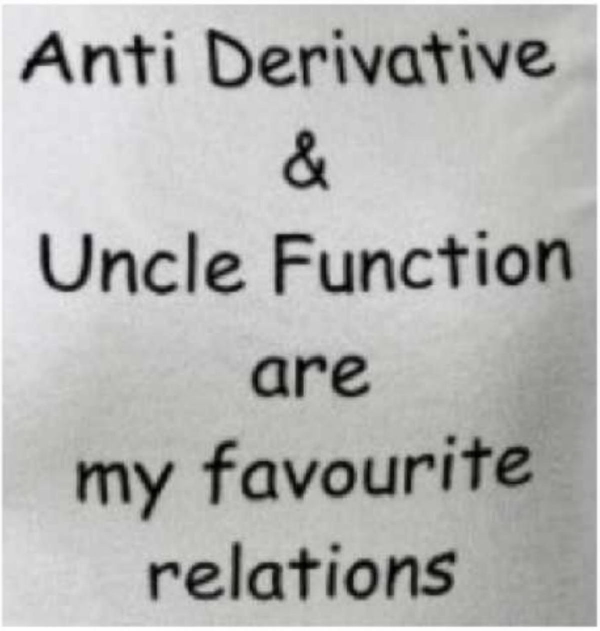
4.1 Antiderivatives and Indefinite Integration



- Antidifferentiation

Definition of Antiderivative

A function F is an **antiderivative** of f on an interval I when $F'(x) = f(x)$ for all x in I.

- Antidifferentiation by Trial and Error!

ex: Find a function, f(x), given its derivative f'(x).

a) $f'(x) = 2x$

$f(x) = x^2$ or

$f(x) = x^2 + 3$

ex: Find a function, f(x), given its derivative f'(x).

b) $f'(x) = 3x^2$

$f(x) = x^3$ $\quad$ $f(x) = x^3 - 1000$

c) $f'(x) = x^4$

$f(x) = \frac{1}{5}x^5$

d) $f'(x) = -50\sin x$ $\quad$ $f(x) = 50\cos x$

ex: Find a function, f(x), given its derivative f'(x).

e) $f'(x) = 2\cos 2x$

$f(x) = \sin 2x$

Representation of Antiderivatives

If $\frac{d}{dx}\left[f(x)\right] = f'(x)$ then $f(x)$ is called the "general antiderivative" of $f'(x)$.

ex: Find the antiderivative: $f'(x) = 12x^7$

$f(x) = \frac{3}{2}x^8 + C$

- Indefinite Integration

If F(x) is any anti-derivative of f(x) than the most general <u>antiderivative</u> of f(x) is called an <u>indefinite integral</u> and denoted,

$$y = \int \underbrace{f(x)}_{\text{Integrand}} \, \overbrace{dx}^{\text{Variable of integration}} = \underbrace{F(x)}_{\text{An antiderivative of } f(x)} + \overbrace{C}^{\text{Constant of integration}}.$$

$\int x \, dx$

$\int 3 \, dx$

Differentiation and Integration are INVERSE Operations

The inverse nature of integration and differentiation can be verified by substituting $F'(x)$ for $f(x)$ in the indefinite integration definition to obtain

$$\int F'(x)\, dx = F(x) + C.$$

Integration is the "inverse" of differentiation.

Moreover, if $\int f(x)\, dx = F(x) + C$, then

$$\frac{d}{dx}\left[\int f(x)\, dx\right] = f(x).$$

Differentiation is the "inverse" of integration.

These two equations allow you to obtain integration formulas directly from differentiation formulas, as shown in the following summary.

- Basic Rules

$\int x^2 dx = \frac{1}{3}x^3$

Differentiation Rules	Integration Rules
$\frac{d}{dx}[kx] =$ K	$\int k\,dx =$ $Kx + C$
$\frac{d}{dx}[kf(x)] =$ $Kf'(x)$	$\int kf'(x)\,dx =$ $Kf(x) + C$
$\frac{d}{dx}[f(x) \pm g(x)] =$ $f'(x) \pm g'(x)$	$\int [f'(x) \pm g'(x)]\,dx =$ $f(x) \pm g(x) + C$
$\frac{d}{dx}[x^n] =$ nx^{n-1}	$\int x^n\,dx =$ $\frac{x^{n+1}}{n+1} + C$

$n \neq -1$

*See printout.

Differentiation Rules	Integration Rules
$\frac{d}{dx}[\sin x] =$ $\cos x$	$\int \sin x\, dx =$ $-\cos x + C$
$\frac{d}{dx}[\cos x] =$ $-\sin x$	$\int \cos x\, dx =$ $\sin x + C$
$\frac{d}{dx}[\tan x] =$ $\sec^2 x$	$\int \sec^2 x\, dx =$ $\tan x + C$
$\frac{d}{dx}[\csc x] =$ $-\csc x \cot x$	$\int \csc x \cot x\, dx =$ $-\csc x + C$
$\frac{d}{dx}[\sec x] =$ $\tan x \sec x$	$\int \sec x \tan x\, dx =$ $\sec x + C$
$\frac{d}{dx}[\cot x] =$ $-\csc^2 x$	$\int \csc^2 x\, dx =$ $-\cot x + C$

Differentiation Rules	Integration Rules
$\frac{d}{dx}[e^x] = e^x$	$\int e^x\,dx = e^x + c$
$\frac{d}{dx}[a^x] = \ln a \cdot a^x$	$\int a^x\,dx = \frac{a^x}{\ln a} + c$
$\frac{d}{dx}[\ln x] = \frac{1}{x}$	$\int \frac{1}{x}\,dx = \ln\lvert x\rvert + c$

ex: Evaluate.

a) $\int x^5\,dx = \frac{1}{6}x^6 + C$

or $\frac{x^6}{6} + C$

b) $\int 5\sec^2 x\,dx = 5\tan x + C$

c) $\int \sqrt{x}\,dx = \int x^{1/2}\,dx = \frac{2}{3}x^{3/2} + C$

ex: Evaluate.

d) $\int \frac{8}{x^2}\,dx = \int 8x^{-2}\,dx = \frac{8x^{-1}}{-1} + C = -\frac{8}{x} + C$

e) $\int \frac{8}{x}\,dx = 8\int \frac{1}{x}\,dx = 8\ln|x| + C$

f) $\int 2x\,dx = x^2 + C$

ex: Evaluate.

g) $\int \frac{1}{(2x)^3} dx =$ $\int \frac{1}{8} x^{-3} dx = \frac{1}{8} \cdot \frac{x^{-2}}{-2}$

$= \frac{1}{-16x^2} + C$

h) $\int \left(4x^3 - 3^x + \sin x - 5e^x\right) dx =$ $x^4 - \frac{3^x}{\ln 3} - \cos x - 5e^x + C$

i) $\int \frac{x^7 - 5x^3 + 2x}{x^4} dx =$ $\int \left(x^3 - \frac{5}{x} + 2x^{-3}\right) dx$

$\frac{x^4}{4} - 5\ln|x| + \frac{2x^{-2}}{-2} + C$

ex: Evaluate.

j) $\int (1+3x)x^2\,dx =$

k) $\int \frac{\sin x}{\cos^2 x}\,dx = \int \frac{\sin x \cdot 1}{\cos x \cdot \cos x}\,dx = \int \tan x \sec x\,dx$

$= \sec x + C$

l) $\int \left(1+\cot^2 x\right)dx = \int \csc^2 x\,dx = -\cot x + C$

- Differential Equations

  A differential equation is an equation involving a derivative.

- Differential Equations Have 2 Types of Solutions

  1. General Solution - general antiderivative (+C)

  2. Particular Solution - an antiderivative that passes through a given initial condition. clues; find C

ex: $f'(x) = 3x^2 - 1$

a) Find the general solution. find $f(x)$

$f(x) = x^3 - x + C$

b) Find the particular solution that satisfies the intitial condition $f(2) = 4$.

$4 = (2)^3 - (2) + C$

$-2 = C$

$\boxed{f(x) = x^3 - x - 2}$

- Total Distance (by hand)

ex: A particles moves on the x-axis so that its position at any time is given by: $x(t) = 4t^3 - 18t^2 + 15t - 1$

Find the total distance traveled by the particle from t=0 to t=3.

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A particle moves on the x-axis so that its velocity at any time $t \geq 0$ is given by $v(t) = 12t^2 - 36t + 15$. At $t = 1$, the particle is at the origin. $x(1) = 0$

(a) Find the position $x(t)$ of the particle at any time $t \geq 0$.

$x(t) = 4t^3 - 18t^2 + 15t + -1$

(b) Find all values of t for which the particle is at rest.

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(c) Find the maximum velocity of the particle for $0 \leq t \leq 2$. go to $a(t)$ make a chart

(d) Find the total distance traveled by the particle from $t = 0$ to $t = 2$.

$v(t) = 3(4t^2 - 12t + 5) = 3(2t-5)(2t-1)$

$t = 2.5, \frac{1}{2}$

t	$x(t)$
0	-1
$1/2$	2.5
2	-11

3.5

13.5

add

$\boxed{17}$

+ pos. − neg.

0 $\frac{1}{2}$ 2

4.1-4.3 Extra Practice WKST

1.

If $f'(x) = 12x^2 - 6x + 1$, $f(1) = 5$, then $f(0)$ equals

(A) 2 (B) 3 (C) 4 (D) -1 (E) 0

*See printout.

4.1-4.3 Extra Practice WKST

2.

Find all functions g such that $g'(x) = \dfrac{5x^2 + 4x + 5}{\sqrt{x}}$

(A) $g(x) = 2\sqrt{x}\left(x^2 + \frac{4}{3}x - 5\right) + C$ (B) $g(x) = 2\sqrt{x}\left(x^2 + \frac{4}{3}x + 5\right) + C$

(C) $g(x) = 2\sqrt{x}\left(5x^2 + 4x - 5\right) + C$ (D) $g(x) = \sqrt{x}\left(x^2 + \frac{4}{3}x + 5\right) + C$

(E) $g(x) = \sqrt{x}\left(5x^2 + 4x + 5\right) + C$

4.1-4.3 Extra Practice WKST

3.

Determine $f(t)$ when $f''(t) = 2(3t+1)$ and $f'(1) = 3$, $f(1) = 5$.

(A) $f(t) = 3t^3 - 2t^2 + 2t + 2$ (B) $f(t) = t^3 - 2t^2 + 2t + 4$

(C) $f(t) = 3t^3 + t^2 - 2t + 3$ (D) $f(t) = t^3 - t^2 + 2t + 3$

(E) $f(t) = t^3 + t^2 - 2t + 5$

4.1-4.3 Extra Practice WKST

4.

Consider the following functions:

I. $F_1(x) = \dfrac{\sin^2 x}{2}$

II. $F_2(x) = -\dfrac{\cos 2x}{4}$

III. $F_3(x) = -\dfrac{\cos^2 x}{2}$

Which are antiderivatives of $f(x) = \sin x \cos x$? (Hint: take the derivative of each and manipulate)

(A) II only (B) I only (C) I & III only (D) I, II, & III (E) I & II only

4.1-4.3 Extra Practice WKST

5.

A particle moves along the x-axis. The velocity of the particle at time t is $6t - t^2$. What is the total distance traveled by the particle from time $t = 0$ to $t = 3$?

(A) 3 (B) 6 (C) 9 (D) 18 (E) 27

4.1-4.3 Extra Practice WKST

6.

A particle moves along the x-axis so that its acceleration at time t is $a(t) = 8 - 8t$ in units of feet and seconds. If the velocity of the particle at $t = 0$ is 12 ft/sec, how many seconds will it take for the particle to reach its furthest point to the right?

(A) 6 seconds (B) 5 seconds (C) 3 seconds (D) 7 seconds (E) 4 seconds