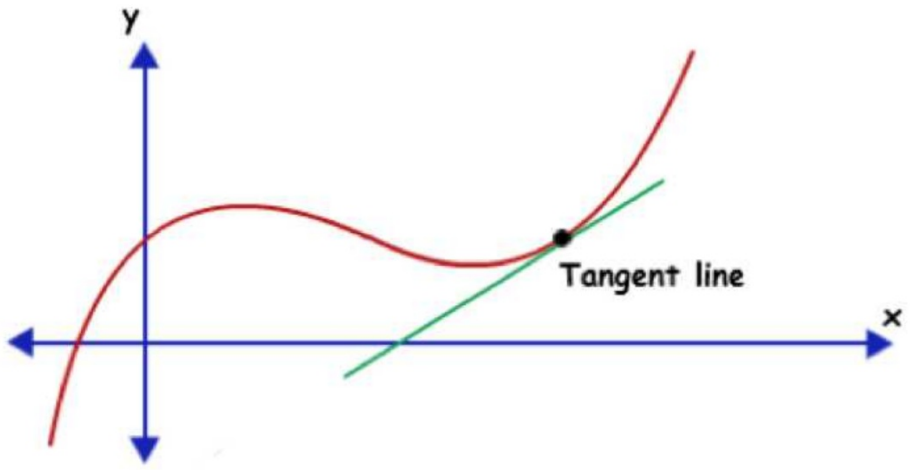
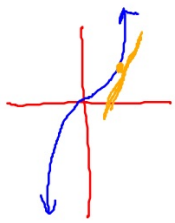
3.7 Linear Approximations

A.K.A. - "Tangent Linear Approximation"



Example.Use local linear approximations to approximate the quantity $(1.98)^3$



Is this an over or under approximation?

$f(x) = x^3$ $\quad (2, 8)$ $\quad (1.98, \quad)$

$f'(x) = 3x^2$

$f'(2) = 12$

$(1.98)^3 = 7.762$

$y - 8 = 12(x - 2)$

$y - 8 = 12(1.98 - 2)$

$y = 12(-.02) + 8 = -.24 + 8$

$\boxed{7.76}$

$y = 12\left(\frac{-2}{100}\right) + 8$

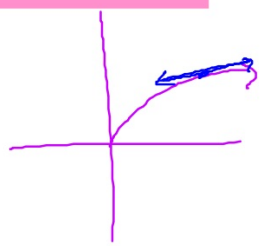
$= 12\left(\frac{-1}{50}\right) + 8 = \frac{-6}{25} + 8$

$= 7\frac{19}{25}$

Under approximation because f''(1.98) > 0

Example. Use local linear approximations to approximate the quantity $\sqrt{80.9}$

Is this an over or under approximation?



$f(x) = \sqrt{x}$ $(81, 9)$

$f'(x) = \frac{1}{2\sqrt{x}}$

$f'(81) = \frac{1}{18}$

$y - 9 = \frac{1}{18}(x - 81)$

$y = \frac{1}{18}(80.9 - 81) + 9$

$= -\frac{1}{180} + 9$

$= 8\frac{179}{180}$

Over approximation because f''(80.9) < 0

Approximate the cube root of -65. $= -4\frac{1}{48}$

Is this an over or under approximation?

$f(x) = \sqrt[3]{x}$

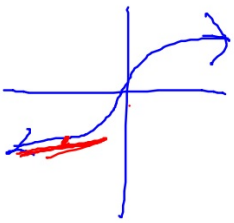
$f'(x) = \frac{1}{3x^{2/3}}$

$f'(-64) = \frac{1}{48}$

$(-64, -4)$

$y + 4 = \frac{1}{48}(-65 + 64)$

$y = \frac{-1}{48} - 4 = -4\frac{1}{48}$

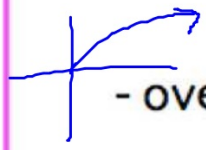


$f''(x) = \frac{1}{3}\left(\frac{-2}{3}\right)x^{-5/3}$

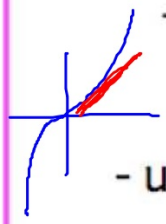
$f''(-64) = \frac{-2}{9}(-64)^{-5/3} > 0$

Underestimate because f''(-65) > 0

Linear Approximations are...



- overestimates when $f''(x) < 0$ (Concave down)



- underestimates when $f''(x) > 0$ (Concave up)