3.6: Optimization

Steps to follow

Guidelines for Solving Optimization Word Problems

1. Identify all given quantities and quantities to be determined.
2. If possible, draw a sketch.
3. Write a primary equation for the quantity that is to be maximized or minimized.
4. If possible, write a secondary equation relating the variables of the primary equation. Using the secondary equation, use substitution to reduce the primary equation to one variable.
5. Determine a feasible domain for the unknown quantities.
6. Determine the desired maximum or minimum values by calculus techniques learned in this chapter. (i.e. find a DERIVATIVE, etc)

#1 Find two positive numbers that satisfy the given requirements.

The sum of the first number squared and the second number is 54 and the product is a maximum.

$D: x > 0$

Find P'

max product: ***(Primary)*** $xy = P$

(Secondary) $x^2 + y = 54$

$y = 54 - x^2$

$P = x(54 - x^2)$

$P'(x) = 54 - 3x^2$

$0 = 54 - 3x^2$

$\pm\sqrt{18} = x$

P': 0 $+$ $\sqrt{18}$ $-$

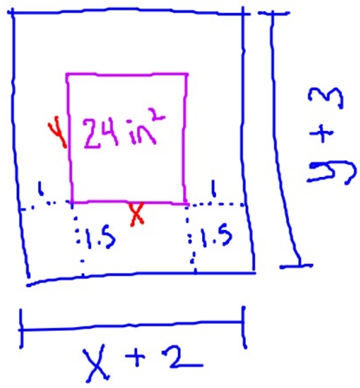
$$\boxed{\sqrt{18},\ 36}$$

#2: A rectangular page is to contain 24 square inches of print. The margins on the top and bottom are to be 1.5 inches and the margins on the left and right are to be 1 inch. what should be the dimensions of the page so that the least amount of paper is used.

Secondary

$$xy = 24$$

$$y = \frac{24}{x}$$



Minimize area (Primary)

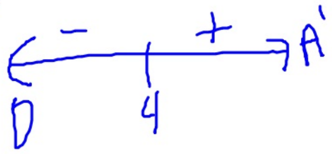
$$A = (x+2)(y+3)$$

$$A(x) = (x+2)\left(\frac{24}{x}+3\right)$$

$$A(x) = 24 + 3x + \frac{48}{x} + 6$$

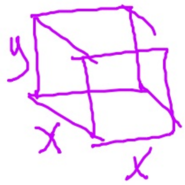
$$A'(x) = 3 - \frac{48}{x^2}$$

$$0 = \frac{3x^2 - 48}{x^2}\ ;\ x = 4$$



$$\boxed{6\text{in},\ 9\text{in}}$$

#3 A manufacturer wants to design an open box having a square base and a surface area of 108 square inches. What dimensions will produce a maximum volume?



Primary (max volume)

$$V = x^2 y$$

$$V(x) = x^2\left(\frac{108 - x^2}{4x}\right)$$

$$V(x) = \frac{1}{4}\left(108x - x^3\right)$$

$$V'(x) = \frac{1}{4}\left(108 - 3x^2\right)$$

$$x = 6$$

Secondary

$$108 = x^2 + 4xy$$

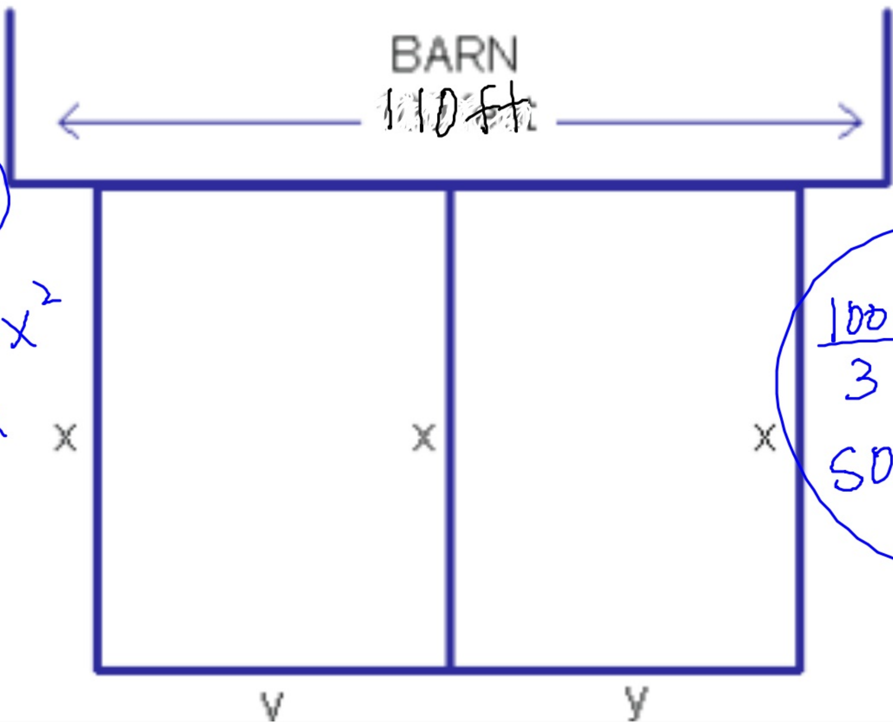
$$\frac{108 - x^2}{4x} = y$$

$$\boxed{6\,\text{in} \times 6\,\text{in} \times 3\,\text{in}}$$

#4: Two pens are to be built alongside a barn as shown. The barn will make up one side of each pen. If 200 ft of fencing are available, what pen size maximizes the area?

$A = 2xy$

$200 = 3x + 2y$



$A(x) = 2x\left(\frac{200-3x}{2}\right)$

$A(x) = 200x - 3x^2$

$A'(x) = 200 - 6x$

$\frac{100}{3} = x$

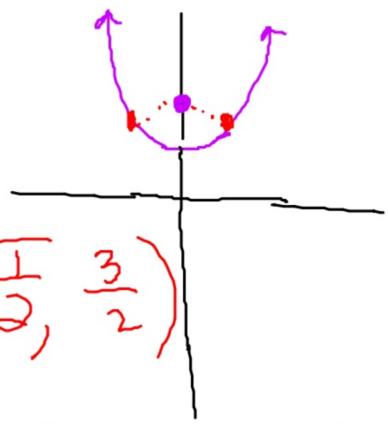
$\frac{100}{3}$ ft

50 ft

Because d is the smallest when the expression inside the radical is smallest, you need only find the critical numbers of the radicand when maximizing or minimizing distance.

#5 Which point(s) on $\underbrace{y = x^2 + 1}_{\text{secondary}}$ are closest to $(\overset{x}{0},\overset{y}{2})$?

Primary: (minimizing distance)

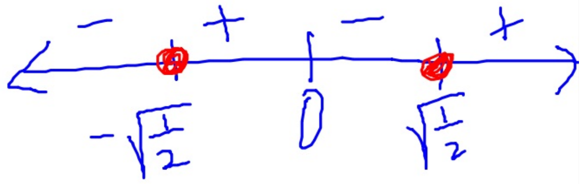


$$\left(\pm\sqrt{\tfrac{1}{2}},\ \tfrac{3}{2}\right)$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(x - 0)^2 + (y - 2)^2}$$

$$d = \sqrt{x^2 + (x^2 + 1 - 2)^2}$$



$$D = x^2 + (x^2 - 1)^2$$

$$D'(x) = 2x + 2(x^2 - 1)^1 \cdot 2x = 2x + 4x^3 - 4x$$

$$= 4x^3 - 2x = 0$$

$$2x(2x^2 - 1) = 0$$

#6 Find the maximum volume of a box that can be made by cutting out squares from the corners of a 14 inch by 10 inch rectangular sheet of cardboard and folding up the sides.

#8 We want to construct a box whose base length is 3 times the base width. The material used to build the top and bottom cost \$10 per square foot and the material used to build the sides cost \$6 per square foot. If the box must have a volume of 50 cubic feet, determine the dimensions that will minimize the cost to build the box.

#6 Find the dimensions of a rectangle of largest area that has its base on the x-axis and its other two vertices above the x-axis and lying on the parabola $y = 8 - x^2$.

secondary

Primary (max area) $A = 2xy$

$$A = 2x(8 - x^2)$$

$$A(x) = 16x - 2x^3$$

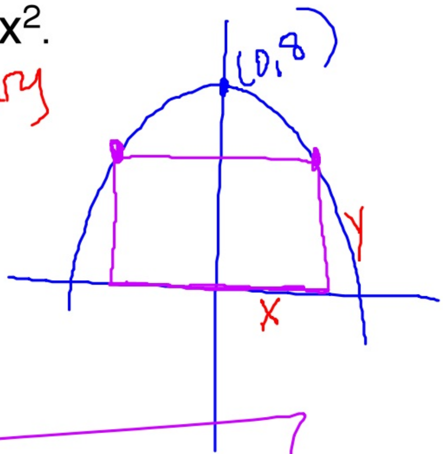
$$A'(x) = 16 - 6x^2$$

$$0 = 2(8 - 3x^2)$$

$$\pm\sqrt{\frac{8}{3}} = x$$

+ −

$\sqrt{\frac{8}{3}}$



$$\boxed{2\sqrt{\frac{8}{3}},\ \frac{16}{3}}$$

base height