5.) $f(x) = \frac{24}{x^2+12}$ $f'(x) = -48x(x^2+12)^{-2}$

$f''(x) = -48x(-2(x^2+12)^{-3}2x) + (x^2+12)^{-2}(-48)$

$= -48(x^2+12)^{-3}(-4x^2 + x^2 + 12)$

$= \frac{(-3x^2+12)(-48)}{(x^2+12)^3} = \frac{-3(x^2-4)(-48)}{(x^2+12)^3}$

53. $y' = 2xe^{-x} - x^2e^{-x}$

$y' = xe^{-x}(2-x)$

$x=0$	$x=2$
$y''(0)>0$	$y''(2)<0$
rel. min $(0,0)$	rel. max $(2,4e^{-2})$

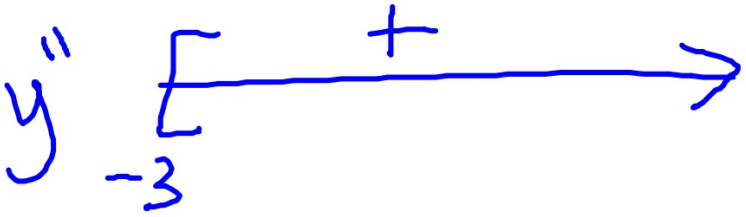
$y'' = xe^{-x}(-1) + (2-x)(x(-e^{-x}) + e^{-x}\cdot 1)$

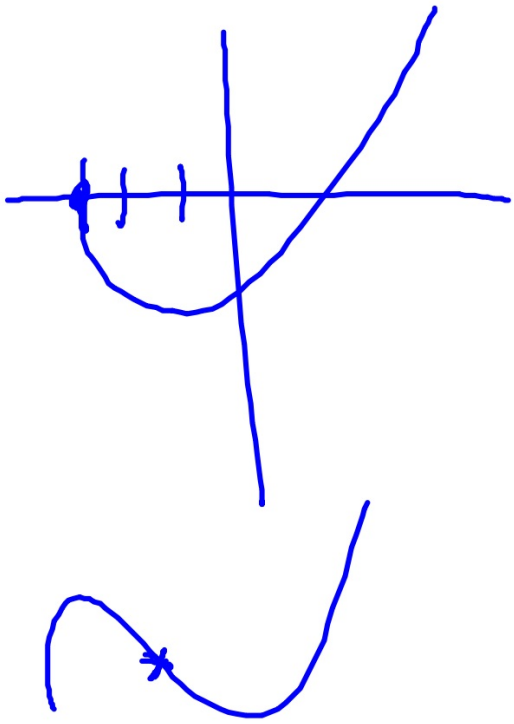
$= -xe^{-x} + (2-x)(e^{-x} - xe^{-x})$

$-xe^{-x} + (2e^{-x} - 2xe^{-x} - xe^{-x} + x^2e^{-x})$

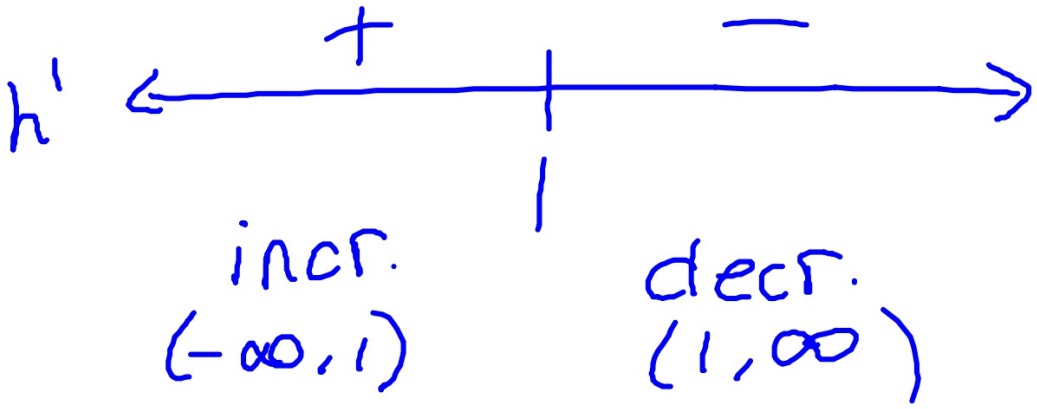
$-4xe^{-x} + 2e^{-x} + x^2e^{-x} = e^{-x}(x^2 - 4x + 2)$

19.) $y'' = \frac{3(x+4)}{4(x+3)^{3/2}}$





$$h'(x) = 4e^{-x}(1-x)$$



incr.
$(-\infty, 1)$

decr.
$(1, \infty)$

3.5 Summary of Curve Sketching

To sketch curves you must find:

- x & y intercepts
- relative extrema
- intervals of increasing/decreasing
- concavity
- points of inflections
- asymptotes

For Rational Functions, also find:

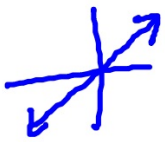
- holes

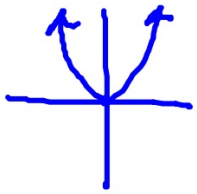
For Radical Functions, also find:

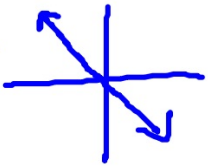
- domain

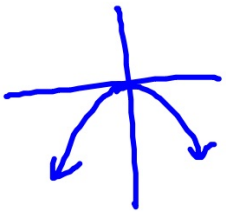
For Polynomial Functions, also find:

- end behavior

$y = x$ 

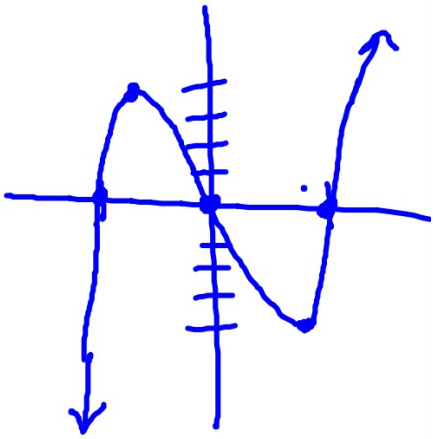
$y = x^2$ 

$y = -x$ 

$y = -x^2$ 

Sketching Polynomial Functions

ex: Sketch. $y = x^5 - 5x$



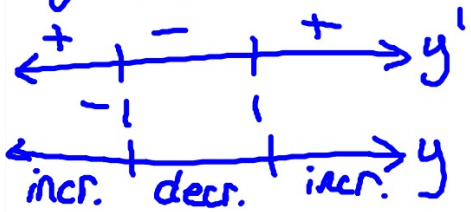
x int: $(0,0)(\pm\sqrt[4]{5}, 0)$

y int $(0,0)$

rel. extrema $(-1, 4)$ max $(1, -4)$ min

$y' = 5x^4 - 5$

$0 = 5(x^4 - 1)$



Sketching Rational Functions

ex: Sketch.

$$y = \frac{x^2 - 2x + 4}{x - 2}$$

xint: none

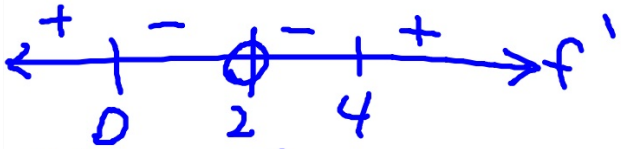
yint: (0, -2)

VA: $x = 2$

~~HA~~: none

SA: $y = x$

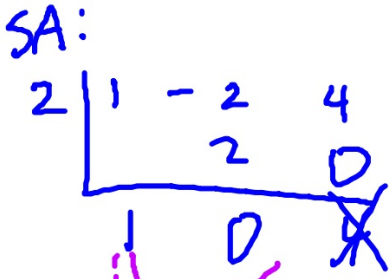
$$f'(x) = \frac{x(x-4)}{(x-2)^2}$$

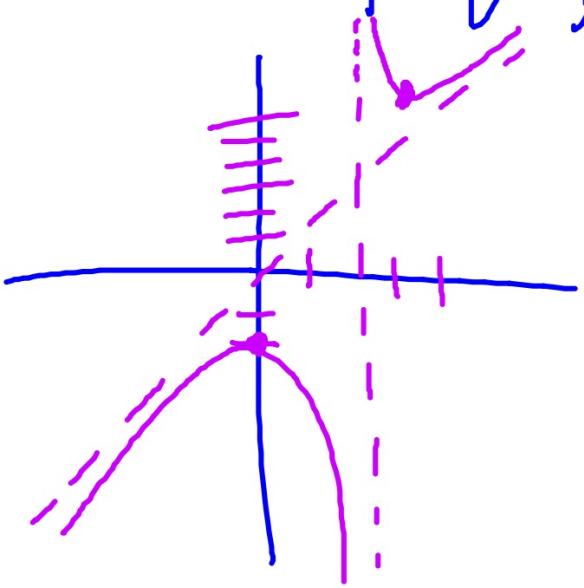


rel. max (0, -2)

rel. min (4, 6)

SA:





Sketching Radical Functions

ex: Sketch.

$$y = x\sqrt{9 - x^2}$$

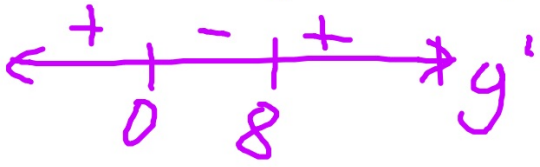
Sketching Radical Functions

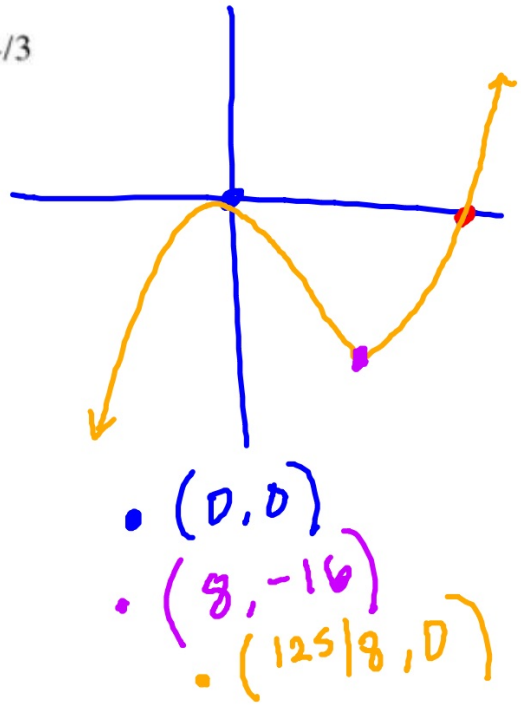
ex: Sketch.

$$y = 2x^{5/3} - 5x^{4/3}$$

$y' = \frac{10}{3}x^{2/3} - \frac{20}{3}x^{1/3}$

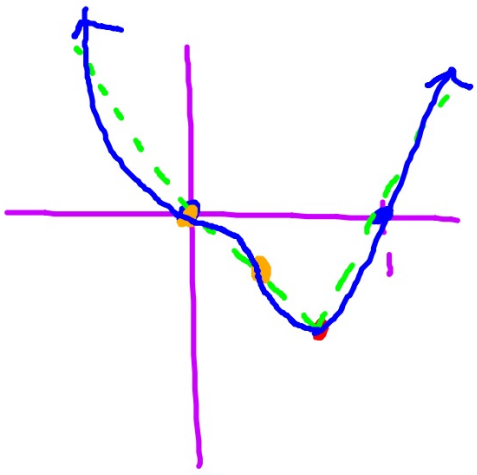
$0 = \frac{10}{3}x^{1/3}\left(x^{1/3} - 2\right)$





- (0, 0)
- (8, −16)
- (125/8, 0)

$y = x^4 - x^3$



x-int $(0,0)$ $(1,0)$

y-int $(0,0)$

rel min: $\left(\frac{3}{4}, \frac{-27}{256}\right)$

POI: $\left(\frac{1}{2}, \frac{-1}{16}\right), (0,0)$