11.) $y = x\sqrt{16 - x^2}$

$$y' = x \cdot \cancel{\tfrac{1}{2}}(16 - x^2)^{-1/2} \cdot -\cancel{2}x + (16 - x^2)^{1/2} \cdot 1$$

$$y' = \frac{-x^2}{(16-x^2)^{1/2}} + \frac{(16-x^2)^{1/2}}{1}$$

$$y' = \frac{-x^2 + 16 - x^2}{\sqrt{16-x^2}}$$

$$y' = \frac{16 - 2x^2}{\sqrt{16-x^2}}$$

$0 = 16 - 2x^2$

$x^2 = 8$

$x = \pm\sqrt{8}$

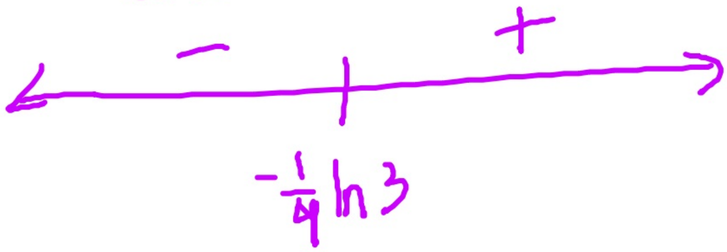
D: $[-4, 4]$

-4		$-\sqrt{8}$		$\sqrt{8}$		4
	$-$		$+$		$-$	

17.) $g(x) = e^{-x} + e^{3x}$

$g'(x) = -e^{-x} + 3e^{3x}$

$0 = -e^{-x}(1 - 3e^{4x})$



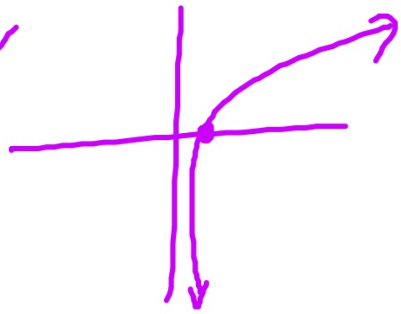
$1 - 3e^{4x} = 0$

$\ln e^{4x} = \ln \frac{1}{3}$

$4x = \ln \frac{1}{3}$

$x = \frac{1}{4} \ln \frac{1}{3}$

$x = -\frac{1}{4} \ln 3$



118.) a.) $\left(0, \frac{5\pi}{4}\right) \cup \left(\frac{7\pi}{4}, 2\pi\right)$

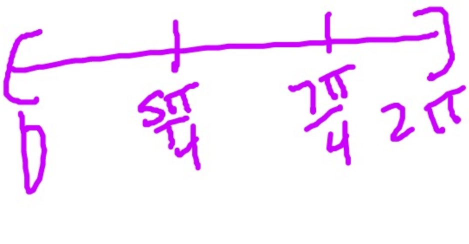
b.) $\left(\frac{5\pi}{4}, \frac{7\pi}{4}\right)$

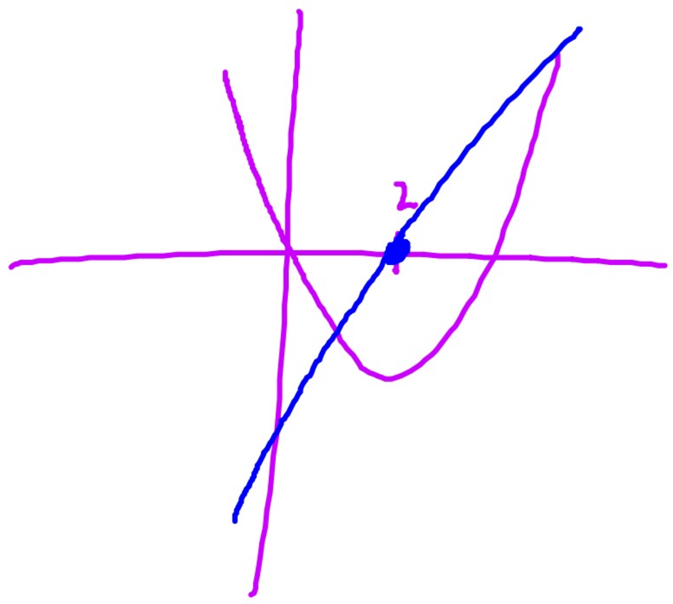
c.) rel. max $\frac{5\pi}{4}$

min $\frac{7\pi}{4}$

118.) $g' = \sqrt{2} + 2\sin x$

$-\frac{\sqrt{2}}{2} = \sin x$

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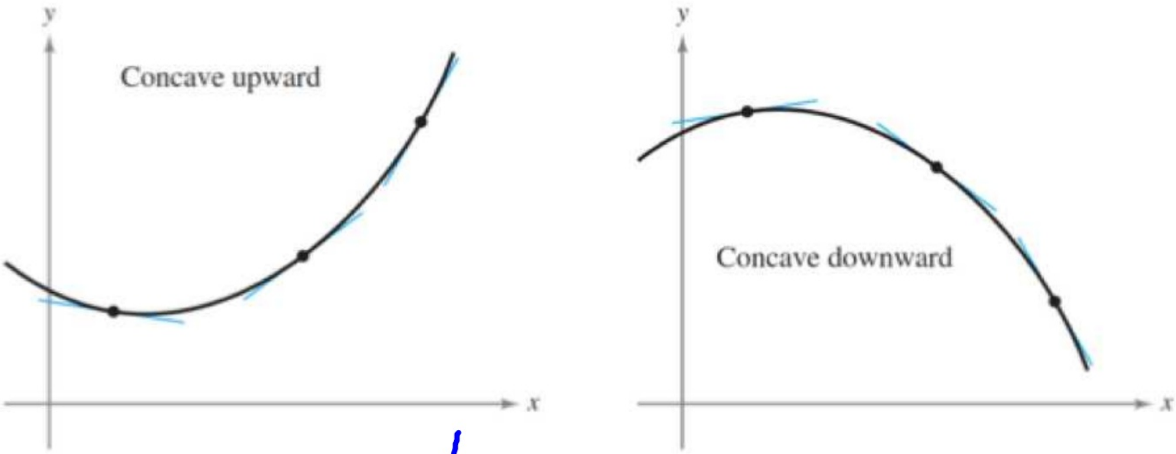
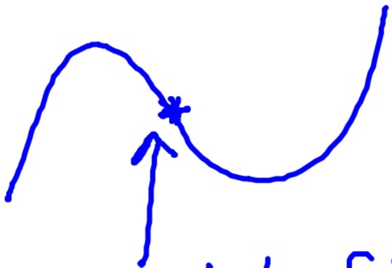
3.4 Concavity and the Second Derivative Test

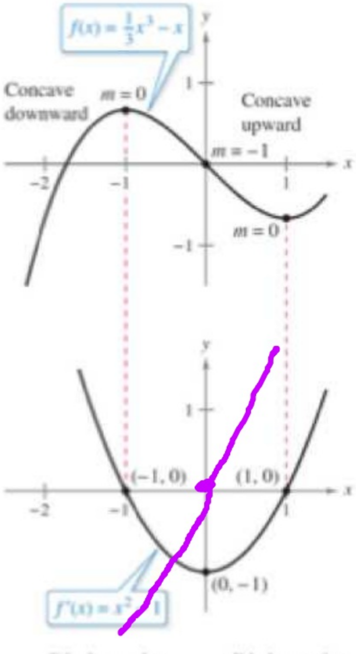




point of inflection

Definition of Concavity

Let f be differentiable on an open interval I. The graph of f is **concave upward** on I when f' is increasing on the interval and **concave downward** on I when f' is decreasing on the interval.



f' is decreasing. f' is increasing.

concave up

concave down

f	CCU	CCD
f'	incr.	decr.
f''	+	−

ex: Determine the open intervals on which the graph of

$$f(x) = e^{-\frac{x^2}{2}}$$

is concave upward and concave downward? Justify your answer.

$$f'(x) = e^{-x^2/2} \cdot (-x)$$

$$f''(x) = e^{-x^2/2}(-1) + (-x)e^{-x^2/2}(-x)$$

$$= -e^{-x^2/2}(1 - x^2)$$

f'': $+$ on $(-\infty, -1)$, $-$ on $(-1, 1)$, $+$ on $(1, \infty)$; marks at -1 and 1

CCU on (-inf, -1) U (1, inf) because f''>0 on these interv

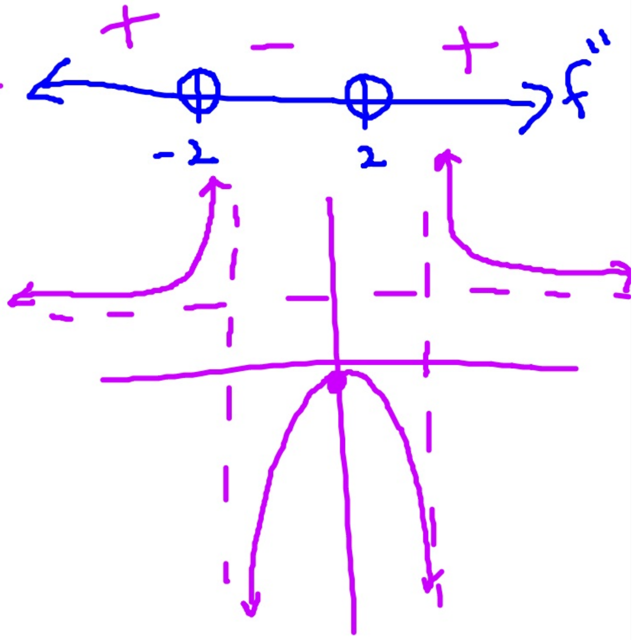
CCD on (-1,1) because f'' < 0 on this interval.

ex: Determine the open intervals on which the graph of

$$f(x)=\frac{x^2+1}{x^2-4} \qquad D: x \neq \pm 2$$

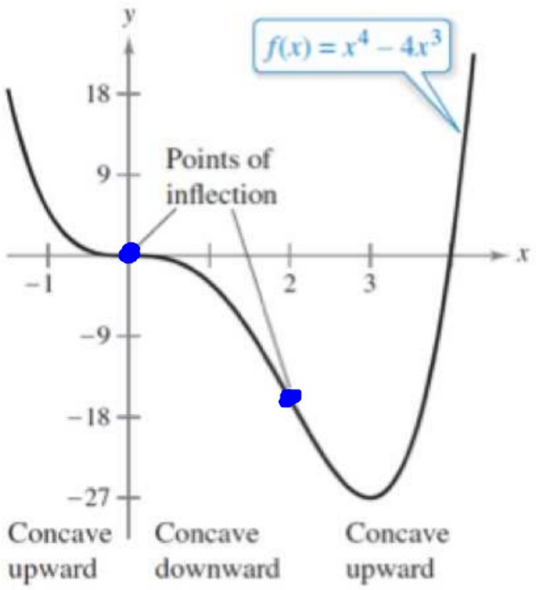
is concave upward and concave downward? Justify your answer.

$$f''(x) = \frac{10(3x^2+4)}{(x^2-4)^3}$$



Definition of Point of Inflection

Let f be a function that is continuous on an open interval, and let c be a point in the interval. If the graph of f has a tangent line at this point $(c, f(c))$, then this point is a **point of inflection** of the graph of f when the concavity of f changes from upward to downward (or downward to upward) at the point.



POI REQUIREMENTS

1. f(c) is defined
2. f''(c) is 0 or undefined
3. f'' changes signs at x=c.

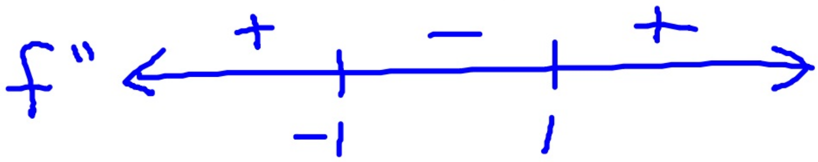
$\frac{1}{3}x^3 - x$

$x^2 - 1$

$2x$

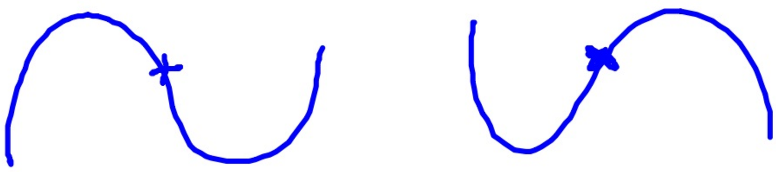
ex: Find all points of inflection on the graph of f(x), if possible.

$$f(x) = e^{-\frac{x^2}{2}}$$



Points of inflection at $(1, e^{-1/2})$ and $(-1, e^{-1/2})$

because f'' changes signs at these points



ex: Find all points of inflection on the graph of f(x), if possible.

$$f(x)=\frac{x^2+1}{x^2-4}$$

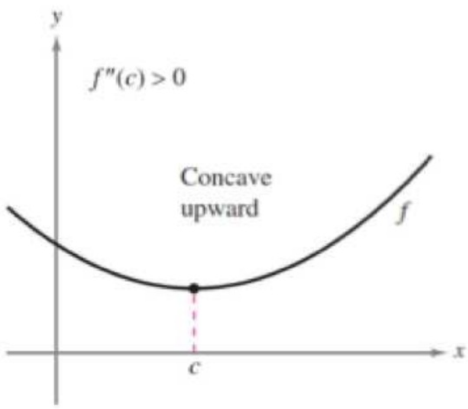
None; there were no points defined where the concavity changed

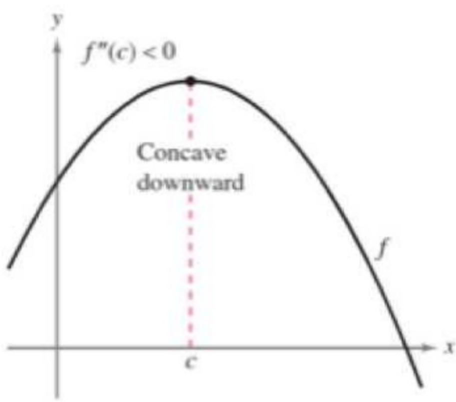
THEOREM 3.9 Second Derivative Test (find rel. extrema)

Let f be a function such that $\boxed{f'(c) = 0}$ and the second derivative of f exists on an open interval containing c.

1. If $f''(c) > 0$, then f has a relative minimum at $(c, f(c))$.
2. If $f''(c) < 0$, then f has a relative maximum at $(c, f(c))$.

If $f''(c) = 0$, then the test fails. That is, f may have a relative maximum, a relative minimum, or neither. In such cases, you can use the First Derivative Test.





ex: Use the 2nd Derivative Test to find all relative extrema of

points

$$f(x) = -3x^5 + 5x^3$$

① find $f'(c) = 0$

$f'(x) = -15x^4 + 15x^2$

$0 = -15x^2(x^2 - 1)$

$c = 0, 1, -1$

② find $f''(x)$ all $f''(c)$

$f''(x) = -60x^3 + 30x$

$f''(0) = 0$; test fails (no rel. ext.)

$f''(1) < 0$ rel. max $(1, 2)$

$f''(-1) > 0$ rel. min $(-1, -2)$

ex: What can be concluded about f(x) at x=2 if

$f(2)=16$ ← (2,16) (defined)

$f'(2)=0$ ← 2 is a crit. point

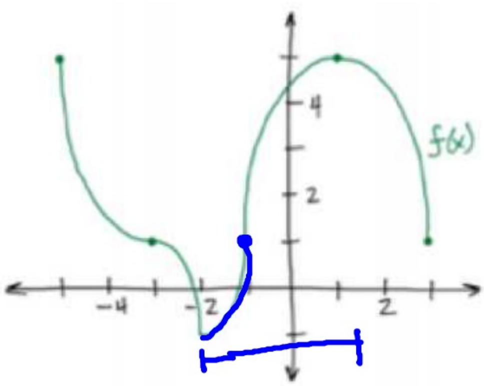
$f''(2)=-300$ ← negative → CCD

$\boxed{\text{x=2 is a rel. max}}$

3.4 WKST

2.

Let f be a continuous function on $[-5,3]$ with a vertical tangent line at $x=-1$, horizontal tangents at $x=-3$ and $x=1$ and a cusp at $x=-2$. The graph of f is given at right. Which of the following properties are satisfied?

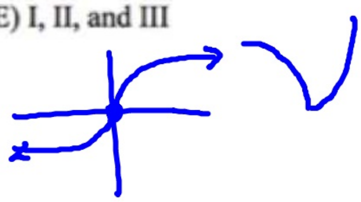


I. $f'(x)<0$ on $(-2,1)$ X

II. f has exactly 2 local extrema ✓

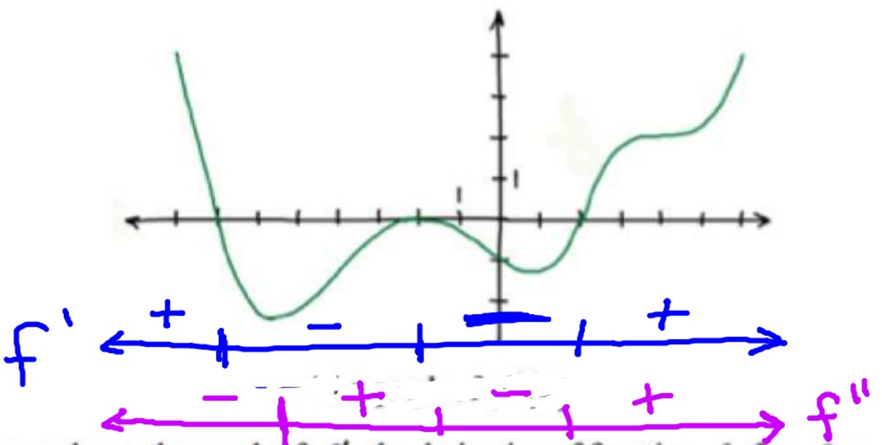
III. f has exactly 4 critical points ✓

(A) I only (B) II only (C) III only (D) II and III only (E) I, II, and III



3.4 WKST

3.



The figure above shows the graph of f', the derivative of function f, for $-8 < x < 6$. Of the following, which best describes the graph of f on the same interval?

(A) 1 local minimum, 1 local maximum, and 3 inflection points

(B) 1 local minimum, 1 local maximum, and 4 inflection points

(C) 2 local minima, 1 local maximums, and 2 inflection points

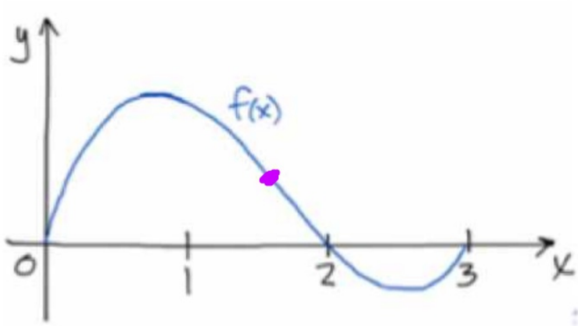
(D) 2 local minima, 1 local maximum, and 4 inflection points

(E) 2 local minima, 2 local maxima, and 3 inflection points

A

3.4 WKST

4.



The graph of a differentiable function $f(x)$ is shown in the figure above and has an inflection point at $x = \frac{3}{2}$. Which of the following correctly orders $f(2)$, $f'(2)$, and $f''(2)$?

0 neg. pos.

f' f''
(A) $f(2) < f'(2) < f''(2)$

f' f''
→ (B) $f'(2) < f(2) < f''(2)$

f' f''
(C) $f'(2) < f''(2) < f(2)$

f'' f'
(D) $f''(2) < f(2) < f'(2)$

f'' f'
(E) $f''(2) < f'(2) < f(2)$

3.4 WKST

6.

Let f be the function defined by $f(x)=2x^3-3x^2-12x+18$. On which of the following intervals is the graph of f both increasing and concave down?

(A) $(-\infty,-1)$ (B) $\left(-1,\frac{1}{2}\right)$ (C) $(-1,2)$ (D) $\left(\frac{1}{2},2\right)$ (E) $(2,\infty)$

3.4 WKST

7.

$f'(x) > 0$ $f''(x) < 0$

If $f'(x) > 0$ for all x and $f''(x) < 0$ for all x, which of the following could be a table of values for f?

(A)

x	$f(x)$
−1	4
0	3
1	1

(B)

x	$f(x)$
−1	4
0	4
1	4

(C)

x	$f(x)$
−1	4
0	5
1	6

(D)

x	$f(x)$
−1	4
0	5
1	7

(E)

x	$f(x)$
−1	4
0	6
1	7

f is incr.

decr. rate

3.4 WKST

8.

(**Calculator Permitted**) The derivative of the function f is given by $f'(x) = x^2 \sin\left(x^2\right)$. How many points of inflection does the graph of f have on the open interval $(-2, 2)$?

(A) One (B) Two (C) Three (D) Four (E) Five

3.4 WKST

9.

The second derivative of a function g is given by $g''(x) = 2^{-x^2} + \cos x + x$. For $-5 < x < 5$, on what open intervals is the graph of g concave up?

(A) $-5 < x < -1.016$ only

(B) $-1.016 < x < 5$ only

(C) $0.463 < x < 2.100$ only

(D) $-5 < x < 0.463$ and $2.100 < x < 5$