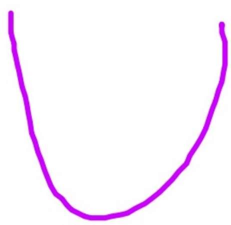
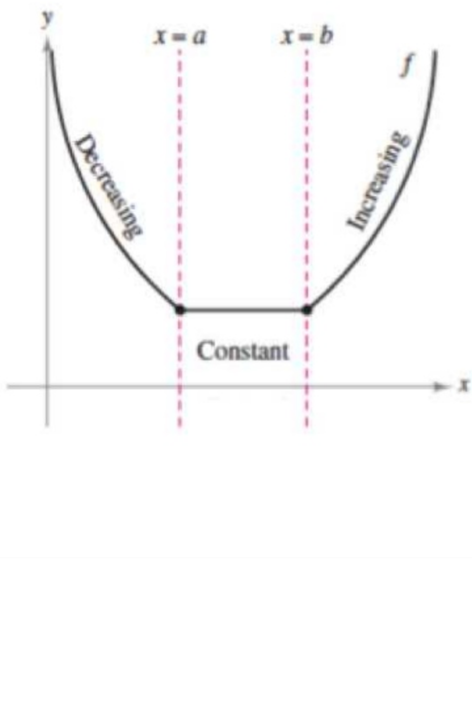
3.3 Increasing and Decreasing Functions and The First Derivative Test

Definitions of Increasing and Decreasing Functions

A function f is **increasing** on an interval when, for any two numbers x_1 and x_2 in the interval, $x_1 < x_2$ implies $f(x_1) < f(x_2)$.

A function f is **decreasing** on an interval when, for any two numbers x_1 and x_2 in the interval, $x_1 < x_2$ implies $f(x_1) > f(x_2)$.

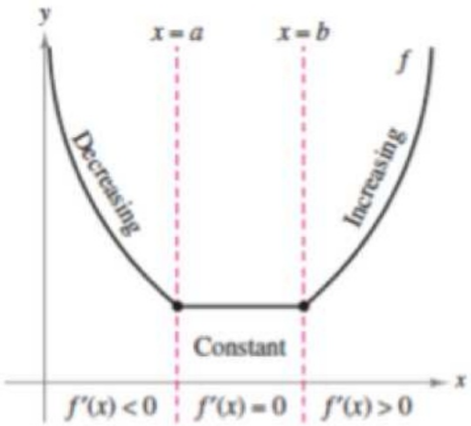




THEOREM 3.5 Test for Increasing and Decreasing Functions

Let f be a function that is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b).

1. If $f'(x) > 0$ for all x in (a, b), then f is increasing on $[a, b]$.
2. If $f'(x) < 0$ for all x in (a, b), then f is decreasing on $[a, b]$.
3. If $f'(x) = 0$ for all x in (a, b), then f is constant on $[a, b]$.



f	Inc	Dec
f'	+	−

ex: On what interval(s) is f(x) increasing and decreasing? Justify your answer.

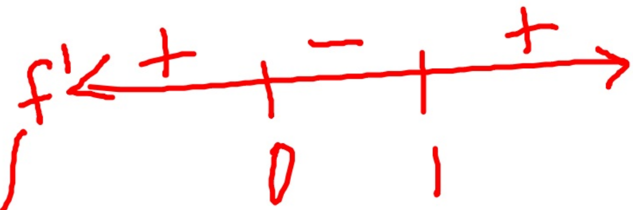
D: $(-\infty, \infty)$

a) $f(x) = x^3 - \frac{3}{2}x^2$

$f'(x) = 3x^2 - 3x$

$0 = 3x(x-1)$

$x = 0, 1$



f is incr. on (-inf,0) U (1, inf) because f'>0 on these intervals

f is decr. on (0,1) because f'<0 on this interval

ex: On what interval(s) is f(x) increasing and decreasing? Justify your answer.

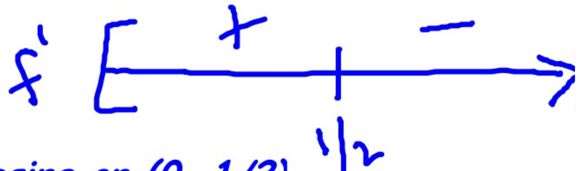
$D: [0, \infty)$

b) $f(x) = \sqrt{x}e^{-x}$

$$f'(x) = \sqrt{x}\left(-e^{-x}\right) + e^{-x} \cdot \frac{1}{2\sqrt{x}}$$

$$= \frac{-2xe^{-x} + e^{-x}}{2\sqrt{x}}$$

$$= \frac{e^{-x}(-2x+1)}{2\sqrt{x}}$$

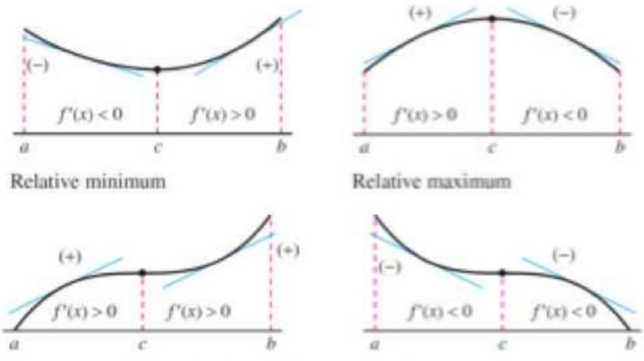


f is increasing on (0, 1/2)
because f'>0 on this interval
f is decreasing on (1/2, inf)
because f' < 0 on this interval.

THEOREM 3.6 The First Derivative Test

Let c be a critical number of a function f that is continuous on an open interval I containing c. If f is differentiable on the interval, except possibly at c, then $f(c)$ can be classified as follows.

1. If $f'(x)$ changes from negative to positive at c, then f has a *relative minimum* at $(c, f(c))$.
2. If $f'(x)$ changes from positive to negative at c, then f has a *relative maximum* at $(c, f(c))$.
3. If $f'(x)$ is positive on both sides of c or negative on both sides of c, then $f(c)$ is neither a relative minimum nor a relative maximum.



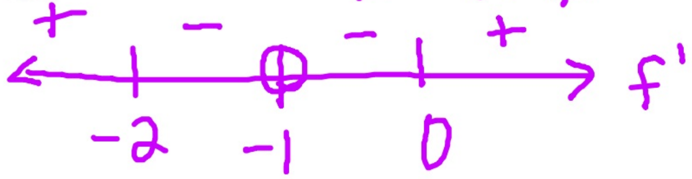
Relative minimum

Relative maximum

Neither relative minimum nor relative maximum ex: Determine the value(s) of x in which f(x) has local extrema. Justify your answer.

$D: \{x \mid x \neq -1\}$

a) $f(x) = \dfrac{x^2}{x+1}$



$$f'(x) = \frac{x^2+2x}{(x+1)^2}$$

$$= \frac{x(x+2)}{(x+1)^2}$$

Local max at x = -2 because f' changes from pos. to neg. at this point.

Local min at x= 0 becaue f' changes from neg. to to pos. at this point

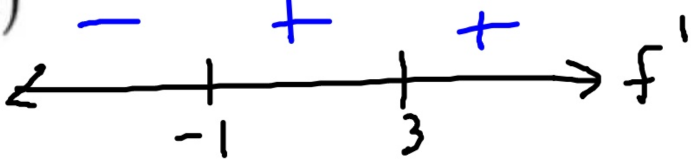
ex: $f(x) = \begin{cases} -x^3 + 1, & x \leq 1 \\ x^2 - 4x, & x > 1 \end{cases}$

$f'(x) = \begin{cases} -3x^2, & x < 1 \\ 2x - 4, & x > 1 \end{cases}$

a) At what x-values does f(x) have relative extrema? Justify your answer.

Relative min. at x = 2 because f' is changing from negative to positive at this point.

ex: $f'(x) = (x-3)^{4/5}(x+1)^{1/5}$



a) On what intervals is f(x) increasing and decreasing? Justify your answer.

incr: $(-1, 3) \cup (3, \infty)$

decr: $(-\infty, -1)$

3.3 WKST

The figures below show the graph of f'. For each of the functions find:

a) all x-values of critical numbers of f.

b) intervals of increasing and decreasing on f.

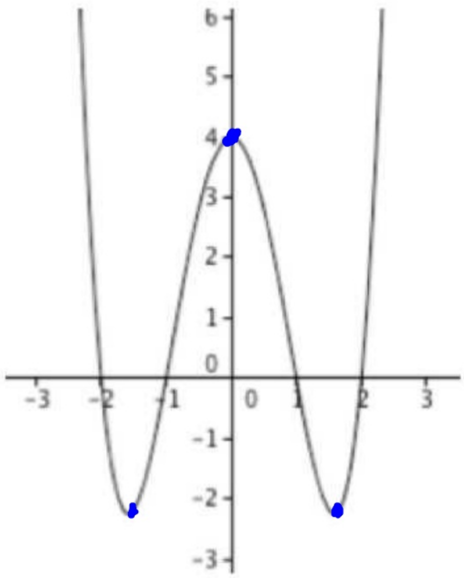
c) all x-values at which f has relative extrema.

JUSTIFY all of your answers using f'.

*See printout.

3.3 WKST

1.



Crit. numb: $x = -1.5, 0, 1.5$

rel. min: $(-1.5, -2)$

$(1.5, -2)$

rel. max $(0, 4)$

Decr: $(-\infty, -1.5) \cup (0, 1.5)$

Incr: $(-1.5, 0) \cup (1.5, \infty)$

ex:

The derivative of a function f is given by $\boxed{f'(x) = e^{\sin x} - \cos x - 1}$ for $0 < x < 9$. On what intervals is f decreasing?

(A) $0 < x < 0.633$ and $4.115 < x < 6.916$

(B) $0 < x < 1.947$ and $5.744 < x < 8.230$

(C) $0.633 < x < 4.115$ and $6.916 < x < 9$

(D) $1.947 < x < 5.744$ and $8.230 < x < 9$

ex:

The derivative of the function f is given by $f'(x) = -\frac{x}{3} + \cos\left(x^2\right)$. At what values of x does f have a relative minimum on the interval $0 < x < 3$?

(A) 1.094 and 2.608

(B) 1.798

(C) 2.372

(D) 2.493