49.) $f(x) = x\log_2 x \qquad [1, 2]$

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

$$\cancel{x}\cdot\frac{1}{\ln 2}\cdot\frac{1}{\cancel{x}} + \log_2 x = 2$$

$$\frac{1}{\ln 2} + \frac{\ln x}{\ln 2} = 2$$

$$1 + \ln x = 2\cdot\ln 2$$

$$\underbrace{\ln e + \ln x} = \ln 4$$

$$e^{\ln(ex)} = e^{\ln(4)}$$

$$ex = 4$$

$$x = \frac{4}{e}$$

$$y = \log_a u$$

$$y' = \frac{1}{\ln a} \cdot \frac{1}{u} \cdot u'$$

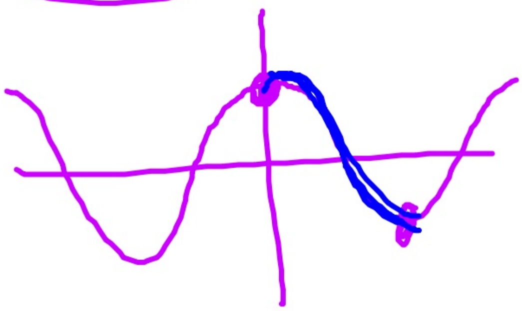
57.) $\frac{s(3)-s(0)}{3-0} = -14.7 \text{ m/sec}$

b) $-14.7 = -9.8t$

$1.5 = t$

Domain

$y=\cos x$	$y=\cos^{-1} x$
D: $(-\infty,\infty)$	D: $[-1,1]$
R: $[-1,1]$	R: $[0,\pi]$



Tabular Data

Review: 4 Existence Theorems

1. IVT

Conditions: *Continuity on [a,b] and f(a) $\neq$ f(b) c is between a and b and k is between f(a) and f(b)*

Conclusion:

f(c) = k at least once on [a,b]

Review: 4 Existence Theorems

2. EVT

Conditions: Continuity on [a,b]

Conclusion: The function will have a minimum and a maximum on the interval.

Review: 4 Existence Theorems

3. MVT

Conditions:

Continuity on [a, b]
Differentiability on (a, b)

Conclusion:

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Review: 4 Existence Theorems

4. Rolle's Theorem

Conditions: Continuity on [a,b]
Differentiability on (a,b)
$f(a) = f(b)$

Conclusion:

$f'(c) = 0$

ex: Consider the differentiable function v(t) with select values given in the table below.

t (min)	0	5	10	15	20	25	30
v(t) (meters/min)	7	9.2	9.5	7	4.5	2.4	2.4

a) Estimate a(7). Indicate units of measure.

$$\frac{v(10)-v(5)}{10-5} = \frac{.3}{5} = \frac{3}{50}\ m/min^2$$

t (min)	0	5	10	15	20	25	30
v(t) (meters/min)	7	9.2	9.5	7	4.5	2.4	2.4

b) Estimate a(20). Indicate units of measure.

(15,7) (25, 2.4)

or

* (15, 7) (20, 4.5) $\Rightarrow \dfrac{4.5-7}{20-15} = \dfrac{-2.5}{5}$ m/min^2

or

* (20, 4.5) (25, 2.4)

v(t) is differentiable.

t (min)	0	5	10	15	20	25	30
v(t) (meters/min)	7	9.2	9.5	7	4.5	2.4	2.4

c) What is the smallest number of instances in which v(t)=8 on (0, 30)? Justify your answer. $\boxed{2}$

Since v(t) is differentiable, then v(t) is also continuous. and v(0) < 8 < v(5), by IVT there must exist a value c in (0,5) such that v(c) = 8

t (min)	0	5	10	15	20	25	30
v(t) (meters/min)	7	9.2	9.5	7	4.5	2.4	2.4

d) What is the smallest number of instances in which $a(t)=0$ on $(0, 30)$? Justify your answer. 2

Since v(t) is differentiable and therefore continuous on [0,15] and v(0) = v(15) by Rolle's Theorem there must exist a value c in (0,15) such that f'(c) = 0

t (min)	0	5	10	15	20	25	30
v(t) (meters/min)	7	9.2	9.5	7	4.5	2.4	2.4

e) On the interval (0, 20) must there be a time when a(t)=-1/8? Justify your answer.

$$\frac{v(20)-v(0)}{20-0} = \frac{4.5-7}{20} = \frac{-2.5}{20} = \frac{-25}{200} = -\frac{1}{8}$$

Since v(t) is differentiable and therefore continuous, by MVT there must exist a value c such that v'(c) = -1/8

In general, when given a differentiable function f(x) and asked to show....

1. $f(x)=c$

Use: *IVT*

Show: *State the continuity, state the inequality say that $f(k) = c$ at some k-value.*

In general, when given a differentiable function f(x) and asked to show....

2. $f'(x)=c$

Use: *MVT*

Show: *Continuity, differentiability and slope of tangent equals slope of secant*

In general, when given a differentiable function f(x) and asked to show....

3. $f'(x)=0$

Use: Rolle's or MVT

Show:
Rolle's (3 criteria)
MVT (2 criteria)

Existence Theorems - AP Style Questions

$f(x) = x^3 - x - 1 \quad [-1, 2]$

1.

Let $f(x) = x^3 - x - 1$. On the interval $[-1,2]$ where does the instantaneous rate of change f equal the average rate of change of f on that interval?

a) $-\frac{1}{2}$
b) -1, 1
c) 0
d) 1
e) $\frac{1}{2}$

$m_{sec} = 2$

$3x^2 - 1 = 2$

$x = \pm 1$

*See printout.

2.

Which of the following functions below satisfy the conditions of the MVT?

I. $f(x)=\frac{1}{x+1}$, $[0,2]$ II. $f(x)=x^{1/3}$, $[0,1]$ III. $f(x)=|x|$, $[-1,1]$ X

a) I only
b) I and II only
c) I and III only
d) II only
e) II and III only

3.

Consider the following statements:

I. $f(x)$ is continuous on $[a,b]$

II. $f(x)$ is differentiable on (a,b)

III. $f(a)=f(b)$

Which of the above statements are required in order to guarantee $c\in(a,b)$ such that $f'(c)(\cancel{b-a})=\frac{f(b)-f(a)}{b-a}$?

a) I only
b) I and II only
c) I, II and III
d) III only
e) I and III only

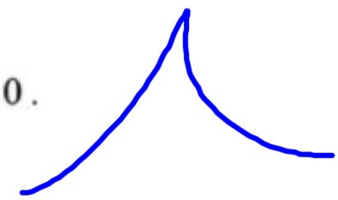
4.

If f is a continuous function on $[a,b]$, which of the following is necessarily true?

(A) f' exists on (a,b).

(B) If $f(x_0)$ is a maximum of f, then $f'(x_0)=0$.

(C) $\lim_{x\to x_0} f(x) = f\left(\lim_{x\to x_0} x\right)$ for $x_0 \in (a,b)$



(D) $f'(x)=0$ for some $x\in[a,b]$

(E) The graph of f' is a straight line.

5.

If f is continuous for $a \le x \le b$ and differentiable for $a < x < b$, which of the following could be false?

be

(A) $f'(c) = \dfrac{f(b) - f(a)}{b - a}$ for some c such that $a < c < b$.

(B) $f'(c) = 0$ for some c such that $a < c < b$.

(C) f has a minimum value on $a \le x \le b$.

(D) f has a maximum value on $a \le x \le b$.

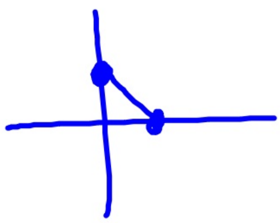
6.

Let g be a continuous function on the closed interval $[0,1]$. Let $g(0)=1$ and $g(1)=0$. Which of the following is NOT necessarily true?

(A) There exists a number h in $[0,1]$ such that $g(h) \geq g(x)$ for all x in $[0,1]$. EVT

(B) For all a and b in $[0,1]$, if $a=b$, then $g(a)=g(b)$.

(C) There exists a number h in $[0,1]$ such that $g(h)=\frac{1}{2}$. IVT



(D) There exists a number h in $[0,1]$ such that $g(h)=\frac{3}{2}$.

(E) For all h in the open interval $(0,1)$, $\lim_{x \to h} g(x) = g(h)$.

7.

x	0	1	2
$f(x)$	1	k	2

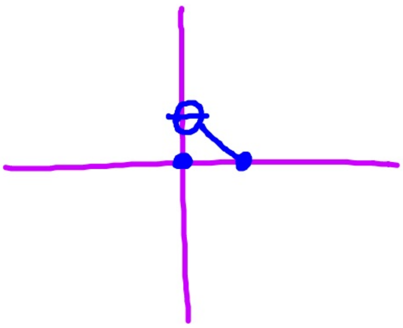
0

The function f is continuous on the closed interval $[0,2]$ and has values that are given in the table above. The equation $f(x)=\frac{1}{2}$ must have at least two solutions in the interval $[0,2]$ if $k=$

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) 3

8.

The function $f(x)=\begin{cases}0, & x=0\\ 1-x, & 0<x\le 1\end{cases}$ is differentiable on $(0,1)$ and satisfies $f(0)=f(1)$. However, its derivative is never zero on $(0,1)$. Does this contradict the Mean Value Theorem? Explain why or why not.



Not contradicting MVT because $f(x)$ is not continuous on $[0,1]$

9.

Determine the values of a, b, and c such that the function f satisfies the hypothesis of the MVT on the interval $[0,3]$.

$$f(x)=\begin{cases}1, & x=0\\ ax+b, & 0<x\le 1\\ x^2+4x+c, & 1<x\le 3\end{cases}$$