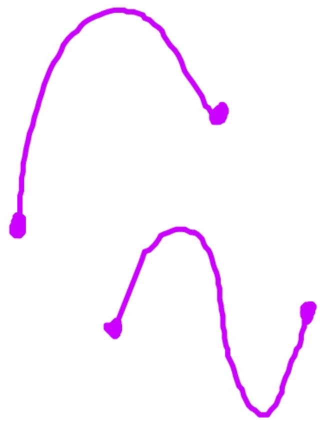
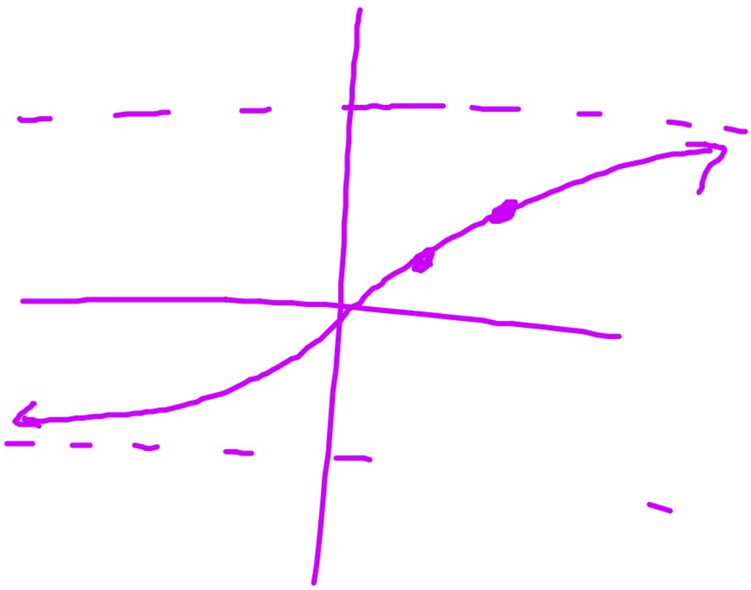
17) $f(t) = te^{-2t}$

$f'(t) = t \cdot -2e^{-2t} + e^{-2t} \cdot 1$

$0 = e^{-2t}(-2t+1)$ $[1, 10]$

$t = +\frac{1}{2}$





30.) $h(t) = \dfrac{t}{t+3}$ $\quad [-1, 6]$

$$h'(t) = \frac{3}{(t+3)^2}$$

t	$h(t)$
-1	
6	

40.) $g(x) = \dfrac{\ln x}{x}$ $[1, 4]$

$x > 0$

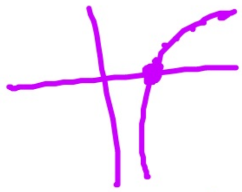
$$g'(x) = \frac{1 - \ln x}{x^2}$$

$$1 - \ln x = 0$$

$$1 = \ln x$$

$$e^1 = e^{\ln x}$$

	x	$g(x)$
min	1	0
max	e	$\frac{1}{e}$
	4	$\frac{\ln 4}{4}$



$\frac{1}{2}$ $\frac{1}{4}$

41.) $h(x) = 5e^{x} - e^{2x}$ $\quad [-1, 2]$

$h'(x) = 5e^{x} - 2e^{2x}$

$0 = e^{x}(5 - 2e^{x})$

$5 = 2e^{x}$

$\frac{5}{2} = e^{x}$

$\ln\frac{5}{2} = x$

$e^{2\ln\frac{5}{2}}$

$e^{\ln\frac{25}{4}}$

	x	$h(x)$
	-1	$\frac{5}{e} - \frac{1}{e^{2}}$
max	$\ln\frac{5}{2}$	$\frac{25}{2} - \frac{25}{4}$
min	2	$5e^{2} - e^{4}$

3.2 Rolle's Theorem and the Mean Value Theorem

HW p.224

ex: List the critical numbers of f(x).

$$f(x) = x^{4/5}(x-5)^2$$

$f'(x) = x^{4/5} \cdot \frac{10}{5}(x-5) + (x-5)^2 \frac{4}{5} x^{-1/5}$

$= x^{-1/5}(x-5)\frac{2}{5}\left(5x + (x-5)\cdot 2\right)$

$(7x-10)$

$x = \frac{10}{7}$

$x = 0$

$x = 5$

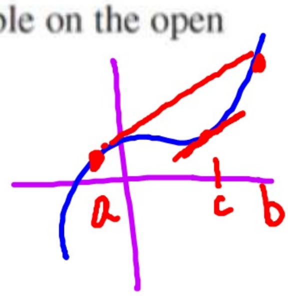
ex: Find the maximum and miniumum values of f(x) on the indicated interval.

$$f(x) = \sin x + \cos x, \quad \left[0, \frac{\pi}{2}\right]$$

THEOREM 3.4 The Mean Value Theorem

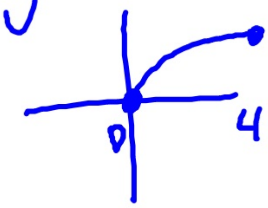
If f is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b), then there exists a number c in (a, b) such that

$$\underbrace{f'(c)}_{\text{slope of tangent}} = \underbrace{\frac{f(b)-f(a)}{b-a}}_{\text{slope of secant}}$$

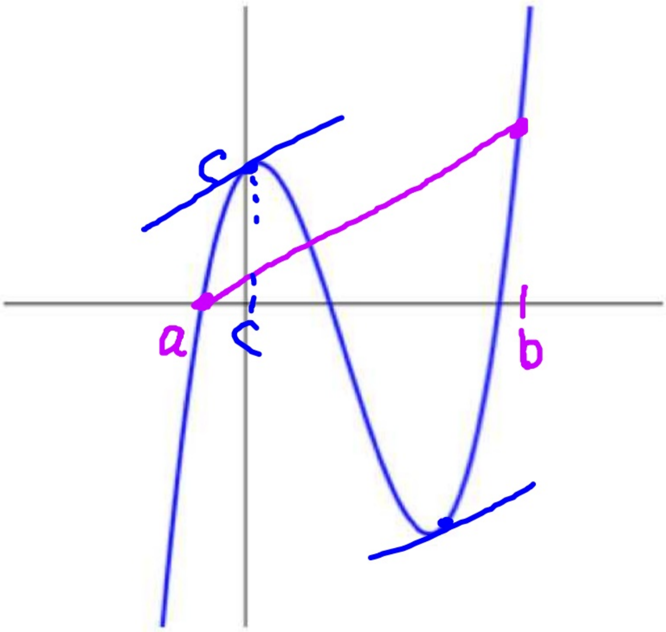


$y = \sqrt{x}$ $[0, 4]$

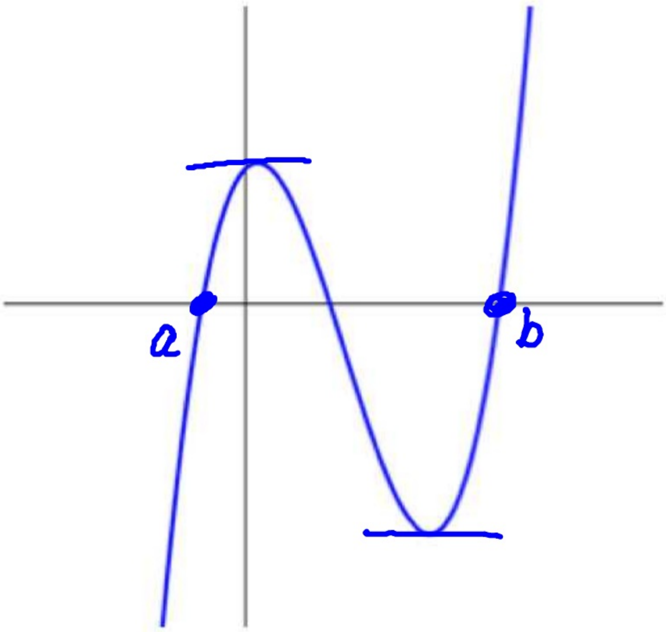
$(0, 4)$



The Graphical Interpretation of the Mean Value Theorem.



The Graphical Interpretation of the ~~Mean Value Theorem.~~ Rolle's



ex: Determine the value(s) of c guaranteed by the conclusion of the MVT on the given interval, if possible.

a) $f(x) = 5 - \frac{4}{x}, \quad [1,4]$

Check if theorem applies:
1) Continuity on [1,4] yes
2) Differentiability on (1,4) yes

$m_{sec} = m_{tan} (f'(x)$

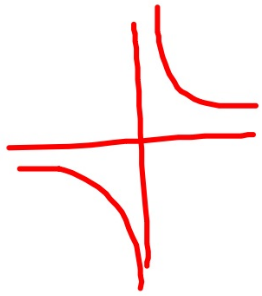
$\frac{f(4)-f(1)}{4-1} = \frac{4}{x^2}$

$\frac{4-1}{3} = \frac{4}{x^2}$

$x^2 = 4$

$x = \pm 2$

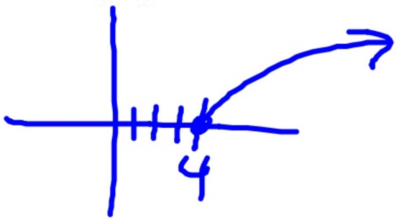
$\boxed{c = 2}$



ex: Determine the value(s) of c guaranteed by the conclusion of the MVT on the given interval, if possible.

b) $f(x) = \sqrt{x-4}, \quad [4,8]$

1) *Continuity [4,8] yes*
2) *Differentiability (4,8) yes*



$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$\frac{1}{2\sqrt{c-4}} = \frac{1}{2}$$

$$\sqrt{c-4}^{\,2} = 1^2$$

$$c - 4 = 1 \qquad c = 5$$

ex: Determine the value(s) of c guaranteed by the conclusion of the MVT on the given interval, if possible.

c) $f(x) = \frac{1}{x}, \quad [-1,1]$

1) Continuity on [-1,1] NO!!!

MVT does not apply; the function has a discontinuity at x = 0

ex: Determine the value(s) of c guaranteed by the conclusion of the MVT on the given interval, if possible.

d) $f(x) = |x|, \quad [-2,2]$

1) Continuous on [-2,2] Yes
2) Differentiable on (-2,2) NO!!

MVT does not apply. This function is not differentiable at x = 0.

$$f(x) = \sqrt[3]{x} \qquad [0, 8]$$

1) Continuity on [0,8] yes

2) Differentiability on (0,8) yes

$$\frac{1}{3x^{2/3}} = \frac{1}{4}$$

$$3x^{2/3} = 4$$

$$x^{2/3} = \left(\frac{4}{3}\right)^{3/2}$$

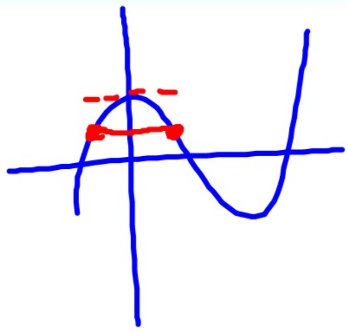
$$x = \pm \frac{\sqrt{64}}{\sqrt{27}} = \frac{8}{3\sqrt{3}}$$

$$c = \frac{8}{3\sqrt{3}}$$

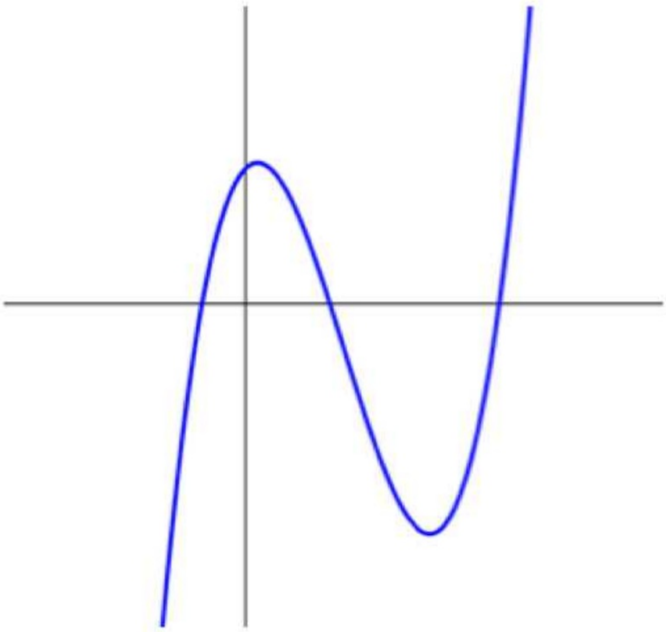
THEOREM 3.3 Rolle's Theorem

Let f be continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b). If $f(a) = f(b)$, then there is at least one number c in (a, b) such that

$$f'(c) = 0$$



The Graphical Interpretation of Rolle's Theorem.



ex: Determine the value(s) of c guaranteed by the conclusion of Rolle's Theorem on the given interval, if possible.

a) $f(x) = \cos\frac{x}{3}, \quad [0, 6\pi]$

1) Continuity on [0,6pi]
2) Differentiability (0, 6pi)
3) f(0) = f(6pi) = 1 yes

$f'(x) = \frac{-\sin\frac{x}{3}}{3}$

$0 = -\frac{1}{3}\sin\frac{x}{3}$

$0 = \sin\frac{x}{3}$

$0, \pi, 2\pi, 3\pi, \ldots = \frac{x}{3}$

$\cancel{0}, 3\pi, \cancel{6\pi}, \cancel{9\pi} = x$

$\boxed{c = 3\pi}$

$0 = \sin 2x$

$\frac{0, \pi, 2\pi, 3\pi \ldots}{2} = \frac{2x}{2}$

$0, \frac{\pi}{2}, \pi, \frac{3\pi}{2} \ldots = x$

ex: Determine the value(s) of c guaranteed by the conclusion of Rolle's Theorem on the given interval, if possible.

b) $f(x) = x, \quad [1, 20]$

Rolle's Theorem does not apply. f(1) is not equal to f(20) ex:

$f(x) = x^3 - 3x^2$

Let f be the function given by $f(x) = x^3 - 3x^2$. What are all values of c that satisfy the conclusion of the Mean Value Theorem of differential calculus on the closed interval $[0,3]$?

$[0,3]$

(A) 0 only (B) 2 only (C) 3 only (D) 0 and 3 (E) 2 and 3

	old		new	
no	28Q 55min		30Q 60 min	no
Calc	17Q 50min		15Q 45min	yes
	6Q			

FR:

Let f be the function given by $f(x) = x^3 - 7x + 6$. $f(x)=x^3-7x+6$

(a) Find the zeros of f. $x = 2, 1, -3$

(b) Write an equation of the line tangent to the graph of f at $x = -1$. $y - 12 = -4(x+1)$

(c) Find the number c that satisfies the conclusion of the Mean Value Theorem for f on the closed interval $[1, 3]$.

$c = \sqrt{\frac{13}{3}}$