3.1 Extrema on an Interval

Definition of Extrema

Let f be defined on an interval I containing c.

1. $f(c)$ is the **minimum of f on I** when $f(c) \leq f(x)$ for all x in I.
2. $f(c)$ is the **maximum of f on I** when $f(c) \geq f(x)$ for all x in I.

Types of Extrema

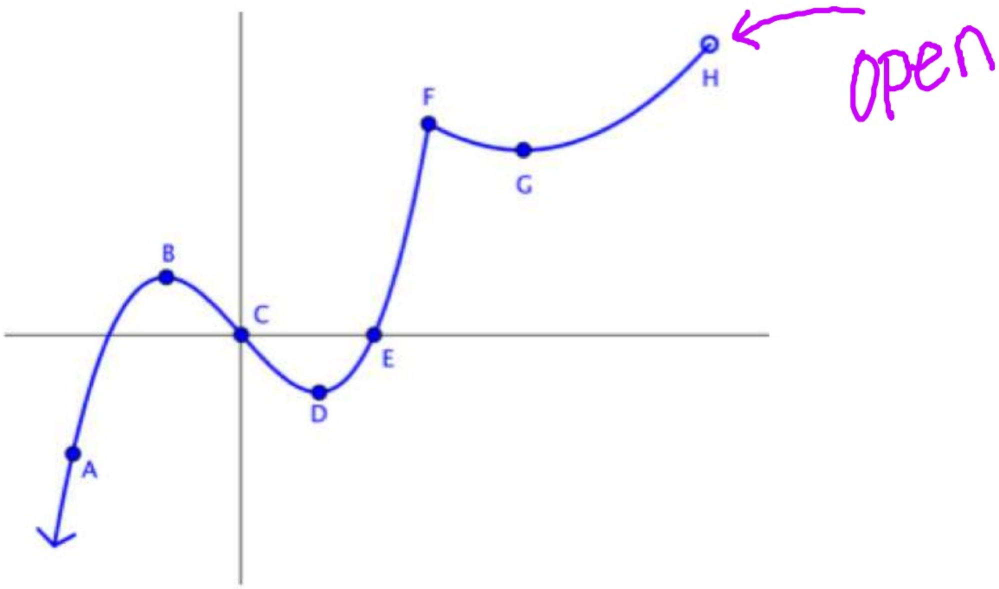
1. Absolute Extrema - the highest and lowest points on a curve (can occur ANYWHERE on a curve)

2. Relative Extrema - the highest and lowest points on a curve "in a neighborhood" (can occur ANYWHERE on an OPEN interval...no endpoints)
(a.k.a. "Local Extrema)

When asked "where" does f(x) have extrema answer with an x-value.

When asked the extreme "value" of f(x) answer with a y-value.

ex:



At what point(s), if any, does f(x) have a(n)

a) Absolute Maximum none

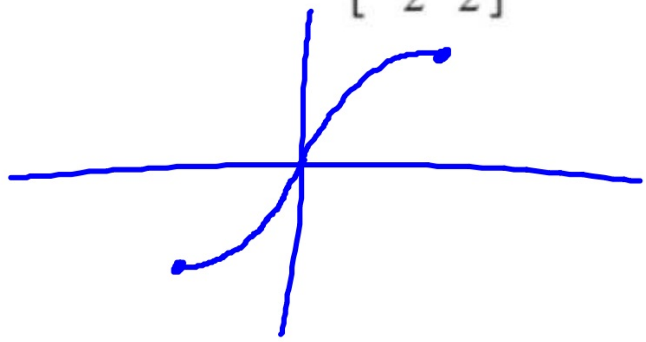
b)Absolute Minimum none

c) Relative Maximum B, F

d) Relative Minimum D, G

ex: $y = \sin x$

a) Sketch on the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



At what point(s), if any, does f(x) have a(n)

b) Absolute Maximum $\left(\frac{\pi}{2}, 1\right)$

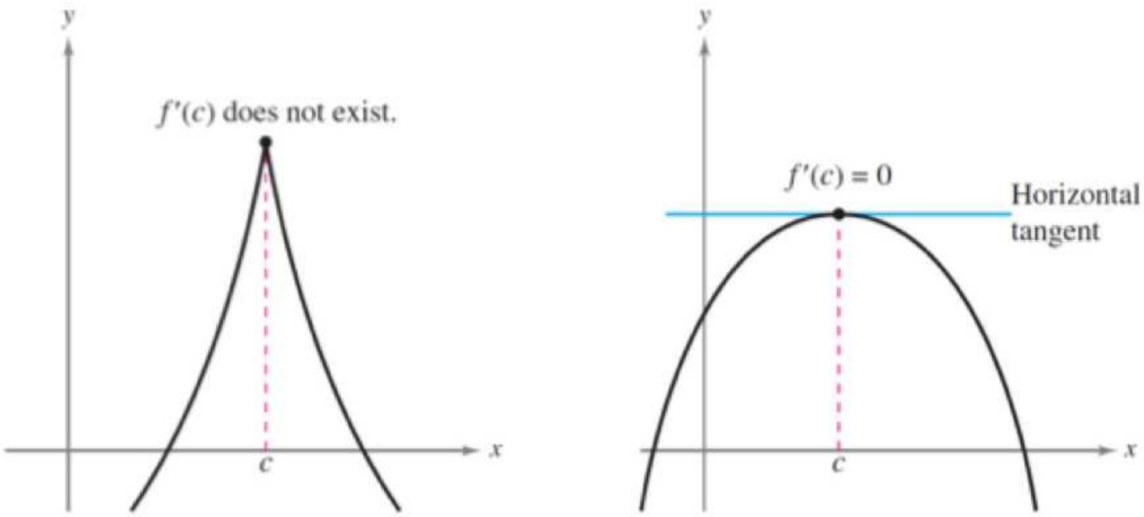
c)Absolute Minimum $\left(-\frac{\pi}{2}, -1\right)$

d) Relative Maximum none

e) Relative Minimum none

Definition of a Critical Number

Let f be defined at c. If $f'(c) = 0$ or if f is not differentiable at c, then c is a **critical number** of f.



ex: Find the critical numbers.

a) $f(x) = x^2 + 2x - 4$

$f'(x) = 2x + 2$

$0 = 2x + 2$

$\boxed{-1 = x}$

$x = \frac{-b}{2a} = \frac{-2}{2} = -1$

$f(x) = ax^2 + bx + c$

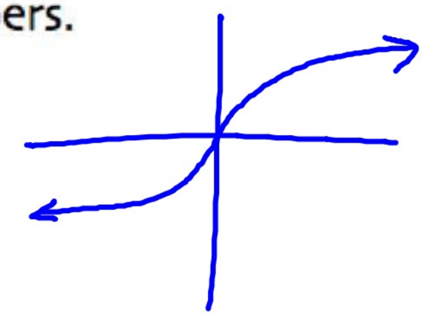
$f'(x) = 2ax + b$

$0 = 2ax + b$

$\frac{-b}{2a} = x$ ☺

ex: Find the critical numbers.

b) $f(x) = \sqrt[3]{x}$



$f'(x) = \frac{1}{3x^{2/3}}$

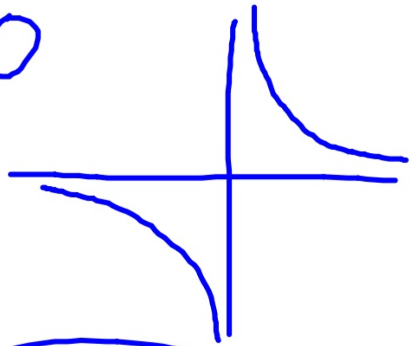
$3x^{2/3} = 0$

$\boxed{x = 0}$

ex: Find the critical numbers.

c) $f(x)=\frac{1}{x}$

$D: x \neq 0$



$f'(x)=\frac{-1}{x^2}$

$\cancel{x=0}$ not in the domain!

no critical numbers

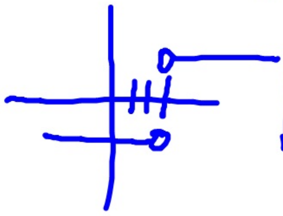
ex: Find the critical numbers.

$f(x) = |x|$

$f'(x) = \frac{|x|}{x}$

d) $f(x) = |x-3|$

$f'(x) = \frac{|x-3|}{x-3}$



$\boxed{x = 3}$

ex: Find the critical numbers.

e) $f(x) = (2x)(\ln x)$

$D: x > 0$

$f'(x) = 2x \cdot \frac{1}{x} + \ln x \cdot 2$

$0 = 2 + 2\ln x$

$0 = 2(1 + \ln x)$

make the derivative useful

$0 = 1 + \ln x$

$e^{-1} = e^{\ln x}$

$\boxed{x = \frac{1}{e}}$

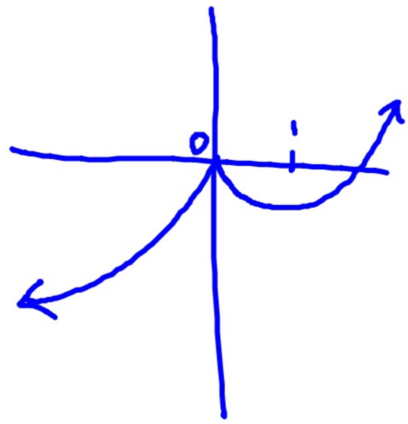
f.) $f(x) = 2x - 3x^{2/3}$

D: All reals

$f'(x) = 2 - 2x^{-1/3}$

$f'(x) = 2 - \frac{2}{x^{1/3}}$

$f'(x) = \frac{2x^{1/3} - 2}{x^{1/3}}$



$f'(x) = 0$ (top)

$2x^{1/3} - 2 = 0$

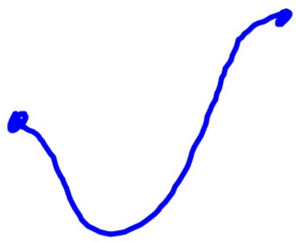
$x = 1$

$f'(x)$ does not exist (bottom)

$x^{1/3} = 0$; $x = 0$

THEOREM 3.1 **The Extreme Value Theorem** (EVT)

If f is continuous on a closed interval $[a, b]$, then f has both a minimum and a maximum on the interval.



ex: Find the maximum and minimum values of f(x) on the indicated interval.

a) $f(x) = 3x^4 - 4x^3, \quad [-1,2]$

$f'(x) = 12x^3 - 12x^2$

$0 = 12x^2(x-1)$

$x = 0, 1$

Absolute extrema : points

1. Find f'(x)
2. Find the critical numbers
3. Set up a chart with the crit. numbers and the endpoints.
4. Write the coordinate(s) of the max and min

x	f(x)
-1	7
0	0
1	-1
2	16

* min value is -1
* max value is 16

y-value

ex: Find the maximum and minimum values of f(x) on the indicated interval.

$\sin 2x = 2\sin x\cos x$

b) $f(x) = 2\sin x - \cos 2x, \quad [0, \pi]$

$f'(x) = 2\cos x + 2\sin 2x$

$f'(x) = 2\cos x + 4\sin x\cos x$

$0 = 2\cos x(1 + 2\sin x)$

$\cos x = 0 \qquad 1 + 2\sin x = 0$

$x = \frac{\pi}{2} \qquad x =$ not in interval

x	f(x)
0	-1
π/2	3
π	-1

max value is 3
min value is -1

ex: What is the maximum acceleration on the interval [0,3] if the velocity is modeled by the equation

$$v(t) = t^3 - 3t^2 + 12t + 4$$

$$v'(t) = a(t) = 3t^2 - 6t + 12$$

$$a'(t) = 6t - 6$$

$$1 = t$$

t	a(t)
0	12
1	9
3	21

$\boxed{\text{max accel: } 21}$

ex: Sketch a function with the given characteristics.

Relative minimum at $x = -1$

Critical number (but no extremum) at $x = 0$

Absolute maximum at $x = 2$

Absolute minimum at $x = 5$