2.5 Implicit Differentiation

Implicit Form	*Explicit Form*
$xy = 1$	$y = \frac{1}{x} = x^{-1}$

ex: If $x^2 - y^2 = 16$ find dy/dx.

$$y = \pm\sqrt{x^2-16}$$

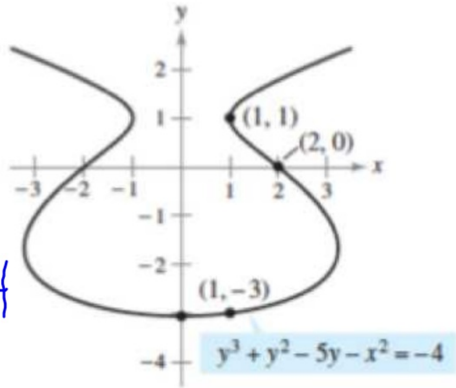
$$y' = \pm\frac{1}{2}(x^2-16)^{-1/2}\cdot 2x$$

$$y' = \frac{\pm x}{\sqrt{x^2-16}}$$

2.5: Implicit Differentiation

Implicit differentiation is nessecary to derive equations that can only be expressed implicitly.

$y^3 + y^2 - 5y - x^2 = -4$



$y = \pm\sqrt{x^2-16}$

ex: If $x^2 - y^2 = 16$ find dy/dx implicitly.

$$\frac{d}{dx}\left(x^2 - y^2 = 16\right)$$

$$2x \cdot \cancel{\frac{dx}{dx}} - 2y \cdot \frac{dy}{dx} = 0$$

$$x - y\frac{dy}{dx} = 0$$

$$\boxed{\frac{dy}{dx} = \frac{x}{y}}$$

explicit

$$y' = \frac{\pm x}{\sqrt{x^2-16}}$$

$$\frac{x}{y} = \frac{x}{\pm\sqrt{x^2-16}}$$

ex: Find dy/dx.

$$\frac{d}{dx}[f(x)]$$

a) $\left(x^2 - 2y^3 + 4y = 2\right)\frac{d}{dx}$

$$2x\cdot\cancel{\frac{dx}{dx}} - 6y^2\frac{dy}{dx} + 4\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}\left(-6y^2+4\right) = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{-6y^2+4} = \frac{2x}{6y^2-4} = \frac{x}{3y^2-2}$$

ex: Find dy/dx.

$$\frac{d}{dx}\left[x^3\right] = 3x^2 \cdot \frac{\cancel{dx}}{\cancel{dx}}$$

b) $\left(x^2 y - 2\cos 3x = 3\right)\frac{d}{dx}$

$$\underbrace{x^2 \cdot 1 \cdot \frac{dy}{dx}} + y \cdot 2x\frac{\cancel{dx}}{\cancel{dx}} + 6\sin 3x \cdot \frac{\cancel{dx}}{\cancel{dx}} = 0$$

$$\frac{dy}{dx} = \frac{-6\sin 3x - 2xy}{x^2}$$

ex: Find dy/dx.

c) $\left(3e^{2x} + \cos y = 1111^3\right)\frac{d}{dx}$

$$6e^{2x} - \sin y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{6e^{2x}}{\sin y}$$

ex: Find dy/dx.

e) $\left(y = \sin(xy)\right) \frac{d}{dx}$

$\frac{dy}{dx} = y'$

$\frac{dy}{dx} = \cos(xy) \cdot \left(x \cdot \frac{dy}{dx} + y \cdot 1\right)$

$\frac{dy}{dx} = \cos(xy)\, x \frac{dy}{dx} + \cos(xy) \cdot y$

$\frac{dy}{dx} - \cos(xy)\, x \frac{dy}{dx} = \cos(xy) y$

$\frac{dy}{dx} = \frac{y\cos(xy)}{1 - x\cos(xy)}$

ex: If $x^2 + y^2 = 1$ find d^2y/dx^2.

$$-y^2 - x^2 = -(y^2 + x^2) = -1$$

$$\frac{dy}{dx} = \frac{-x}{y}$$

$$\frac{d}{dx}\left(\frac{dy}{dx} = \frac{-x}{y}\right)$$

$$\frac{d^2y}{(dx)^2} = \frac{y(-1) - (-x)\left(\frac{dy}{dx}\right)}{y^2} = \left(\frac{-y + x\left(\frac{-x}{y}\right)}{y^2}\right)\frac{\cdot y}{\cdot y}$$

$$= \frac{-y^2 - x^2}{y^3} = \frac{-1}{y^3}$$

ex: $x^2 - (xy) + y^2 = 7$

$$2x - \left(x\frac{dy}{dx} + y\right) + 2y\frac{dy}{dx} = 0$$

a) Find dy/dx.

$$2x - x\frac{dy}{dx} - y + 2y\frac{dy}{dx} = 0$$

$$-x\frac{dy}{dx} + 2y\frac{dy}{dx} = y - 2x$$

b) Find the slope at the point (-1, 2).

$$\frac{dy}{dx} = \frac{y-2x}{2y-x}$$

$$\left.\frac{dy}{dx}\right|_{(-1,2)} = \frac{4}{5}$$

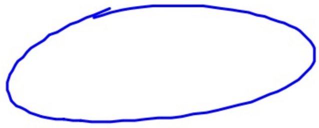
c) Write an equation of the tangent line to the graph at the point (-1, 2).

$$y - 2 = \frac{4}{5}(x+1)$$

d) Write an equation of the normal line to the graph at the point (-1, 2).

ex: $4x^2 + y^2 - 8x + 4y + 4 = 0$

$$\frac{dy}{dx} = \frac{-8x+8}{2y+4}$$



a) Find the points, if any, at which the equation has a horizontal tangent line.

set num = 0

$-8x+8=0$

$x=1$

$(1, 0)$ $(1, -4)$

b) Find the points, if any, at which the equation has a vertical tangent line.

set den. = 0

$2y+4=0$

$y=-2$

$(0, -2)$ $(2, -2)$

Logarithmic Differentiation

When given a complicated equation it is often convenient to use logarithms as aids in differentiating nonlogarithmic functions. This process is called logarithmic differentiation.

$y = x^n$

Candidates for Logarithmic Differentiation:

- $y = \dfrac{(x-2)^2}{\sqrt{x^2+1}}$

- $y = x^{x-1}$

$$\left(\ln y\right) = \ln\left(5x\sqrt[3]{(3x+1)^2}\right)$$

$$\frac{d}{dx}\left(\ln y = \ln 5x + \frac{2}{3}\ln(3x+1)\right)$$

$$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{1}{x} + \frac{2}{\cancel{3}}\cdot\frac{\cancel{3}}{3x+1}$$

$$\frac{dy}{dx} = \left(\frac{1}{x} + \frac{2}{3x+1}\right)\cdot y$$

$$= \left(\frac{1}{x} + \frac{2}{3x+1}\right)5x\sqrt[3]{(3x+1)^2}$$

$$= \frac{5x+1}{x(3x+1)}\cdot 5x(3x+1)^{2/3} = \frac{5(5x+1)}{\sqrt[3]{3x+1}}$$

27.) $\ln(y) = \ln\left(\frac{x}{\sqrt{x^2+1}}\right)$

$$\frac{d}{dx}\left(\ln y = \ln x - \frac{1}{2}\ln(x^2+1)\right)$$

$$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{1}{x} - \frac{x}{x^2+1}$$

$$\boxed{\frac{1}{(x^2+1)^{3/2}}}$$

$$\frac{dy}{dx} = \frac{x^2+1-x^2}{x(x^2+1)}\left(\frac{x}{\sqrt{x^2+1}}\right)$$

$$= \frac{1}{(x^2+1)^{3/2}}$$

ex: Differentiate.

a) $y = \dfrac{(x-2)^2}{\sqrt{x^2+1}}$

ex: Differentiate.

b) $y = x^{x-1}$

$$\ln y = \ln x^{x-1}$$

$$\frac{d}{dx}\left(\ln y = (x-1)\ln x\right)$$

$$\frac{1}{y}\cdot\frac{dy}{dx} = (x-1)\frac{1}{x} + \ln x \cdot 1$$

$$\frac{dy}{dx} = \left(\frac{x-1}{x} + \ln x\right)\cdot x^{x-1} = \left(\frac{x-1+x\ln x}{x^{1}}\right)x^{x-1}$$

$$= x^{x-2}\left(x-1+x\ln x\right)$$