23) $f(x) = x^2 (x-2)^4$

$$f'(x) = \underbrace{x^2 \cdot 4(x-2)^3} + \underbrace{(x-2)^4 \cdot 2x}$$

$$= 2x(x-2)^3\big(2x + (x-2)\big)$$

$$= 2x(x-2)^3(3x-2)$$

55.) $f(x) = \dfrac{\dfrac{\cos x}{\sin x}}{\sin x} = \dfrac{\cos x}{\sin^2 x}$

$$f'(x) = \frac{\sin^2 x(-\sin x) - \cos x \cdot 2\sin x\cos x}{\sin^4 x}$$

$$= \frac{-\sin^3 x - 2\cos^2 x\sin x}{\sin^4 x}$$

$$= \frac{-\sin^2 x - 2\cos^2 x}{\sin^3 x} = \frac{-(1-\cos^2 x) - 2\cos^2 x}{\sin^3 x} = \frac{-1-\cos^2 x}{\sin^3 x}$$

63) $f(t) = 3(\sec(\pi t - 1))^2$

$f'(t) = 6\pi(\sec(\pi t - 1))^1 \cdot \sec(\pi t - 1)\tan(\pi t - 1)$

27) $y = \frac{x}{\sqrt{x^2+1}} = x\left(x^2+1\right)^{-1/2}$

$$y = x \cdot \frac{-1}{\cancel{2}}\left(x^2+1\right)^{-3/2} \cdot \cancel{2}x + \left(x^2+1\right)^{-1/2} \cdot 1$$

$$= \left(x^2+1\right)^{-3/2}\left(-x^2 + \left(x^2+1\right)^{1}\right) = \frac{1}{\left(x^2+1\right)^{3/2}}$$

2.4 Chain Rule Cont.

ex: Differentiate.

a) $y = \csc(7x)$

$y' = -\csc(7x)\cot(7x) \cdot 7$

b) $y = \dfrac{6}{(3x^2-5)^5} = 6(3x^2-5)^{-5}$

$y' = -30(3x^2-5)^{-6} \cdot 6x$

THEOREM 2.13 **Derivative of the Natural Logarithmic Function**

Let u be a differentiable function of x.

1. $\frac{d}{dx}[\ln x] = \frac{1}{x}$

2. $\frac{d}{dx}[\ln u] = \frac{1}{u} \cdot u'$

ex: Differentiate.

a) $y = \ln(2x)$

$y' = \frac{1}{2x} \cdot 2 = \frac{1}{x}$

$y = \ln 2 + \ln x$

$y' = 0 + \frac{1}{x} = \frac{1}{x}$

ex: Differentiate.

b) $y = \underbrace{x}\underbrace{\ln x}$

$y' = x \cdot \frac{1}{x} + \ln x \cdot 1$

$y' = 1 + \ln x$

ex: Differentiate.

c) $y = \ln\sqrt{x^2 + 8e^x}$

$$y = \frac{1}{2}\ln(x^2 + 8e^x)$$

$$y' = \frac{1}{2} \cdot \frac{1}{x^2 + 8e^x} \cdot (2x + 8e^x) = \frac{x + 4e^x}{x^2 + 8e^x}$$

ex: Differentiate.

d) $f(x) = \ln(x \sin x)$

$\ln 4$

$\ln 2^2$

$2 \ln 2$

$f(x) = \ln x + \ln(\sin x)$

$f'(x) = \frac{1}{x} + \frac{1}{\sin x} \cdot \cos x$

$= \frac{1}{x} + \frac{\cos x}{\sin x} = \frac{\sin x + x\cos x}{x \sin x}$

THEOREM 2.14 Derivative Involving Absolute Value

If u is a differentiable function of x such that $u \neq 0$, then

$$\frac{d}{dx}[\ln|u|] = \frac{1}{u} \cdot u'$$

ex: Differentiate.

a) $f(x) = \ln|\csc x|$

$$\ln \frac{1}{\sin x} = \ln 1 - \ln \sin x$$

$$0 - \frac{1}{\sin x} \cos x$$

$$f'(x) = \frac{1}{\csc x} \cdot (-\csc x \cot x)$$

$$= -\cot x$$

ex: Differentiate.

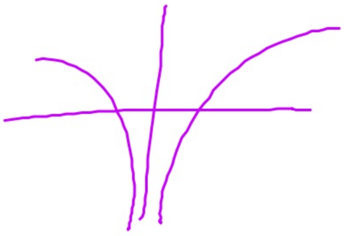
b) $f(x) = \ln\left|\dfrac{\cos x - 2}{e^x}\right|$

$f(x) = \ln|\cos x - 2| - \cancel{\ln e^{x}}\ x$

$f'(x) = \dfrac{-\sin x}{\cos x - 2} - 1$

ex: Differentiate.

c) $y = \left|\ln x\right|$



$y = |x|$

$$y' = \begin{cases} -\dfrac{1}{x}, & x < 0 \\ \dfrac{1}{x}, & x > 0 \end{cases}$$

$$y' = \begin{cases} -1, & x < 0 \\ 1, & x > 0 \end{cases}$$

Definition of Logarithmic Function to Base a

If a is a positive real number ($a \neq 1$) and x is any positive real number, then the **logarithmic function to the base a** is denoted by $\log_a x$ and is defined as

$$\log_a x = \frac{\log x}{\log a} = \frac{\ln x}{\ln a} = \frac{1}{\ln a}\ln x$$

Differentiate

$$y = \log_a x$$

$$y' = \frac{1}{\ln a} \cdot \frac{1}{x}$$

ex: Differentiate.

a) $y = \log_7 x$

$$y' = \frac{1}{\ln 7} \cdot \frac{1}{x}$$

$$= \frac{1}{x \ln 7}$$

ex: Differentiate.

b) $y = \log_6 \dfrac{x\sqrt{x-2}}{5}$

$y = \log_6 x + \frac{1}{2}\log_6(x-2) - \log_6 5$

$y' = \frac{1}{\ln 6} \cdot \frac{1}{x} + \frac{1}{2} \cdot \frac{1}{\ln 6} \cdot \frac{1}{x-2}$

$= \frac{1}{\ln 6}\left(\frac{1}{x} + \frac{1}{2(x-2)}\right)$

Derivative of the Exponential Function

Let u be a differentiable function of x.

$$\frac{d}{dx}\left[a^x\right] = \ln a \cdot a^x \qquad \frac{d}{dx}\left[e^x\right] = e^x$$

$$\frac{d}{dx}\left[a^u\right] = \ln a \cdot a^u \cdot u' \qquad \frac{d}{dx}\left[e^u\right] = e^u \cdot u'$$

ex: Differentiate.

a) $y = 3^x$

$y' = \ln 3 \cdot 3^x$

$y = e^{5x}$

$y' = \cancel{\ln e} \cdot e^{5x} \cdot 5$

b) $y = e^{5x}$

$y' = e^{5x} \cdot 5$

$y' = 5e^{5x}$

ex: Differentiate.

c) $y = 5^{\ln x}$

$y' = \ln 5 \cdot 5^{\ln x} \cdot \frac{1}{x}$

$= \frac{\ln 5 \cdot 5^{\ln x}}{x}$

d) $f(x) = 4^{\log_4 x}$

$f(x) = x$

$f'(x) = 1$

ex: Differentiate.

e) $y = \frac{3}{e^x} = 3e^{-x}$

$y = 3e^{-x}(-1)$

$y = -3e^{-x}$

f) $y = \ln\left(\frac{1+e^x}{1-e^x}\right) = \ln(1+e^x) - \ln(1-e^x)$

$y' = \frac{1}{1+e^x} \cdot e^x - \frac{1}{1-e^x} \cdot (-e^x) = \frac{e^x}{1+e^x} + \frac{e^x}{1-e^x}$

$= \frac{e^x(1-e^x) + e^x(1+e^x)}{1-e^{2x}} = \frac{2e^x}{1-e^{2x}}$

ex: Differentiate.

g) $y = e^x \ln x$

$y' = e^x \cdot \frac{1}{x} + \ln x \cdot e^x$

$= e^x\left(\frac{1}{x} + \ln x\right)$

h) $f(x) = \ln e^{777x} = 777x$

$f'(x) = 777$

$e^{\ln x} = x$

ex: Write the equation of the tangent line to

$y = e^{4x}$ at $x = \ln 2$.

$(\ln 2, 16)$

$y' = 4e^{4x}$

$y'(\ln 2) = 64$

$4e^{4\ln 2}$

$4(16)$

$y - 16 = 64(x - \ln 2)$

ex: Write an equation of the tangent line to $y = \frac{\ln x}{4x} = \frac{1}{4} \cdot \frac{\ln x}{x}$ at x=1.

$(1, 0)$

$$y' = \frac{4x \cdot \frac{1}{x} - \ln x \cdot 4}{16x^2}$$

$$y - 0 = \frac{1}{4}(x-1)$$

$$y' = \frac{4 - 4\ln x}{16x^2}$$

$$y'(1) = \frac{1}{4}$$

ex: Write an equation of the normal line to $y = \frac{\ln x}{4x}$ at x=1.

$$y = -4(x-1)$$

Summary of Differentiation Rules

General Differentiation Rules	Let u and v be differentiable functions of x.	
	Constant Rule:	*(Simple)Power Rule:*
	$\frac{d}{dx}[c] = 0$, c is a real number.	$\frac{d}{dx}[x^n] = nx^{n-1}, \frac{d}{dx}[x] = 1$, n is a rational number.
	Constant Multiple Rule:	*Sum or Difference Rule:*
	$\frac{d}{dx}[cu] = cu'$, c is a real number.	$\frac{d}{dx}[u \pm v] = u' \pm v'$
	Product Rule:	*Quotient Rule:*
	$\frac{d}{dx}[uv] = uv' + vu'$	$\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{vu' - uv'}{v^2}$
	Chain Rule:	*General Power Rule:*
	$\frac{d}{dx}[f(u)] = f'(u)u'$	$\frac{d}{dx}[u^n] = nu^{n-1}u'$, n is a rational number.
Derivatives of Trigonometric Functions	$\frac{d}{dx}[\sin x] = \cos x$	$\frac{d}{dx}[\tan x] = \sec^2 x$
	$\frac{d}{dx}[\sec x] = \sec x \tan x$	$\frac{d}{dx}[\cos x] = -\sin x$
	$\frac{d}{dx}[\cot x] = -\csc^2 x$	$\frac{d}{dx}[\csc x] = -\csc x \cot x$
Derivatives of Exponential and Logarithmic Functions	$\frac{d}{dx}[e^x] = e^x$	$\frac{d}{dx}[\ln x] = \frac{1}{x}, \quad x > 0$
	$\frac{d}{dx}[a^x] = (\ln a)a^x$,	$\frac{d}{dx}[\log_a x] = \frac{1}{(\ln a)x}$,
	a is a positive real number ($a \neq 1$).	a is a positive real number ($a \neq 1$).

ex: Find the derivative.

a) $y = 5x - 1$

b) $f(x) = x\log_5 x$

c) $g(x) = \cos^2 x$

d) $f(x) = \dfrac{e^{7x}}{3^x}$

e) $y = \ln(\sec 2x)$

f) $y = e^5 - 5e^4 + 7$

ex:

If $f(x) = \sin(\ln(2x))$, then $f'(x) =$

(A) $\dfrac{\sin(\ln(2x))}{2x}$

(B) $\dfrac{\cos(\ln(2x))}{x}$

(C) $\dfrac{\cos(\ln(2x))}{2x}$

(D) $\cos\left(\dfrac{1}{2x}\right)$

$\sin(\ln(2x))$

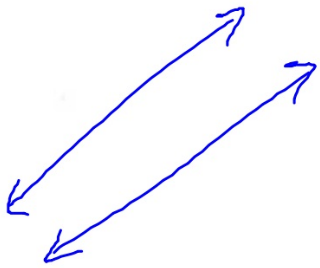
$\cos(\ln(2x)) \cdot \dfrac{1}{2x} \cdot 2$

ex:

$f(x) = 3e^{2x}$ $g(x) = 6x^3$

Let f be the function given by $f(x) = 3e^{2x}$ and let g be the function given by $g(x) = 6x^3$. At what value of x do the graphs of f and g have parallel tangent lines?

(A) −0.701
(B) −0.567
(C) −0.391
(D) −0.302
(E) −0.258



$f'(x) = 6e^{2x}$

$g'(x) = 18x^2$