43. $f(x) = \dfrac{x^3 - 3x^2 + 4}{x^2}$

$f(x) = x - 3 + 4x^{-2}$

$f'(x) = 1 - 8x^{-3}$

19. $y = \frac{\pi}{2}\sin\theta - \cos\theta$

$y' = \frac{\pi}{2}\cos\theta + \sin\theta$

90.) Line: $y = 2x - 4$

$\cancel{x^2 - 4}$

$\cancel{x^2 - 2x + 1}$

$\cancel{x^2 + 4x + 1}$

33. $f(x) = -\frac{1}{4}x^2$

$f'(x) = -\frac{1}{2}x$

$-1 = -\frac{1}{2}x$

$2 = x$

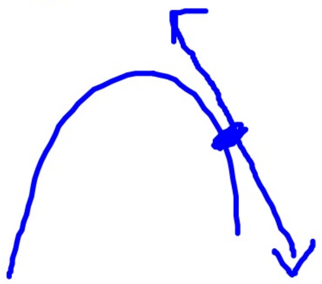
$x + y = 0$

$y = -x$

$y' = -1$

$(2, -1)$

69.) $y = k - x^2$ $\quad$ $y = -6x + 1$



$$k - x^2 = -6x + 1$$

$$k - 9 = -18 + 1$$

$$k = -8$$

$$-2x = -6$$

$$x = 3$$

III.) $\cos x = ax + b$

$1 = b$

$-\sin x = a$

$0 = a$

2.2 Basic Differentiation Rules Cont.

ex: Differentiate.

a) $f(x)=\begin{cases} x^2+3, & x \leq 0 \\ e^x - x, & x > 0 \end{cases}$

First ask: Is the function continuous at x = 0?

discontinuity at $x=0$

$f'(x)=\begin{cases} 2x, & x<0 \\ e^x-1, & x>0 \end{cases}$

no diff. at $x=0$

ex: Differentiate.

b) $f(x) = |x+4|$

$$f(x) = \begin{cases} -x-4, & x \le -4 \\ x+4, & x > -4 \end{cases}$$

$$f'(x) = \begin{cases} -1, & x < -4 \\ 1, & x > -4 \end{cases}$$

ex: Differentiate.

c) $g(x) = \begin{cases} 2, & x < 7 \\ 3 - 5x, & x \geq 7 \end{cases}$

Continuity at $x = 7$?

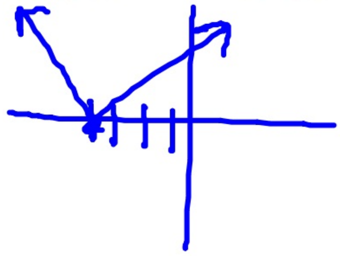
No

$$g'(x) = \begin{cases} 0, & x < 7 \\ -5, & x > 7 \end{cases}$$

$$f(x) = \begin{cases} x^2, & x \leq 1 \\ 2x - 1, & x > 1 \end{cases}$$

$$f'(x) = \begin{cases} 2x, & x \leq 1 \\ 2, & x > 1 \end{cases}$$

ex: Find the slope at the indicated x-value or explain why it does not exist.



a) $f(x)=|x+4|, \quad x=0$

$f'(0) = 1$

b) $f(x)=|x+4|, \quad x=-5$

$f'(-5) = -1$

c) $f(x)=|x+4|, \quad x=-4$ slope undefined

$\lim_{x \to -4^-} f'(x) \neq \lim_{x \to -4^+} f'(x)$

ex:

$$f(x)=\begin{cases}2x-2 & \text{for } x<3\\ 2x-4 & \text{for } x\geq 3\end{cases}$$

↑ Discont. at $x = 3$

$$f'(x) = \begin{cases} 2, & x<3 \\ 2, & x>3 \end{cases}$$

Let f be the piecewise-linear function defined above. Which of the following statements are true?

$f'(3) = ?$

I. $\lim\limits_{h\to 0^-} \dfrac{f(3+h)-f(3)}{h} = 2$

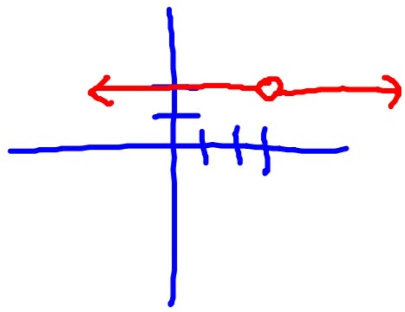
$h \to 0^-$

II. $\lim\limits_{h\to 0^+} \dfrac{f(3+h)-f(3)}{h} = 2$ ✓

$h \to 0^+$

III. $f'(3) = 2$

B



(A) None

(B) II only

(C) I and II only

x (D) I, II, and III

ex: Find a and b so that f(x) is differentiable. → implies continuity

$$g(x)=\begin{cases} 2x-x^2, & x\le 1 \\ x^2+ax+b, & x>1 \end{cases}$$

$2-2x$

$2x+a$

derivatives

$$\lim_{x\to 1^-} g(x)=\lim_{x\to 1^+} g(x)=g(1)$$

$$1=1+a+b$$

$$0=a+b$$

$$\lim_{x\to 1^-} g'(x)=\lim_{x\to 1^+} g'(x)=g'(1)$$

$$0=2+a$$

$$-2=a$$

$$2=b$$

FR 1

Let $f(x) = 4x^3 - 3x - 1$.

(a) Find the x-intercepts of the graph of f. $x = 1, -\frac{1}{2}$

(b) Write an equation for the tangent line to the graph of f at $x = 2$.

a.) $0 = 4x^3 - 3x - 1$

$$\begin{array}{r|rrrr} 1 & 4 & 0 & -3 & -1 \\ & & 4 & 4 & 1 \\ \hline & 4 & 4 & 1 & 0 \end{array}$$

$4x^2 + 4x + 1 = 0$

$(2x+1)^2 = 0$ $x = -\frac{1}{2}$

b.) $f'(x) = 12x^2 - 3$ $(2, 25)$ $\boxed{y - 25 = 45(x-2)}$

$f'(2) = 45$

FR 4

Let f be the function defined as follows:

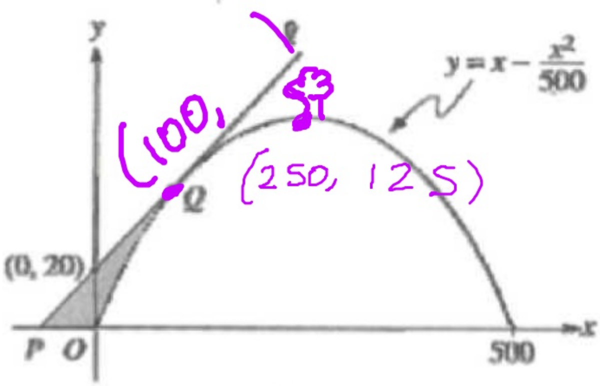
$$f(x) = \begin{cases} |x-1|+2, & \text{for } x < 1 \\ ax^2 + bx, & \text{for } x \geq 1, \text{ where } a \text{ and } b \text{ are constants.} \end{cases}$$

(a) If $a = 2$ and $b = 3$, is f continuous for all x? Justify your answer.

(b) Describe all values of a and b for which f is a continuous function.

(c) For what values of a and b is f both continuous and differentiable?

FR 16



$y' = 1 - \frac{x}{250}$

$y'(100) = \frac{3}{5}$

Line ℓ is tangent to the graph of $y = x - \frac{x^2}{500}$ at the point Q, as shown in the figure above.

(a) Find the x-coordinate of point Q. (100, 80)

(b) Write an equation for line ℓ. $y - 20 = \frac{3}{5}(x - 0)$

175 ft

(c) Suppose the graph of $y = x - \frac{x^2}{500}$ shown in the figure, where x and y are measured in feet, represents a hill. There is a 50-foot tree growing vertically at the top of the hill. Does a spotlight at point P directed along line ℓ shine on any part of the tree? Show the work that leads to your conclusion.

$$x - \frac{x^2}{500} = mx + 20 \; ; \; 1 - \frac{x}{250} = m$$

$$x - \frac{x^2}{500} = \left(1 - \frac{x}{250}\right)x + 20$$

$$\cancel{x} - \frac{x^2}{500} = \cancel{x} - \frac{x^2}{250} + 20$$

$$\frac{x^2}{250} - \frac{x^2}{500} = 20$$

$$\frac{x^2}{500} = 20 \qquad x^2 = 10000$$

$$x = 100$$

ex:

At $x = 3$, the function given by $f(x) = \begin{cases} x^2 & , \quad x < 3 \\ 6x - 9, & x \geq 3 \end{cases}$ is

$f'(x) = \begin{cases} 2x, & x < 3 \\ 6, & x \geq 3 \end{cases}$

(A) undefined.

(B) continuous but not differentiable.

(C) differentiable but not continuous.

(D) neither continuous nor differentiable.

(E) both continuous and differentiable.