2.2 Basic Differentiation Rules

*AKA "Short Cut Rules"

THEOREM 2.2 The Constant Rule

The derivative of a constant function is 0. That is, if c is a real number, then

$$\frac{d}{dx}[c] = 0$$

ex: Differentiate.

$$y = 234897\pi$$

$$y' = 0$$

THEOREM 2.3 The Power Rule

If n is a rational number, then the function $f(x) = x^n$ is differentiable and

$$\frac{d}{dx}[x^n] = n \cdot x^{n-1}$$

ex: Differentiate.

a) $f(x) = x^3$ $\quad f'(x) = 3x^2$

b) $f(x) = x^{20}$ $\quad f'(x) = 20x^{19}$

ex: Differentiate.

c) $g(x) = \frac{1}{x} = x^{-1}$

$g'(x) = -1 \cdot x^{-2} = \frac{-1}{x^2}$

d) $h(x) = \sqrt{x}$

$h(x) = x^{1/2} \quad h'(x) = \frac{1}{2} x^{-1/2} = \frac{1}{2\sqrt{x}}$

e) $y = \frac{1}{\sqrt{x}} \quad y = x^{-1/2} \quad y' = -\frac{1}{2} x^{-3/2} = \frac{-1}{2x^{3/2}}$

THEOREM 2.4 The Constant Multiple Rule

If f is a differentiable function and c is a real number, then cf is also differentiable and

$$\frac{d}{dx}[cf(x)] = c \cdot f'(x)$$

ex: Differentiate.

a) $y = 30x^7$ $\quad y' = 30(7x^6) = 210x^6$

b) $g(x) = \frac{4}{\sqrt[3]{x}}$

$g(x) = 4x^{-1/3}$

$g'(x) = 4 \cdot \frac{-1}{3} x^{-4/3} = \frac{-4}{3} x^{-4/3}$

ex: Differentiate.

c) $g(x) = \pi$

$g'(x) = 0$

d) $m(x) = 4x$

$m'(x) = 4 \cdot 1x^0 = 4$

THEOREM 2.5 The Sum and Difference Rules

The sum (or difference) of two differentiable functions f and g is itself differentiable. Moreover, the derivative of $f + g$ (or $f - g$) is the sum (or difference) of the derivatives of f and g.

$$\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$$

$$\frac{d}{dx}[f(x) - g(x)] = f'(x) - g'(x)$$

ex: Differentiate.

a) $y = 3x^4 - 2x + \pi$

$y' = 12x^3 - 2$

ex: Differentiate.

b) $f(x) = \pi^2 + \frac{1}{\pi} + \sqrt{\pi}$

$f'(x) = 0$

c) $s(x) = \frac{3x^2 - x^4}{x^9}$

$s(x) = 3x^{-7} - x^{-5}$

$s'(x) = -21x^{-8} + 5x^{-6}$

d.) $g(x) = (2x^2 - 5x)^2$

$g(x) = 4x^4 - 20x^3 + 25x^2$

$g'(x) = 16x^3 - 60x^2 + 50x$

THEOREM 2.6 **Derivatives of Sine and Cosine Functions**

$$\frac{d}{dx}[\sin x] = \cos x \qquad \frac{d}{dx}[\cos x] = -\sin x$$

$$f'(x) = \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h}$$

$$= \lim_{h \to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$

$$= \lim_{h \to 0} \left(\frac{\sin x (\cos h - 1)}{h} + \frac{\cos x \sin h}{h} \right) = 0 + \cos x$$

ex: Differentiate.

a) $y = 4\cos x - 2\sin x + 3$

$y' = -4\sin x - 2\cos x$

THEOREM 2.7 **Derivative of the Natural Exponential Function**

$$\frac{d}{dx}[e^x] = e^x$$

ex: Differentiate.

a) $y = 3e^x$ $\quad y' = 3e^x$

ex: Differentiate.

b) $y = x^2 - e^x + e^2$

$y' = 2x - e^x$

c) $f(x) = \cos x + 5e^x$

$f'(x) = -\sin x + 5e^x$

ex: Differentiate.

d) $y = x\left(x^2 + 5\right) = x^3 + 5x$

$y' = 3x^2 + 5$

ex: Differentiate.

d) $f(x)=\dfrac{x^2+5}{x}$

$f(x) = x + 5x^{-1}$

$f'(x) = 1 - 5x^{-2}$

ex: Find the slope at the given point.

a) $f(x) = -5x^4 - 2x^3 + 3\pi, \quad x = -1$

$f'(x) = -20x^3 - 6x^2$

$\boxed{f'(-1) = 14}$

ex: Find the slope at the given point.

b) $g(x) = -e^x, \quad x = 0$

$g'(x) = -e^x$

$g'(0) = -1$

equation of a tangent line at $x = 0$

$y - y_1 = m(x - x_1)$

$(0, -1) \quad m = -1$

$y + 1 = -1(x - 0)$

ex: Write an equation of the tangent line at the given point.

a) $y = \cos x, \quad x = \frac{3\pi}{4}$ $\quad \left(\frac{3\pi}{4}, -\frac{\sqrt{2}}{2}\right)$

$y' = -\sin x$

$y'\left(\frac{3\pi}{4}\right) = -\frac{\sqrt{2}}{2}$ $\quad \boxed{y + \frac{\sqrt{2}}{2} = -\frac{\sqrt{2}}{2}\left(x - \frac{3\pi}{4}\right)}$

numerical slope

ex: Write an equation of the tangent line at the given point.

b) $f(x) = 3 - \frac{3}{5x}, \quad x = \frac{3}{5}$

$\left(\frac{3}{5}, 2\right)$

$f(x) = 3 - \frac{3}{5}x^{-1}$

$f'(x) = 0 + \frac{3}{5}x^{-2}$

$f'\left(\frac{3}{5}\right) = \frac{5}{3}$

$\boxed{y - 2 = \frac{5}{3}\left(x - \frac{3}{5}\right)}$

ex: Find all points, if any, at which f(x) has a horizontal tangent line.

slope 0

a) $f(x)=\sin x, \quad [0,2\pi)$

$f'(x)=\cos x$

$0=\cos x$

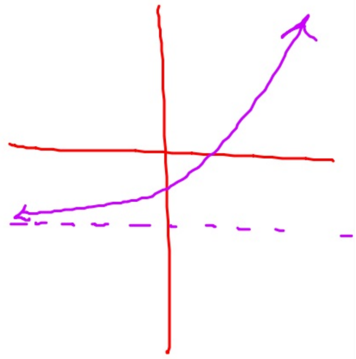
$x=\frac{\pi}{2}, \frac{3\pi}{2}$

ex: Find all points, if any, at which f(x) has a horizontal tangent line.

b) $y = e^x - 2$

$$\ln 2 = \ln e^x$$

$$\ln 2 = x$$



$$y' = e^x$$

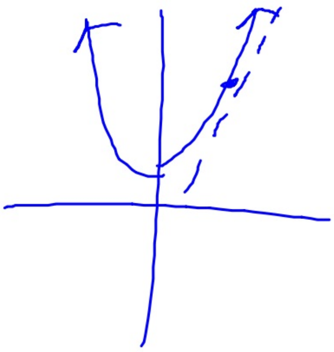
$$0 = e^x$$

No points

(No horiz. tangent

ex: Find an equation of a line that is tangent to $f(x) = 5x^2 + 3$ and parallel to $5x - y = 4$.

$m = 5$



$f'(x) = 10x$

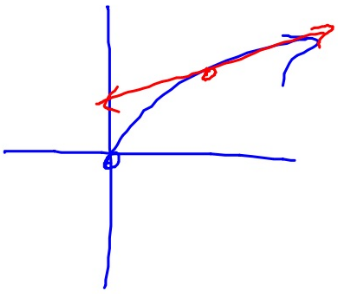
$5 = 10x$

$\frac{1}{2} = x$

$\left(\frac{1}{2}, \frac{17}{4}\right)$

$$y - \frac{17}{4} = 5\left(x - \frac{1}{2}\right)$$

ex: Find the value of k such that the line $y = x + 4$ is tangent to $f(x) = k\sqrt{x}$.



set functions $=$	set slopes $=$
$k\sqrt{x} = x + 4$	$f'(x) = \frac{k}{2\sqrt{x}}$ $\quad y' = 1$
$2\sqrt{x}\sqrt{x} = x + 4$	$\frac{k}{2\sqrt{x}} = 1$
$2x = x + 4$	$k = 2\sqrt{x}$
$x = 4$	$k = 4$