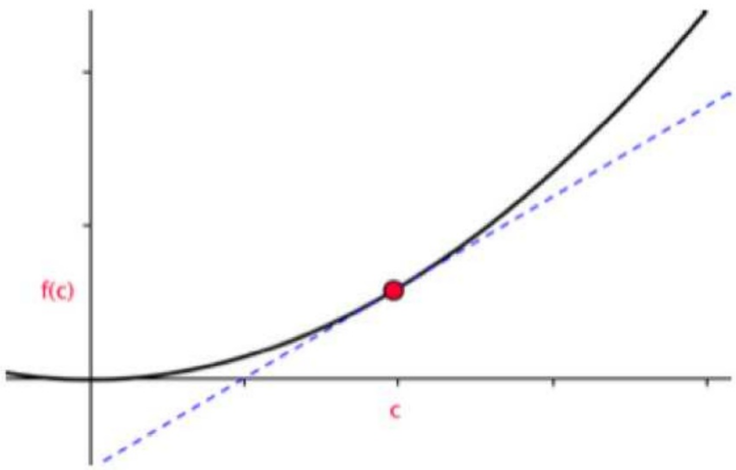
A	12
B	11
C	4
D	1
F	0

2.1 Definition Of A Derivative

The Tangent Line Problem

Task: Write the equation of the tangent line to f(x) at x=c.



DEFINITION OF TANGENT LINE WITH SLOPE m

If f is defined on an open interval containing c, and if the limit

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(c + \Delta x) - f(c)}{\Delta x} = m$$

exists, then the line passing through $(c, f(c))$ with slope m is the **tangent line** to the graph of f at the point $(c, f(c))$.

*The slope of the tangent line to the graph of f at the point (c, f(c)) with slope m is also known as the <u>slope of the graph of f at x=c.</u>

ex: Find the slope of the tangent line at the given point.

a) $f(x) = x^2 + 1, \quad (2,5)$ $\quad f(2+h) = (2+h)^2 + 1$

$$m = \lim_{h \to 0} \frac{(2+h)^2 + 1 - 5}{h}$$

$$= \lim_{h \to 0} \frac{h^2 + 4h + 4 + 1 - 5}{h} = \lim_{h \to 0} (h+4) = 4$$

ex: Find the slope of the tangent line at the given point.

b) $f\left(x\right)=x^2+1, \quad \left(-1,2\right)$

ex: Interpret the expression.

$$\lim_{h\to 0}\frac{f(6+h)-f(6)}{h}=100$$

The derivative of f(x) at x = 6 is 100

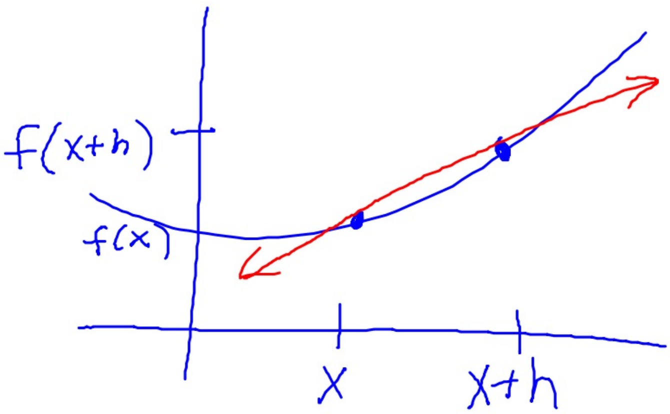
The slope of f(x) at x = 6 is 100

The rate of change...

Derivative - a formula used to find the slope of a tangent line

Vocab:

- differentiation - the process of finding a derivative
- differentiate - to find a derivative
- differntiable - a derivative exists



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{f(x+h) - f(x)}{x + h - x}$$

$$m = \frac{f(x+h) - f(x)}{h}$$

$h \to 0$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = f'(x)$$

$$f'(x), \quad \frac{dy}{dx}, \quad y', \quad \frac{d}{dx}[f(x)], \quad D_x[y].$$

Notation for derivatives

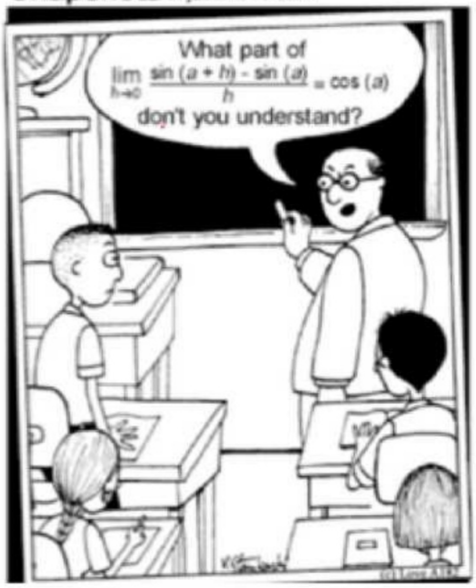
DEFINITION OF THE DERIVATIVE OF A FUNCTION

The **derivative** of f at x is given by

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

provided the limit exists. For all x for which this limit exists, f' is a function of x.

Snapshots at jasonlove.com



Derivative Synonyms:

- rate of change
- slope of a tangent line

ex: Find the derivative using the limit process.

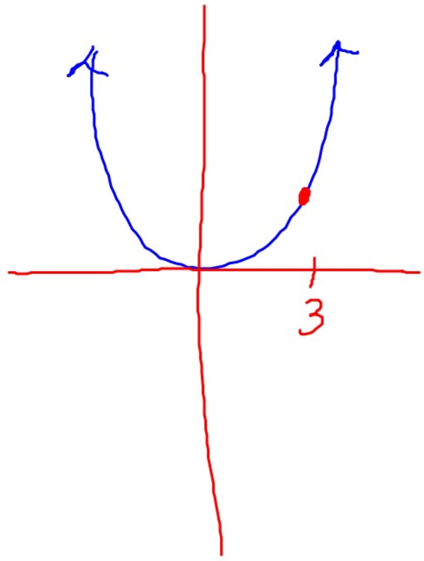
a) $f(x) = x^2$

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$$

$$= 2x$$

$f'(x) = 2x$

$f'(3) = 6$



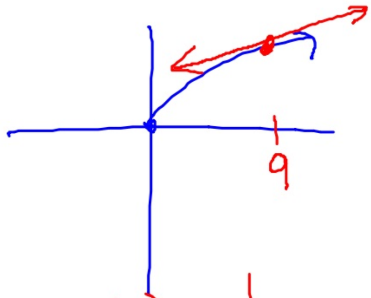
ex: Find the derivative using the limit process.

b) $f(x) = \sqrt{x}$

$$f'(x) = \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{(\sqrt{x+h} + \sqrt{x})}{(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{\cancel{x} + \cancel{h} - \cancel{x}}{\cancel{h}(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$



$f'(9) = \frac{1}{6}$

ex: Find the derivative using the limit process.

c) $f(x)=\frac{1}{x}$

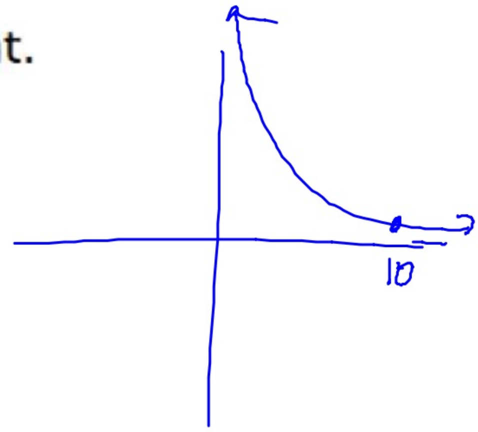
$$f'(x)=\lim_{h\to 0}\frac{\frac{1}{x+h}-\frac{1}{x}}{h}$$

$$=\lim_{h\to 0}\frac{x-(x+h)}{x(x+h)}\cdot\frac{1}{h}$$

$$=\frac{-1}{x^2}$$

ex: Find the slope at the given point.

a) $f(x) = \frac{1}{x}, \quad \left(10, \frac{1}{10}\right)$



$f'(x) = \frac{-1}{x^2}$

$f'(10) = \frac{-1}{100}$

ex: Interpret the expression.

$$g'(22) = 5$$

The slope of g(x) at x = 22 is 5.

The derivative of g(x) at x = 22 is 5.

- Alternate Form Of The Derivative

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

ex: Use the alternate form to find the slope of the curve at the given x-value.

a) $f(x) = x^2 + 1, \quad (\overset{c}{\underline{2}}, 5)$

$f(c) = f(2) = 5$

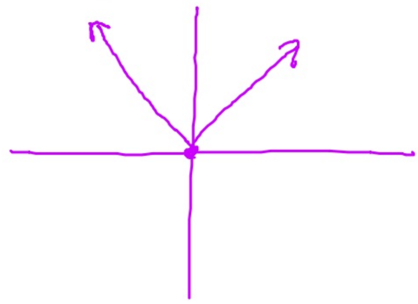
$$f'(2) = \lim_{x \to 2} \frac{x^2 + 1 - 5}{x - 2}$$

$$= \lim_{x \to 2} (x + 2)$$

$$= 4$$

ex: Use the alternate form to find the slope of the curve at the given x-value.

c) $f(x)=|x|, \quad (0,0)$

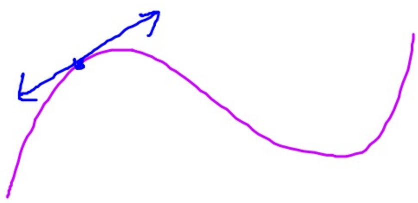


$$f'(0)=\lim_{x\to 0}\frac{|x|-0}{x-0}$$

$$=\lim_{x\to 0}\frac{|x|}{x}$$

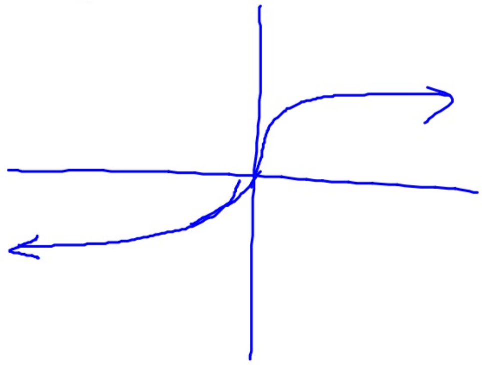
DNE

$f'(0)$ is undefined



ex: Use the alternate form to find the slope of the curve at the given x-value.

d) $f(x) = \sqrt[3]{x}, \quad (0,0)$

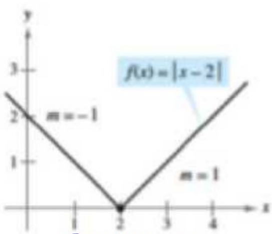


$$f'(0) = \lim_{x \to 0} \frac{\sqrt[3]{x} - 0}{x - 0}$$

$$= \lim_{x \to 0} \frac{1}{x^{2/3}}$$

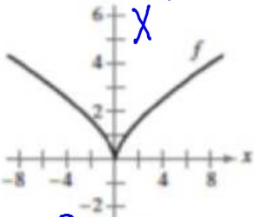
$f'(0)$ is undefined

A function f is NOT DIFFERENTIABLE at...



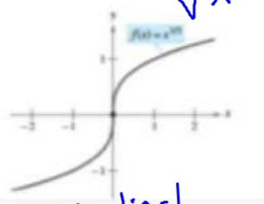
corners

$x^{2/3}$

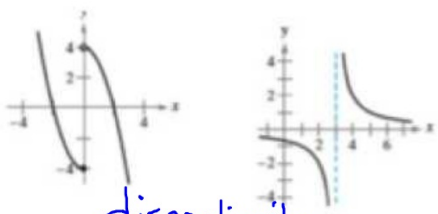


cusp

$\sqrt[3]{x}$



vertical tangents



discontinuity

If f is differentiable at x=c, then f must be continuous at x=c.

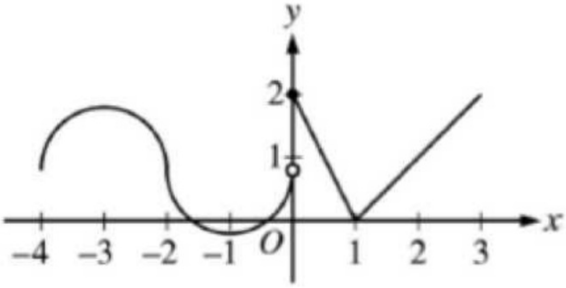
BUT...

If f is continuous at x=c, f may or MAY NOT be differentiable at x=c.

MORAL OF THE STORY:

DIFFERNTIABILITY IMPLIES CONTINUITY

ex:



Graph of f

The graph of the piecewise-defined function f is shown in the figure above. The graph has a vertical tangent line at $x = -2$ and horizontal tangent lines at $x = -3$ and $x = -1$. What are all values of x, $-4 < x < 3$, at which f is continuous but not differentiable?

(A) $x = 1$

(B) $x = -2$ and $x = 0$

(C) $x = -2$ and $x = 1$

(D) $x = 0$ and $x = 1$

If $\lim_{x \to a} f(x) = L$, where L is a real number, which of the following must be true?

$x \to a$

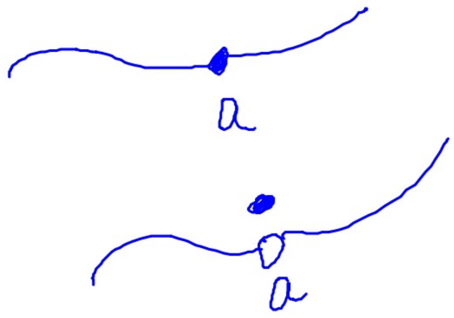
(A) $f'(a)$ exists.

(B) $f(x)$ is continuous at $x = a$.

(C) $f(x)$ is defined at $x = a$.

(D) $f(a) = L$

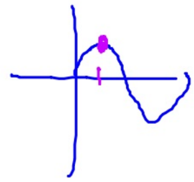
(E) None of the above



ex: The limit below represents the derivative of f at x=c for a function f and a number c. Find f and c.

a) $$\lim_{h\to 0}\frac{\sin\left(\frac{\pi}{2}+h\right)-1}{h}$$

$f(x)=\sin x$



$C=\frac{\pi}{2}$

$$\lim_{h\to 0}\frac{\sin\frac{\pi}{2}\cos h+\cancel{\sin h\cos\frac{\pi}{2}}-1}{h}=\lim_{h\to 0}\frac{\cos h-1}{h}=\lim_{h\to 0}-\frac{(1-\cos h)}{h}=0$$

b) $$\lim_{x\to e}\frac{\ln x-1}{x-e}$$

$f(x)=\ln x$

$C=$

ex:

What is $\lim_{h \to 0} \dfrac{\cos\left(\frac{3\pi}{2} + h\right) - \cos\left(\frac{3\pi}{2}\right)}{h}$?

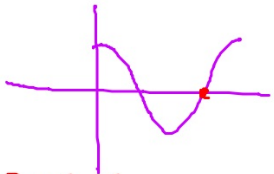
(A) 1

(B) $\dfrac{\sqrt{2}}{2}$

~~(C)~~ 0

~~(D)~~ −1

~~(E)~~ The limit does not exist.



$f(x) = \cos x$

$C = \dfrac{3\pi}{2}$

$f'\left(\dfrac{3\pi}{2}\right) = \_\_\_$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$