11.) $$\lim_{x \to -3^-} \frac{x}{\sqrt{x^2-9}} \quad -\infty$$

"-3.1"

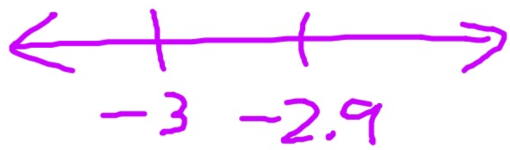
$$\frac{-3.1}{\sqrt{(-3.1)^2-9}}$$

23.) $\lim_{x \to 3} \left(2 - [\underset{2.9}{-x}]\right)$

dne

⋮

justify



21.) $\lim_{x \to 4^-} (5[x] - 7)$

$5[3.9] - 7$

$5 \cdot 3 - 7$

1.5/1.6 Infinite Limits and Limits at Infinity

REVIEW:

Finding Horizontal Asymptotes - if f(x) is a rational function...

$$f(x) = \frac{ax^n + \ldots}{bx^m + \ldots}$$

← **nth degree** polynomial

← **mth degree** polynomial

$\boxed{1}$ If $n < m$, then the x-axis is the horizontal asymptote.

$\boxed{2}$ If $n = m$, then the horizontal asymptote is the line $y = \frac{a}{b}$

$\boxed{3}$ If $n > m$, then there is no horizontal asymptote.

REMEMBER THE ACRONYM: BOBO BOTN EATSDC

*If f(x) is not a rational function but comes in the form of a fraction compare the magnitudes of the numerator and denominator and use "BOBO."

Finding Vertical Asymptotes - Vertical asymptotes are vertical lines which correspond to the zeroes of the denominator of a simplified rational function. (They can also arise in other types of functions.)

*WATCH OUT FOR HOLES!

ex: State the horizontal and vertical asymptotes.

a) $f(x)=\frac{x-1}{x^2+7x-8}$

$f(x)=\frac{\cancel{(x-1)}}{(x+8)\cancel{(x-1)}}$

$=\frac{1}{x+8}$

HA: $y=0$

VA: $x=-8$

b) $f(x)=\frac{x^2-4}{x-5}$

VA: $X=5$
HA: none

2 HA!

c) $f(x)=\frac{5x}{\sqrt{4x^2+1}}$

Eats dc.

HA: $y=5/2$
$y=-5/2$

d) $f(x)=\dfrac{\cos x}{2^x}$

Bobo

VA: none
HA: y=0

e) $f(x)=\dfrac{5\cdot 3^x+2}{3^x}$

VA: none
HA: y=5

Infinite Limits:

$$\lim_{x \to c} f(x) = \infty \qquad \lim_{x \to c} f(x) = -\infty$$

ex: Find the limit. If the limit does not exist, explain why.

a) $\lim_{x \to 4} \frac{1}{x-4}$ dne $\quad \lim_{x \to 4^-} \frac{1}{x-4} \neq \lim_{x \to 4^+} \frac{1}{x-4}$

b) $\lim_{x \to 4} \frac{1}{(x-4)^2}$ ∞

c) $\lim_{x \to 4} \frac{1}{(x-4)^3}$ dne $\lim_{x \to 4^-} \frac{1}{(x-4)^3} \neq \lim_{x \to 4^+} \frac{1}{(x-4)^3}$

d) $\lim_{x \to 4} \frac{1}{(x-4)^4}$ ∞

e) $\lim_{x \to 7} \frac{x-9}{x-7}$

dne ....

f) $\lim_{x \to 7} \frac{x-9}{(x-7)^2}$ ∞

g) $\lim_{x \to 6} \frac{x}{x^2 - 36}$

$\frac{x}{(x-6)(x+6)}$

dne

$\lim_{x \to 6^-} \frac{x}{x^2 - 36} = -\infty$

"5.9"

h) $\lim_{x \to 1} \frac{x-1}{x^2 - 7x + 6}$

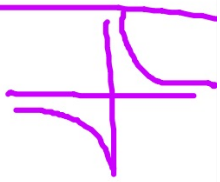
$\lim_{x \to 1} \frac{\cancel{x-1}}{\cancel{(x-1)}(x-6)} = \frac{1}{-5}$

i) $\lim_{x \to 6} \frac{x-1}{x^2 - 7x + 6}$

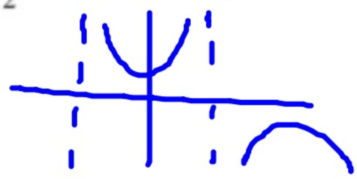
j) $\lim_{x \to 2} \frac{x^2 + 8x + 15}{x^2 + 3x - 10}$

k) $\lim_{x\to 0}\left(x^2-\frac{1}{x}\right) = \lim_{x\to 0} x^2 - \lim_{x\to 0}\frac{1}{x}$ dne

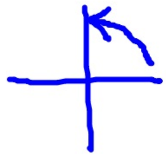
$\lim_{x\to 0^-}\left(x^2-\frac{1}{x}\right) = 0-(-\infty) = \infty$



l) $\lim_{x\to\frac{\pi}{2}} -2\sec x$ dne



$\lim_{x\to\frac{\pi}{2}^-}(-2\sec x)$



$-\infty$

m) $\lim_{x\to -3^-}\frac{x}{\sqrt{x^2-9}}$

In general, if $\lim_{x \to c} f(x) = \frac{n}{0}$, $n \neq 0$, then f(x) must have a

vertical asymptote at x=c.

- If the multiplicity of the factor that produces the vertical asymptote is odd, the limit will not exist.

- If the multiplicity of the factor that produces the vertical asymptote is even, the limit exists and is either $-\infty$ or ∞.

Limits at Infinity:

$$\lim_{x\to\infty} f(x) \qquad \lim_{x\to-\infty} f(x)$$

*The existence or nonexistence of horizontal asymptotes will affect limits at infinity.

**KNOW YOUR LIBRARY OF FUNCTIONS!!!

ex: Find the limit. If the limit does not exist, explain why.

a) $\lim_{x\to\infty} \tan^{-1} x = \frac{\pi}{2}$

b) $\lim_{x \to \infty} \sin x$

dne (oscillating function)

c) $\lim_{x \to \infty} \frac{5x^2 - 4}{x^2 + 2} = 5$

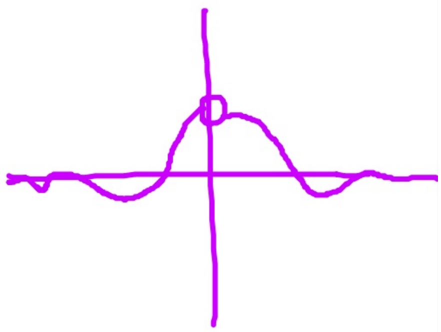
d) $\lim_{x \to -\infty} \frac{5x^2 - 4}{x^2 + 2} = 5$

e) $\lim_{x \to -\infty} \frac{5x-4}{x^2+2} = 0$

$x \to -\infty$

f) $\lim_{x \to \infty} \frac{5x^2-4}{x+2}$ dne or ∞ (increases without bound)

$x \to \infty$



g) $\lim_{x \to \infty} \frac{\sin x}{x} = 0$

$x \to \infty$

h) $\lim_{x \to \infty} \frac{2^{-x}}{x^2} = \lim_{x \to \infty} \frac{1}{x^2 2^x} = 0$

i) $\lim_{x \to \infty} \frac{\sqrt{9x^2+2}}{5x-3} = \frac{3}{5}$

j) $\lim_{x \to -\infty} \frac{\sqrt{9x^2+2}}{5x-3} = \frac{-3}{5}$

k) $\lim_{x\to\infty} \frac{\sqrt{5x^2-2x+3}}{x+4} = \sqrt{5}$

l) $\lim_{x\to\infty} \frac{\sqrt{5x^2-2x+3}}{4-x} = -\sqrt{5}$

Justifying Asymptotes

Horizontal Asymptotes:

If f(x) has a horizontal asymptote at y=c show

$$\lim_{x \to \infty} f(x) = c$$

or

$$\lim_{x \to -\infty} f(x) = c$$

Vertical Asymptotes:

If f(x) has a vertical asymptote at x=c show

$$\lim_{x \to c^-} f(x) = \pm\infty$$

or

$$\lim_{x \to c^+} f(x) = \pm\infty$$

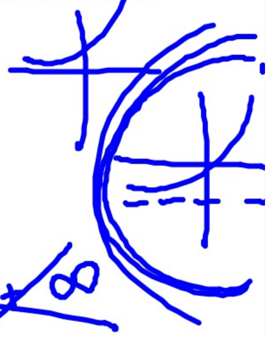
ex: State the horizontal and vertical asymptotes. Then justify your answers using an appropriate limit statement.

a) $f(x) = e^x - 2$

VA: none

HA: $y = -2$ → $\lim\limits_{x \to -\infty} f(x) = -2$

~~or +∞~~



b) $\cancel{y}\; f(x) = \dfrac{\sqrt{6x^2+16}}{x-2}$

$\lim\limits_{x \to 2^+} f(x) = \infty$ VA $x = 2$

$\lim\limits_{x \to \infty} f(x) = \sqrt{6}$ $\lim\limits_{x \to -\infty} f(x) = -\sqrt{6}$

Put it all together...

ex: Find the limit or explain why it does not exist.

a) $\lim_{x \to 0} \frac{1}{3+5^{1/x}}$

b) $\lim_{x \to \infty} \frac{1}{3+5^{1/x}}$

c) $\lim_{x \to -\infty} \frac{1}{3+5^{1/x}}$

ex: Does the graph of y have any vertical or horizontal asymptotes? How do you know?

$$y = \frac{1}{3 + 5^{1/x}}$$

ex: If $\lim_{x \to 6^-} f(x) = \infty$, what can be said about the graph of f(x)?