17.) $\lim\limits_{x \to -2^-} \frac{x+2}{x^2+x-2}$

$\lim\limits_{x \to -2^-} \frac{\cancel{x+2}}{\cancel{(x+2)}(x-1)}$

$\lim\limits_{x \to -2^-} \frac{1}{x-1}$

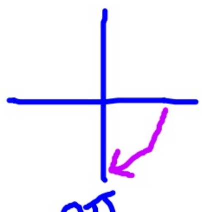
$-\frac{1}{3}$

3.) $\lim\limits_{x \to \frac{3\pi}{4}^+} (-2\tan 2x)$

$-2 \lim\limits_{x \to \frac{3\pi}{4}^+} (\tan 2x)$

$\tan\left(\frac{3\pi}{2}\right)$

$+\infty$



$\frac{3\pi}{2}$

7.) $\lim_{x \to \infty} \frac{x^3}{4x^2+3}$

$+\infty$

5.) $\lim_{x \to \infty} x\cos\left(\frac{1}{x}\right)$

$\cos 0$

$\infty \cdot 1$

∞

20.) $f(x) = \dfrac{x^2-1}{x^2+3x-4}$

$f(x) = \dfrac{(x+1)\cancel{(x-1)}}{(x+4)\cancel{(x-1)}}$

VA @ $x = -4$

$\lim_{x \to -4^+} f(x)$

$-\infty$

HA @ $y = 1$

$\lim_{x \to \infty} f(x) = 1$

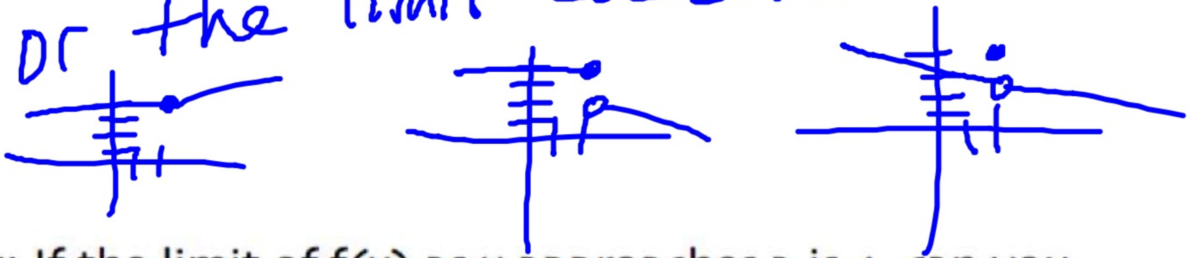
9.) $\lim_{x \to -\infty} \frac{x+2}{x^2+x+1}$

0

1.4 Continuity At A Point & The Intermediate Value Theorem

ex: If f(2)=4, can you conclude anything about the limit of f(x) as x approaches 2? Explain your reasoning.

No. The limit could exist or the limit could not exist

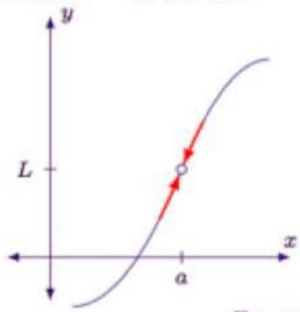


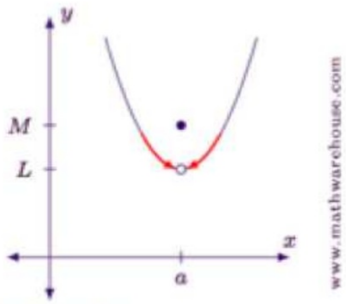
ex: If the limit of f(x) as x approaches 2 is 4, can you conclude anything about f(2)? Explain your reasoning.

Types of Discontinuities

$$y = \frac{x+1}{x^2-1} = \frac{\cancel{x+1}}{\cancel{(x+1)}(x-1)}$$

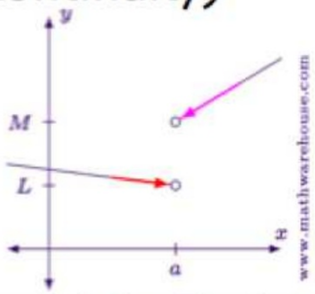
- Removable - holes



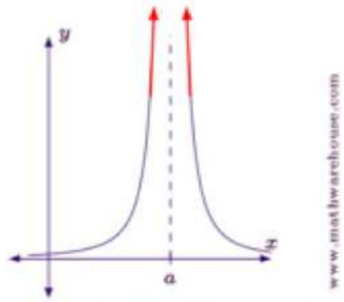


Examples of Removable Discontinuities

- Nonremovable - jumps, vertical asymptotes (a.k.a. infinite discontinuity)



Example of a Jump Discontinuity



Example of an Infinite Discontinuity

ex: At what x-values, if any, is f(x) discontinuous? For each discontinuity state the x-value, the type of discontinuity, and whether the discontinuity is removable or nonremovable.

$$f(x)=\frac{x^2-1}{x^2-4x+3} = \frac{\cancel{(x-1)}(x+1)}{(x-3)\cancel{(x-1)}}$$

Removable
$x=1$

$\lim_{x\to 1} f(x)$ exists
but $\lim_{x\to 1} f(x) \neq f(1)$

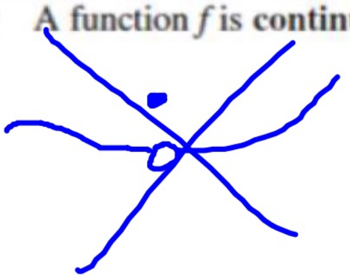
nonremovable
$x=3$

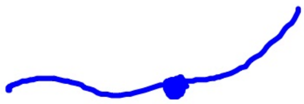
$\lim_{x\to 3^-} f(x) \neq \lim_{x\to 3^+} f(x)$

Continuity At A Point, x=c

DEFINITION OF CONTINUITY

Continuity at a Point: A function f is **continuous at** c if the following three conditions are met.





1. $f(c)$ is defined.
2. $\lim_{x \to c} f(x)$ exists.
3. $\lim_{x \to c} f(x) = f(c)$

Continuity on an Open Interval: A function is **continuous on an open interval** (a, b) if it is continuous at each point in the interval. A function that is continuous on the entire real line $(-\infty, \infty)$ is **everywhere continuous.**

ex: Is f(x) continuous at x=0? Justify your answer.

$$f(x)=\begin{cases} x+1, & x\leq 0 \\ x^2+1, & x>0 \end{cases}$$

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = f(0)$$

$$1 = 1 = 1$$

Yes

ex: Is g(x) continuous at x=3? Justify your answer.

$$g(x) = \frac{x^3 - 27}{x - 3} = \frac{\cancel{(x-3)}(x^2+3x+9)}{\cancel{x-3}}$$

No, because f(3) is not defined.

ex: Find the value of b so that the function f(x) is continuous everywhere.

$$f(x)=\begin{cases} x+3, & x\le 2 \\ bx+7, & x>2 \end{cases}$$

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = f(2)$$

$$\lim_{x\to 2^-}(x+3) = \lim_{x\to 2^+}(bx+7) = 5$$

$$5 = 2b+7$$

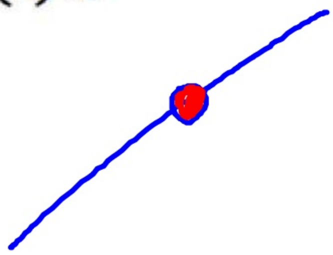
$$\boxed{-1 = b}$$

ex: Find the value of a so that the function h(x) is continuous everywhere.

$$\frac{x^2-14^2}{x-14}$$

$$\frac{(x+a)\cancel{(x-a)}}{\cancel{x-a}}$$

$$h(x)=\begin{cases}\dfrac{x^2-a^2}{x-a}, & x\neq a\\ 28, & x=a\end{cases}$$



$$\lim_{x\to a} h(x) = h(a)$$

$$\lim_{x\to a}(x+a) = 28$$

$$2a = 28 \qquad a=14$$

ex: Find the values of a and b so that the function f(x) is continuous everywhere.

$$f(x)=\begin{cases}2, & x\le -1\\ ax+b, & -1<x<3\\ -2, & x\geq 3\end{cases}$$

$\lim_{x\to -1^-} f(x) = \lim_{x\to -1^+} f(x) = f(-1)$

$2 = -a+b$

$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x) = f(3)$

$3a+b = -2$

$b = 2+a$

$3a+2+a=-2$

$4a=-4$

$a=-1$

$b=1$

ex: Find the values of a and c so that the function f(x) is continuous everywhere.

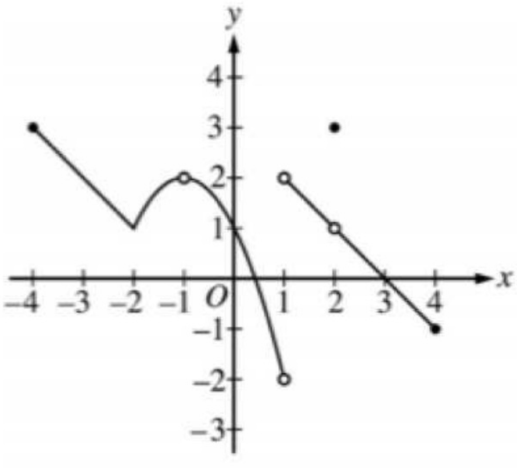
$$f(x)=\begin{cases} 2cx^3-5ax-1, & x>-1 \\ 4ax-1, & x=-1 \\ 5x^2-3cx^3+4ax-2, & x<-1 \end{cases}$$

$$\lim_{x\to -1^+} f(x) = f(-1) = \lim_{x\to -1^-} f(x)$$

$$-2c+5a-1 = -4a-1 = 5+3c-4a-2$$

$$a = \frac{-8}{27} \qquad c = -\frac{4}{3}$$

ex:



Graph of f

The graph of the function f is shown in the figure above. For how many values of x in the open interval $(-4, 4)$ is f discontinuous?

(A) one

(B) two

(C) three

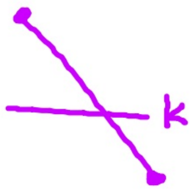
(D) four

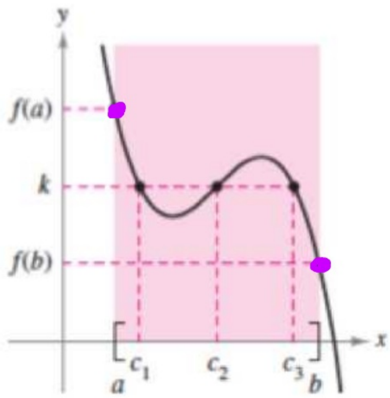
Intermediate Value Theorem

THEOREM 1.13 INTERMEDIATE VALUE THEOREM

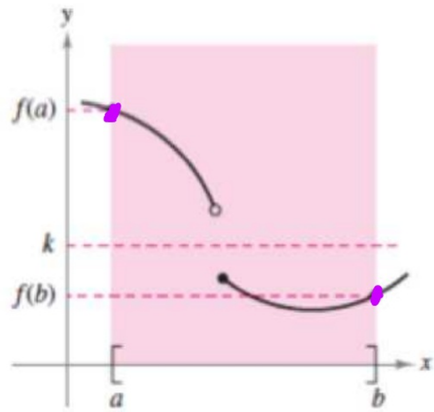
If f is continuous on the closed interval $[a, b]$, $f(a) \neq f(b)$, and k is any number between $f(a)$ and $f(b)$, then there is at least one number c in $[a, b]$ such that

$$f(c) = k.$$



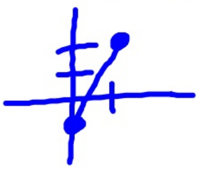


f is continuous on $[a, b]$.
[There exist three c's such that $f(c) = k$.]



f is not continuous on $[a, b]$.
[There are no c's such that $f(c) = k$.]

ex: Use the Intermediate Value Theorem to show a zero exists on f(x) on the given interval.



$$f(x) = x^3 + 2x - 1, \qquad [0,1]$$

$f(0) = -1$

$f(1) = 2$

Since f(x) is continuous on [0,1] and f(0) < 0 < f(1) by IVT there must exist a value c in (0,1) such that f(c) = 0.

ex: Consider the table of values of f(x) given below.

continuous

x	0	2	3	10	20
$f(x)$	-2	3	4	20	-10

What is the least amount of time f(x)=15 on [0, 20]?
Justify your answer. 2

Since f(x) is continuous on [3, 10] and f(3) < 15 < f(10), by IVT there must exist a value c such that f(c) = 15

Given $g(x) = x^2 + 2x - 8$ on $[0, 3]$; $k = 0$.

Find c.

$g(0) = -8$

$g(3) = 7$

$x^2 + 2x - 8 = 0$

$(x+4)(x-2) = 0$

$x = \cancel{-4}, 2$ $\quad c = 2$

Since $g(x)$ is continuous on $[0, 3]$ and $g(0) < 0 < g(3)$ by IVT there exists a value c such that $f(c) = 0$.

ex:

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	-4	4	2
4	-1	3	6	7

The functions *f* and *g* are continuous for all real numbers. The table above gives values of these functions and their first derivatives at selected values of *x*. The function *h* is given by $h(x) = f(g(x)) - 6$. Explain why there must be a value r for $1 < r < 3$ such that $h(r) = -5$.