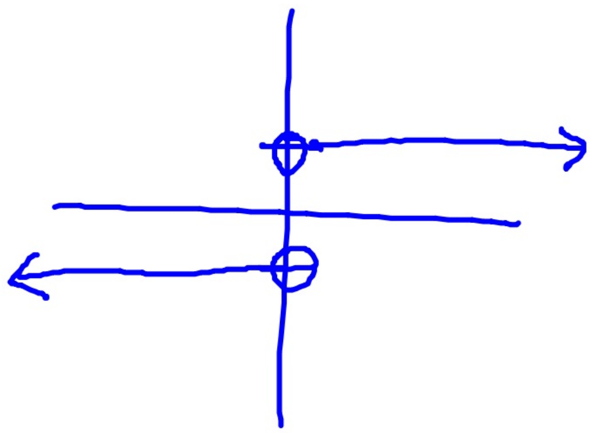
79.) $\lim_{x \to 0} \frac{1-e^{-x}}{e^x - 1} \cdot \frac{e^x}{e^x}$

$\lim_{x \to 0} \frac{\cancel{e^x - 1}}{\cancel{(e^x - 1)}e^x} = 1$

$$\lim_{x \to 1} \frac{(x+1)(x^2 - x + 1)}{x - 1}$$

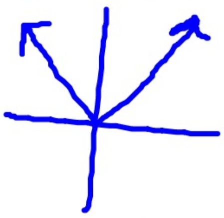
$$\lim_{x \to 0} \frac{|x|}{x}$$



1.4 One-Sided Limits

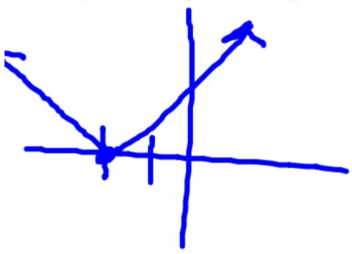
REVIEW: Rewrite each absolute value function as a piecewise function.

a) $y = |x|$



$$|x| = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}$$

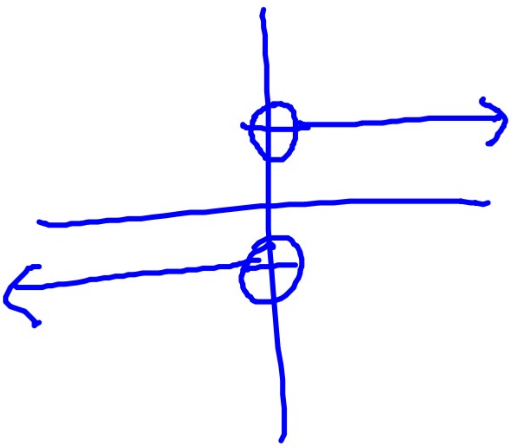
b) $y = |x+2|$



$$|x+2| = \begin{cases} -x-2, & x < -2 \\ x+2, & x \geq -2 \end{cases}$$

c) $y = \frac{|x|}{x}$

$$\frac{|x|}{x} = \begin{cases} -1, & x < 0 \\ 1, & x > 0 \end{cases}$$



One-Sided Limits

$$\lim_{x \to c^-} f(x) \qquad\qquad \lim_{x \to c^+} f(x)$$

Left-Sided Limit Right-Sided Limit

*Use the techniques you learned in 1.2 and 1.3 to find one-sided limits.

ex: Find the limit. If the limit does not exist, explain.

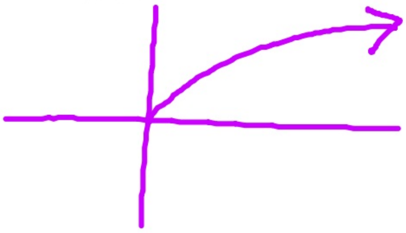
a) $\lim_{x \to 6^-} (x^2 - 21)$ $= 15$

b) $\lim_{x \to 5^+} \frac{x^2 - 25}{x - 5}$

$\lim_{x \to 5^+} \frac{(x+5)\cancel{(x-5)}}{\cancel{x-5}} = 10$

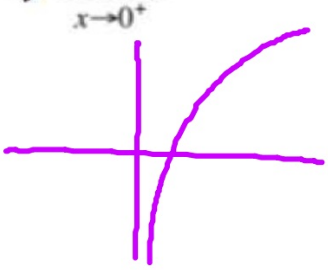
c) $\lim\limits_{x\to 0^-}\sqrt{x}$ dne

The domain is $[0,\infty)$ therefore cannot approach 0^-

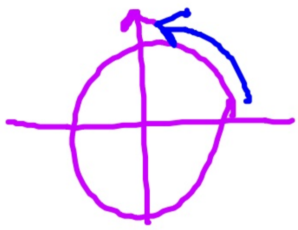


d) $\lim\limits_{x\to 0^+}\ln x$ $-\infty$

As the function approaches, 0^+, the value decrease without bound.



e) $\lim\limits_{x\to \frac{\pi}{2}^-}\tan x$ ∞



ex: Given f(x) find each limit. If the limit does not exist, explain.

$$f(x)=\begin{cases} x^2+4, & x>5 \\ 2x-3, & x\le 5 \end{cases}$$

a) $\lim_{x\to 5} f(x)$ dne

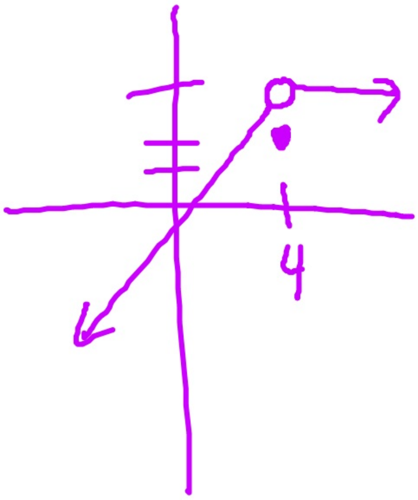
$$\lim_{x\to 5^-} f(x) \neq \lim_{x\to 5^+} f(x)$$

7   29

b) $\lim_{x\to 11} f(x) = 125$

ex: Given g(x) find each limit. If the limit does not exist, explain.

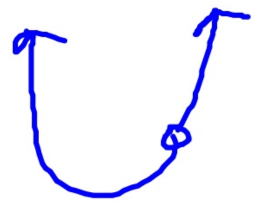
$$g(x)=\begin{cases}3, & x>4\\ 2, & x=4\\ x-1, & x<4\end{cases}$$



$$\lim_{x\to 4} g(x) = 3$$

ex: Given h(x) find each limit. If the limit does not exist, explain.

$$h(x) = \begin{cases} x^2 - 7, & x \neq 3 \\ 1, & x = 3 \end{cases}$$

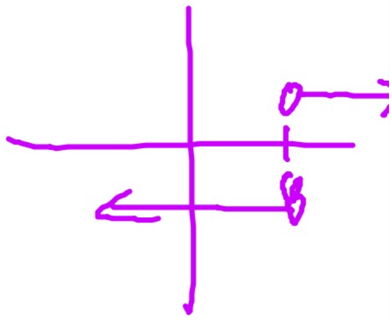


a) $\lim\limits_{x \to 3} h(x) = 2$

b) $\lim\limits_{x \to 0} h(x) = -7$

ex: Find the limit. If the limit does not exist, explain.

a) $\lim_{x \to 6} |x-6| = 0$



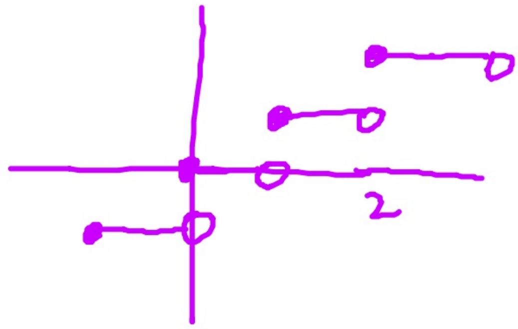
b) $\lim_{x \to 6} \frac{|x-6|}{x-6}$ dne

$$\lim_{x \to 6^-} \frac{|x-6|}{x-6} \neq \lim_{x \to 6^+} \frac{|x-6|}{x-6}$$

c) $\lim_{x \to -5} \frac{|x+5|}{x-3}$ $= 0$

d) $\lim_{x \to 2} [x]$ dne

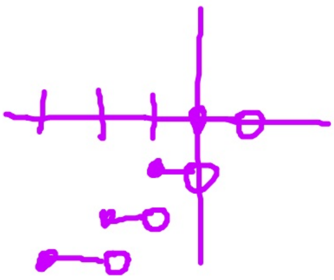
$\lim_{x \to 2^-} [x] \neq \lim_{x \to 2^+} [x]$



e) $\lim_{x \to 2.3} [x]$ $= 2$

f) $\lim_{x \to -2^+} [x] = -2$

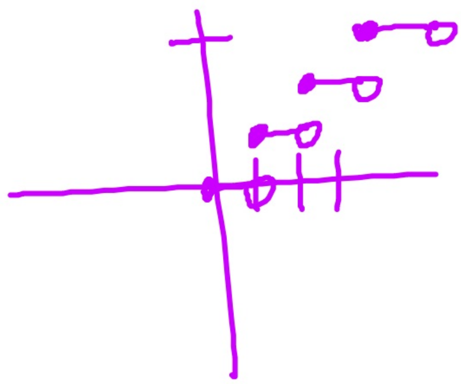
$x \to -2^+$



2.9

g) $\lim_{x \to 3^-} -[5x+6] = -20$

$x \to 3^-$



$$f^*(x) = \lim_{h \to 0} \frac{(x+h)^2 + (x+h) - \left[(x-h)^2 + (x-h)\right]}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 + x + h - \left[x^2 - 2xh + h^2 + x - h\right]}{h}$$

$$= 4x + 2$$