1.3 Evaluating Limits Analytically

- Evaluate a limit using properties of limits.
- Develop and use a strategy for finding limits.
- Evaluate a limit using dividing out and rationalizing techniques.
- Evaluate a limit using the Squeeze Theorem.

THEOREM 1.2 PROPERTIES OF LIMITS

Let b and c be real numbers, let n be a positive integer, and let f and g be functions with the following limits.

$$\lim_{x \to c} f(x) = L \quad \text{and} \quad \lim_{x \to c} g(x) = K$$

1. Scalar multiple: $\lim_{x \to c} [b f(x)] = bL$
2. Sum or difference: $\lim_{x \to c} [f(x) \pm g(x)] = L \pm K$
3. Product: $\lim_{x \to c} [f(x)g(x)] = LK$
4. Quotient: $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{L}{K}$, provided $K \neq 0$
5. Power: $\lim_{x \to c} [f(x)]^n = L^n$

Strategy for finding limits analytically

1. Direct Substitution
2. Algebraic techniques

(factoring or rationalizing or simplifying)

3. Special Cases

THEOREM 1.9 TWO SPECIAL TRIGONOMETRIC LIMITS

1. $\lim_{x \to 0} \frac{\sin x}{x} = 1$ 2. $\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$

Ex 1

$$\lim_{x \to 2} \frac{\sqrt{x+2}}{x-4} = \frac{2}{-2}$$

$$= -1$$

Ex 2

$$\lim_{x \to 7} \sec\left(\frac{\pi x}{6}\right)$$

$$-\frac{2}{\sqrt{3}}$$

Ex 3

$$\lim_{x \to 5\pi/3} \cos x$$

$\frac{1}{2}$

Pythagorean Identities

$\sin^2 x + \cos^2 x = 1$

$1 + \cot^2 x = \csc^2 x$

$\tan^2 x + 1 = \sec^2 x$

Other Trig identities

$\sin 2x = 2 \sin x \cos x$

$\cos 2x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1$

$$\frac{\frac{1}{x+2}-\frac{1}{2}}{x}=\frac{\frac{2-(x+2)}{2(x+2)}}{\left(\frac{x}{1}\right)}$$

$$\frac{\cancel{2}-x\cancel{-2}}{2(x+2)}\cdot\frac{1}{x}$$

$$\frac{-1}{2(x+2)}$$

$$\frac{\left(\frac{1}{x+2}-\frac{1}{2}\right)}{\left(\frac{x}{1}\right)} \cdot \frac{2(x+2)}{2(x+2)}$$

$$\frac{2-(x+2)}{2x(x+2)} = \frac{-1}{2(x+2)}$$

$$\cos 2x = 1 - 2\sin^2 x$$

$$\frac{\cos 2x - 1}{-2} = \frac{-2\sin^2 x}{-2}$$

$$\frac{1 - \cos 2x}{2} = \sin^2 x$$

Ex 4

$$\lim_{x \to -1} \frac{2x^2 - x - 3}{x + 1}$$

Ex 5

$$\lim_{x \to -1} \frac{x^3 + 1}{x + 1}$$

Ex 6

$$\lim_{x \to 3} \frac{\sqrt{x+1} - 2}{x - 3}$$

Ex 7

$$\lim_{x \to 0} \frac{[1/(x+4)] - (1/4)}{x}$$

Ex 8

$$\lim_{x \to 0} \frac{3(1 - \cos x)}{x}$$

Ex 9

$$\lim_{t \to 0} \frac{\sin 3t}{2t}$$

$$\lim_{x \to 0} \frac{\sin 4x}{11x}$$

Ex 10: Given f(x) = 5x - 2,

$$\textbf{find } \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}.$$

Ex. 11

$\lim_{x \to c} f(x) = \frac{3}{2}$

$\lim_{x \to c} g(x) = \frac{1}{2}$

(a) $\lim_{x \to c} [4f(x)]$

(b) $\lim_{x \to c} [f(x) + g(x)]$

(c) $\lim_{x \to c} [f(x)g(x)]$

(d) $\lim_{x \to c} \frac{f(x)}{g(x)}$