1.2 Finding Limits Graphically and Numerically

- Estimate a limit using a numerical or graphical approach.
- Learn different ways that a limit can fail to exist.
- Study and use a formal definition of limit.

1. Numerical approach — Construct a table of values.
2. Graphical approach — Draw a graph by hand or using technology.
3. Analytic approach — Use algebra or calculus.

Ex 1

$$\lim_{x \to 2} \frac{x-2}{x^2-4}$$

x	1.9	1.99	1.999	2.001	2.01	2.1
$f(x)$	.25641	.25063	.25006	.24994	.24938	.2439

$= .25$

Ex 2

$$\lim_{x \to 4} \frac{[x/(x+1)] - (4/5)}{x-4}$$

x	3.9	3.99	3.999	4.001	4.01	4.1
$f(x)$	.04082	.04008	.04001	.03999	.03992	.03922

.04

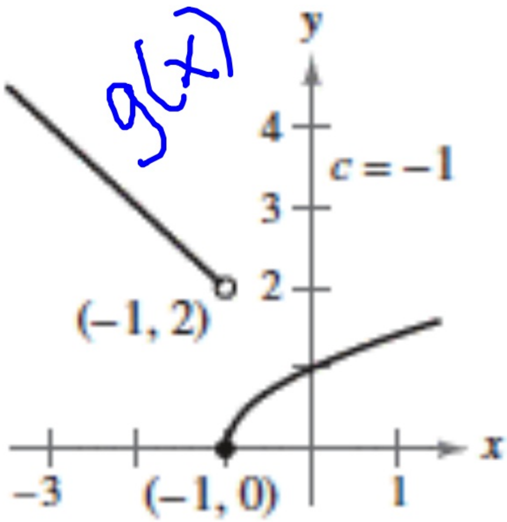
COMMON TYPES OF BEHAVIOR ASSOCIATED WITH NONEXISTENCE OF A LIMIT

1. $f(x)$ approaches a different number from the right side of c than it approaches from the left side.

2. $f(x)$ increases or decreases without bound as x approaches c.

3. $f(x)$ oscillates between two fixed values as x approaches c.

DNE
Type #1



$$\lim_{x \to -1} g(x)$$

$$\lim_{x \to -1^-} g(x) \neq \lim_{x \to -1^+} g(x)$$

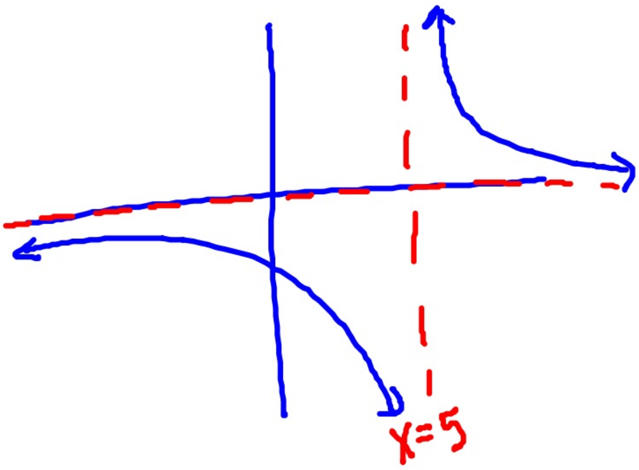
$$2 \neq 0$$

dne

Justification

DNE Type #2

$$\lim_{x\to 5} \frac{2}{x-5}$$



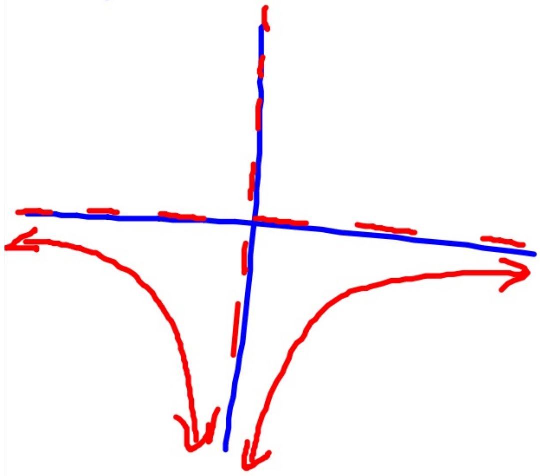
let $\frac{2}{x-5} = f(x)$

$$\lim_{x\to 5^-} f(x) \neq \lim_{x\to 5^+} f(x)$$

$$-\infty \neq \infty$$

dne

$$\lim_{x \to 0} -\frac{1}{x^2} \quad -\infty$$



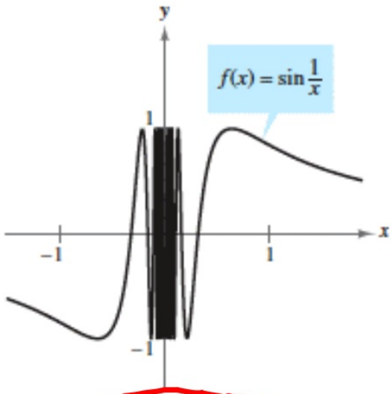
Infinity is a special case of DNE

$$\lim_{x \to 0^-} \frac{-1}{x^2} = \lim_{x \to 0^+} -\frac{1}{x^2}$$

$$-\infty \qquad -\infty$$

DNE Type #3

$f(x) = \sin\frac{1}{x}$



$\lim_{x\to 0} f(x)$ does not exist.

$\lim_{x\to 0^-} f(x) \neq \lim_{x\to 0^+} f(x)$

x	−0.1	−0.01	−0.001	0.001	0.01	0.1
$f(x)$	.544	.506	−.827	.827	−.506	−.544

(a) $f(-2)$ undefined

(b) $\lim\limits_{x\to -2} f(x)$ dne; $\lim\limits_{x\to -2^-} f(x) \neq \lim\limits_{x\to -2^+} f(x)$

(c) $f(0) = 4$

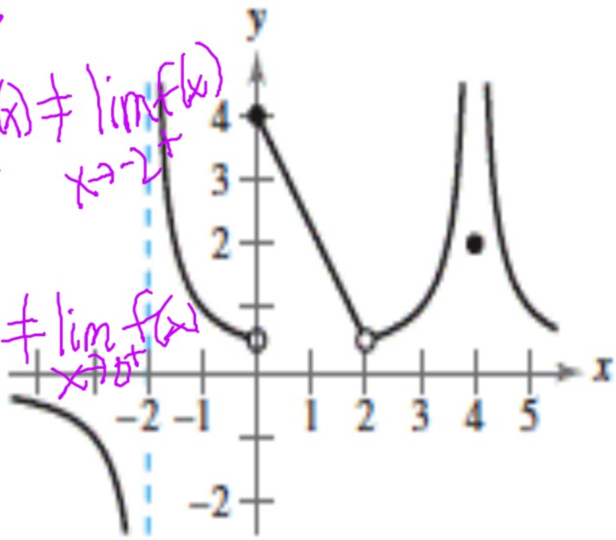
(d) $\lim\limits_{x\to 0} f(x)$ dne; $\lim\limits_{x\to 0^-} f(x) \neq \lim\limits_{x\to 0^+} f(x)$

(e) $f(2)$ undefined

(f) $\lim\limits_{x\to 2} f(x) = \frac{1}{2}$

(g) $f(4) = 2$

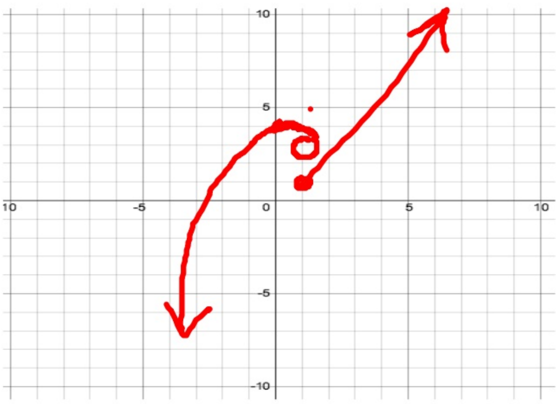
(h) $\lim\limits_{x\to 4} f(x)$ ∞ (dne) $\lim\limits_{x\to 4^-} f(x) \neq \lim\limits_{x\to 4^+} f(x)$



Sketching Piecewise functions

Sketch the graph of the following piecewise function.

$$g(x) = \begin{cases} -x^2+4 & \text{if } x < 1 \\ 2x-1 & \text{if } x \geq 1 \end{cases}$$



$\lim_{x \to 1} g(x)$ dne

$\lim_{x \to 1^-} g(x) \neq \lim_{x \to 1^+} g(x)$

Sketch the graph of the following piecewise function.

$$h(x)=\begin{cases} x+3 & \text{if } x<-2 \\ x^2 & \text{if } -2\le x<1 \\ -x+2 & \text{if } x\ge 1 \end{cases}$$

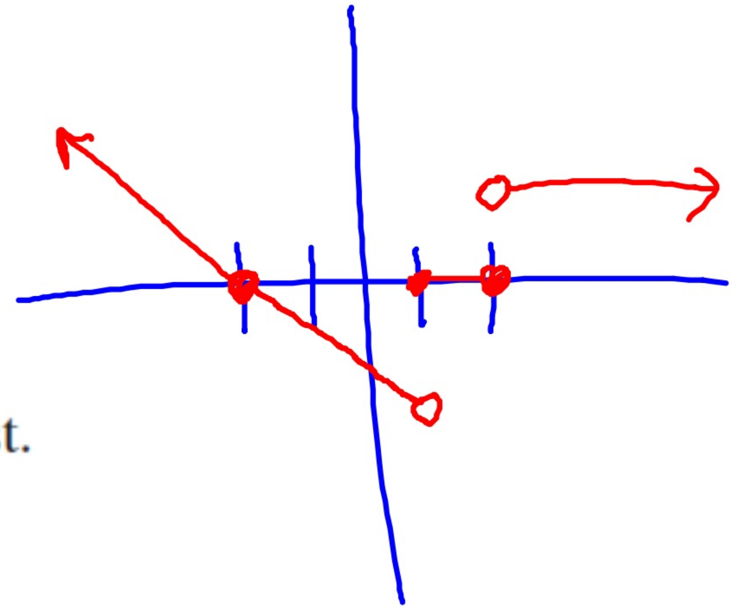
Sketch a graph of a function f that satisfies the given values. (There are many correct answers)

$$f(-2) = 0$$

$$f(2) = 0$$

$$\lim_{x \to -2} f(x) = 0$$

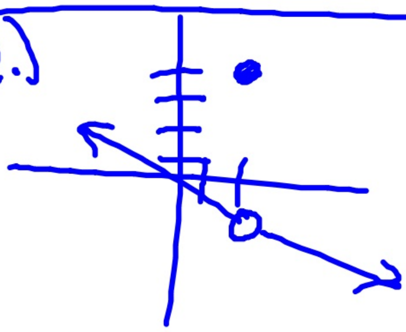
$\lim_{x \to 2} f(x)$ does not exist.



. (a) If $f(2) = 4$, can you conclude anything about the limit of $f(x)$ as x approaches 2? Explain your reasoning.

(b) If the limit of $f(x)$ as x approaches 2 is 4, can you conclude anything about $f(2)$? Explain your reasoning.

a.)



No . . . .

b.)