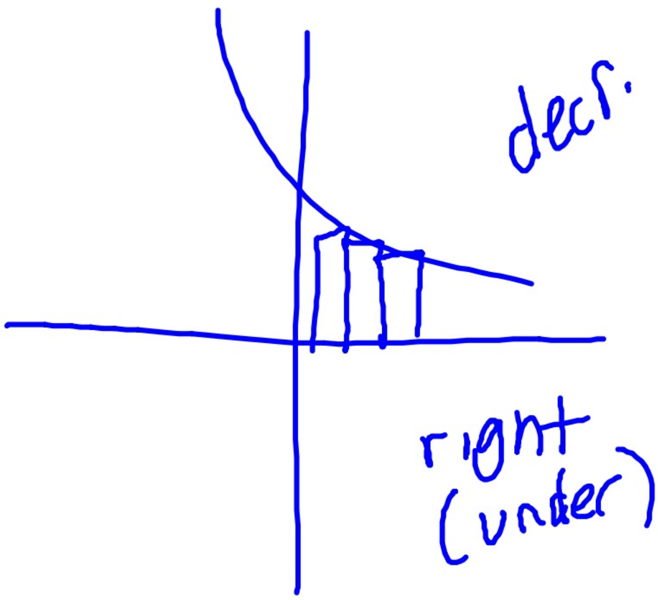
Juniors: Interested in computer programming? Consider taking AP Computer Science A next year.

16.) $-9\int_{1}^{7} f(x)\,dx$

$-9[11-8]$

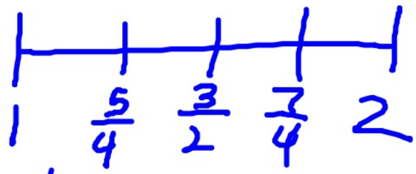
-27


incr.
right
(over)


decr.
right
(under)

3b) $\int_1^2 \frac{1}{x^2}\,dx \quad n=4$ (left)

$\frac{2-1}{4}$



$$\frac{1}{4}\left[1+\frac{1}{\left(\frac{5}{4}\right)^2}+\frac{1}{\left(\frac{3}{2}\right)^2}+\frac{1}{\left(\frac{7}{4}\right)^2}\right.$$

$$\frac{1}{4}\left[1+\frac{16}{25}+\frac{4}{9}+\frac{16}{49}\right]$$

Quiz average: 25/28

Riemann Approximations - cont.

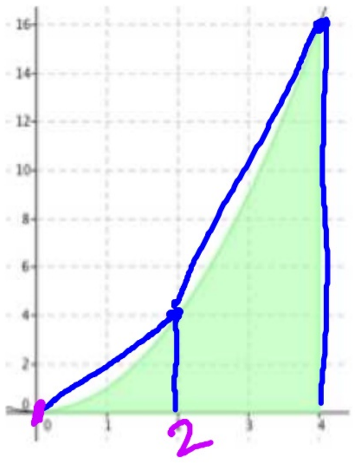
Actual: 21.333

ex: Approximate the integral $\int_0^4 x^2\,dx$ using a trapezoidal approximation with two equal subdivisions. Then determine if the approximation is an over or under estimate.

$\frac{1}{2}h(b_1 + b_2)$

$\frac{1}{2}(2)(\underline{0+4} + \underline{4+16})$

24



CCU trap (over)

ex: Approximate the integral $\int_1^{10} \sqrt{x}\,dx$ using a trapezoidal approximation with three equal subdivisions. Then determine if the approximation is an over or under estimate.

$$\frac{1}{2}(3)\left[1+2+2+\sqrt{7}+\sqrt{7}+\sqrt{10}\right]$$

$$\frac{3}{2}\left[5+2\sqrt{7}+\sqrt{10}\right]$$

20.181

CCD

Actual: 20.415

ex: Approximate the integral $\int_2^{14} \frac{1}{x} dx$ using a trapezoidal approximation with three equal subdivisions. Then determine if the approximation is an over or under estimate.

$$\frac{1}{2}(4)\left[\frac{1}{2}+\frac{1}{6}+\frac{1}{6}+\frac{1}{10}+\frac{1}{10}+\frac{1}{14}\right] = 2.2\dot{0}9$$

or

$$2.210$$

over; $\frac{1}{x}$ is CCD

THEOREM 4.9 The Trapezoidal Rule

Let f be continuous on $[a, b]$. The Trapezoidal Rule for approximating $\int_a^b f(x)\,dx$ is

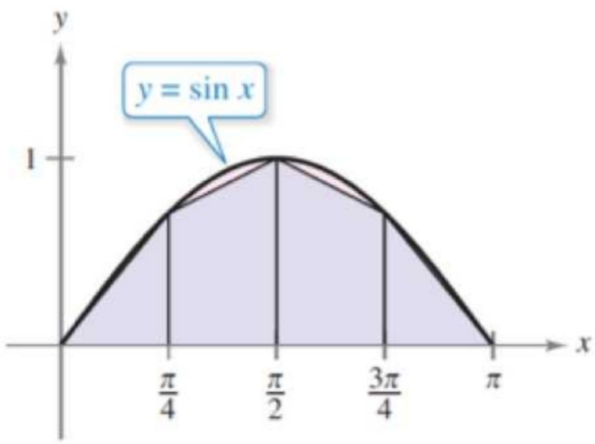
$$\int_a^b f(x)\,dx \approx \frac{b-a}{2n}[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)].$$

Moreover, as $n \to \infty$, the right-hand side approaches $\int_a^b f(x)\,dx$.

Remark

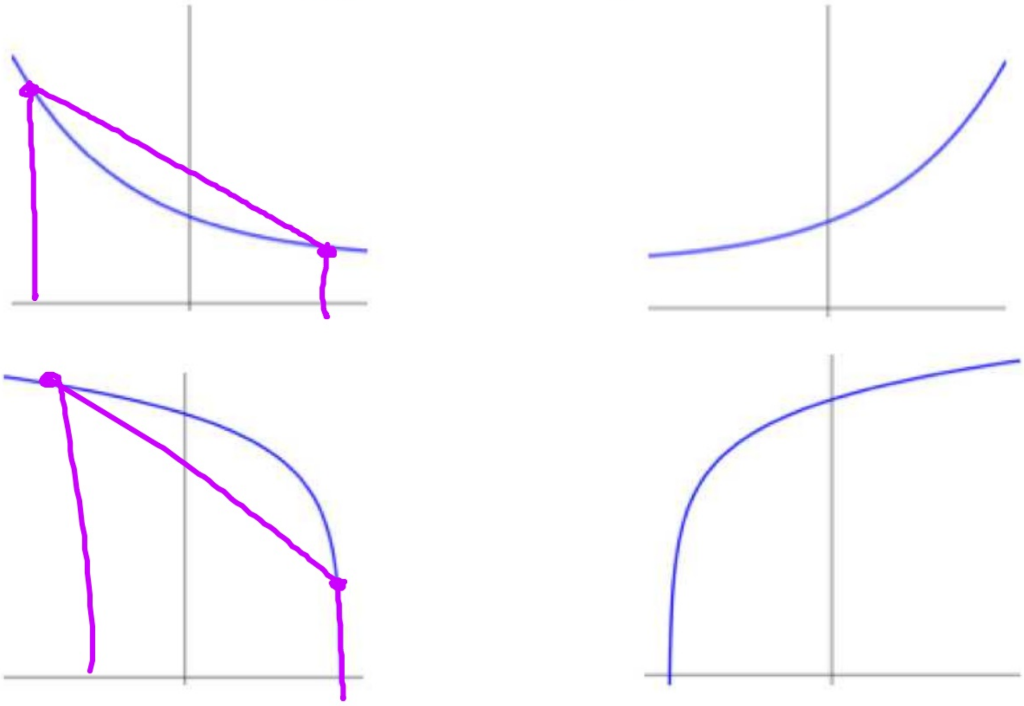
Observe that the coefficients in the Trapezoidal Rule have the following pattern.

1 2 2 2 . . . 2 2 1



- Over and Under Estimates

ex: Determine if Trapezoid Approximation would yield an over or under approximations.

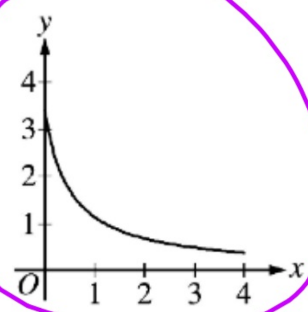


When estimating an integral value using a Trapezoidal Approximation, the approximation will be an over or underestimate depending on whether the curve is

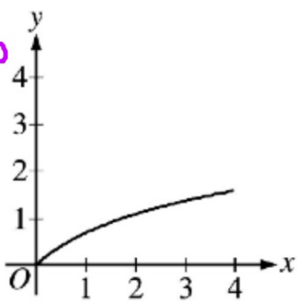
concave up or concave down.

If a trapezoidal sum over approximates $\int_0^4 f(x)dx$, and a right Riemann sum under approximates $\int_0^4 f(x)dx$, which of the following could be the graph of $y = f(x)$?

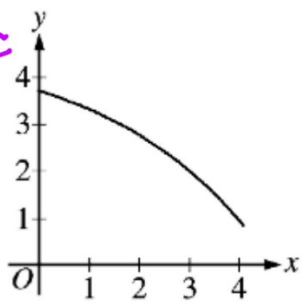
a



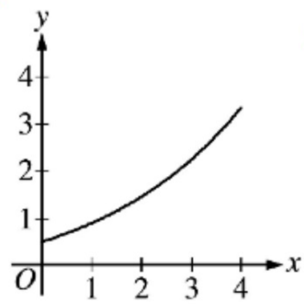
b



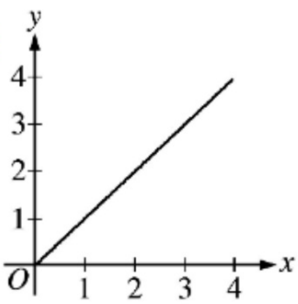
c



d



e



Comments:

If the graph is decreasing, then $\text{Right}(n) < \int_a^b f(x)dx < \text{Left}(n)$ for the right Riemann and left Riemann sums using n subintervals.

If the graph is concave up, then $\text{Mid}(n) < \int_a^b f(x)dx < \text{Trap}(n)$ for the trapezoid sum and the midpoint Riemann sum using n subintervals.

Graph (A) is decreasing and concave up, and therefore could be the graph of $y = f(x)$.

If the graph is increasing or concave down, the respective inequalities are reversed.

- Riemann Approximations and Tabular Data

ex: Estimate the value of the integral using the indicated method and n subdivisions indicated by the data.

right n=4
m.p. n=2
trap n=4

x	2	4	6	8	10
$f(x)$	17	1	-2	8	7

a) $\int_{2}^{10} f(x)dx$ Left Riemann, n=4

$2(17+1+-2+8)$

x	2	4	6	8	10
$f(x)$	$\cancel{17}$	1	-2	8	7

b) $\int_2^{10} f(x)dx$ Right Riemann, n=4

$2[1 + -2 + 8 + 7]$

28

x	2	4	6	8	10
$f(x)$	17	1	-2	8	7

c) $\int_{2}^{10} f(x)dx$ Midpoint Riemann, n=2

$$4\left[1+8\right]=36$$

x	2	4	6	8	10
$f(x)$	17	1	-2	8	7

d) $\int_{2}^{10} f(x)\,dx$ Trapezoid Approximation, n=4

$$\frac{1}{2}(2)\left[\underline{17+1} + \underline{1+-2} + \underline{-2+8} + \underline{8+7}\right]$$

38

ex: Estimate the value of the integral using the indicated method and n subdivisions indicated by the data.

x	1	3	9	12	21
$f(x)$	2	-10	11	5	6

left: n=4
trap: n=4

a) $\int_{1}^{21} f(x)\,dx$ Right Riemann, n=4

$9(6) + 3(5) + 6(11)$
$+ 2(-10)$
115

x	1	3	9	12	21
$f(x)$	2	-10	11	5	6

b) $\int_{1}^{21} f(x)dx$ Trapezoid Approximation, n=~~2~~ 4

$$\frac{1}{2}\left[2(2-10)+6(-10+11)+3(11+5)+9(5+6)\right]$$

$$\frac{137}{2}$$

x	1	3	9	12	21
$f(x)$	2	-10	11	5	6

c) $\int_1^9 f(x)\,dx$ Left Riemann, n=~~2~~ 4

$2(2) + 6(-10) + 3(11) + 9(5)$

22

FR 20

A test plane flies in a straight line with positive velocity $v(t)$, in miles per minute at time t minutes, where v is a differentiable function of t. Selected values of $v(t)$ for $0 \le t \le 40$ are shown in the table above.

t (min)	0	5	10	15	20	25	30	35	40
$v(t)$ (mpm)	7.0	9.2	9.5	7.0	4.5	2.4	2.4	4.3	7.3

(a) Use a midpoint Riemann sum with four subintervals of equal length and values from the table to approximate $\int_0^{40} v(t)\,dt$. Show the computations that lead to your answer. Using correct units, explain the meaning of $\int_0^{40} v(t)\,dt$ in terms of the plane's flight.

(b) Based on the values in the table, what is the smallest number of instances at which the acceleration of the plane could equal zero on the open interval $0 < t < 40$? Justify your answer.

a.) $10(9.2 + 7 + 2.4 + 4.3) = 229$ miles

This represents how far the plane traveled (in miles) from time t = 0 to t = 40 (minutes)

4.2/4.6 Extra Practice WKST

3, 4, 5, 6

1.

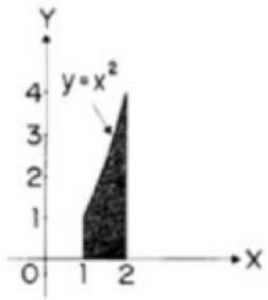
(Calculator Permitted) If the midpoints of 4 equal-width rectangles is used to approximate the area enclosed between the x-axis and the graph of $y = 4x - x^2$, the approximation is

(A) 10 (B) 10.5 (C) 10.666 (D) 10.75 (E) 11

*See printout.

4.2/4.6 Extra Practice WKST

2.



Calculate the approximate area of the shaded region in the figure by the trapezoidal rule, using divisions at $x=\frac{4}{3}$ and $x=\frac{5}{3}$.

(A) $\frac{50}{27}$ (B) $\frac{251}{108}$ (C) $\frac{7}{3}$ (D) $\frac{127}{54}$ (E) $\frac{77}{27}$

4.2/4.6 Extra Practice WKST

$$\frac{1}{2}\int_{-4}^{2} e^{-x}\,dx$$

3.

If three equal subdivisions of $[-4, 2]$ are used, what is the trapezoidal approximation of

$\int_{-4}^{2} \frac{e^{-x}}{2}\,dx$?

(A) $e^2 + e^0 + e^{-2}$

(B) $e^4 + e^2 + e^0$

(C) $e^4 + 2e^2 + 2e^0 + e^{-2}$

(D) $\frac{1}{2}\left(e^4 + e^2 + e^0 + e^{-2}\right)$

(E) $\frac{1}{2}\left(e^4 + 2e^2 + 2e^0 + e^{-2}\right)$

4.2/4.6 Extra Practice WKST

4.

x	0	0.5	1.0	1.5	2.0
$f(x)$	3	3	5	8	13

A table of values for a continuous function f is shown above. If four equal subintervals of $[0, 2]$ are used, which of the following is the trapezoidal approximation of $\int_0^2 f(x)\,dx$?

(A) 8 (B) 12 (C) 16 (D) 24 (E) 32

4.2/4.6 Extra Practice WKST

$2(5+2+8) = 30$ ft/sec

5. $\int_0^6 a(t)\,dt$ initial + net change

t (sec)	0	2	4	6
$a(t)$ (ft/sec^2)	5	2	8	3

The data for the acceleration $a(t)$ of a car from 0 to 6 seconds are given in the table above. If the velocity at $t = 0$ is 11 feet per second, the approximate value of the velocity at $t = 6$, computed using a left-hand Riemann sum with three subintervals of equal length, is

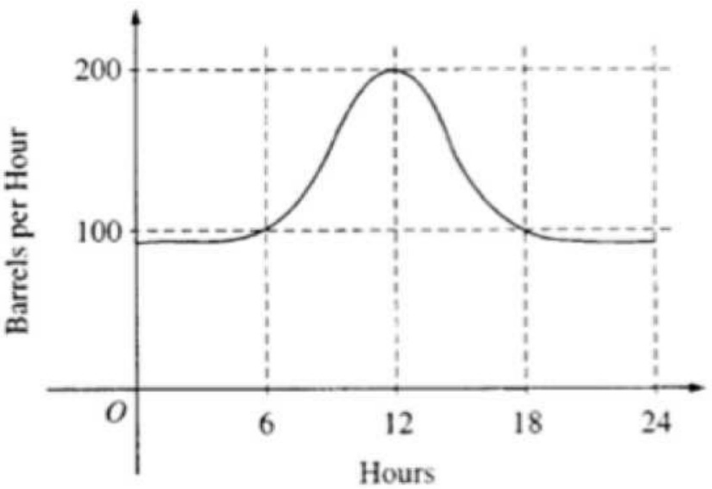
(A) 26 ft/sec (B) ~~30 ft/sec~~ (C) 37 ft/sec (D) 39 ft/sec (E) 41 ft/sec

$11+30$

41

4.2/4.6 Extra Practice WKST

6.

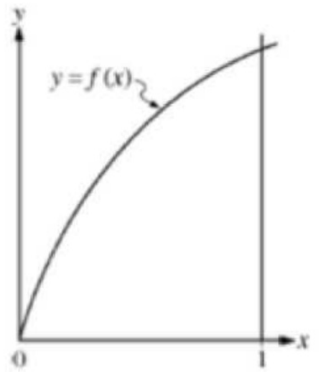


The flow of oil, in barrels per hour, through a pipeline on July 9 is given by the graph shown above. Of the following, which best approximates the total number of barrels of oil that passed through the pipeline that day?

(A) 500 (B) 600 (C) 2,400 (D) 3,000 (E) 4,800

4.2/4.6 Extra Practice WKST

7.



A left Riemann sum, a right Riemann sum, and a trapezoidal sum are used to approximate the value of $\int_0^1 f(x)\,dx$, each using the same number of subintervals. The graph of the function f is shown in the figure above. Which of the sums give an underestimate of the value of $\int_0^1 f(x)\,dx$?

I. Left sum

II. Right sum

III. Trapezoidal sum

(A) I only

(B) II only

(C) III only

(D) I and III only

(E) II and III only