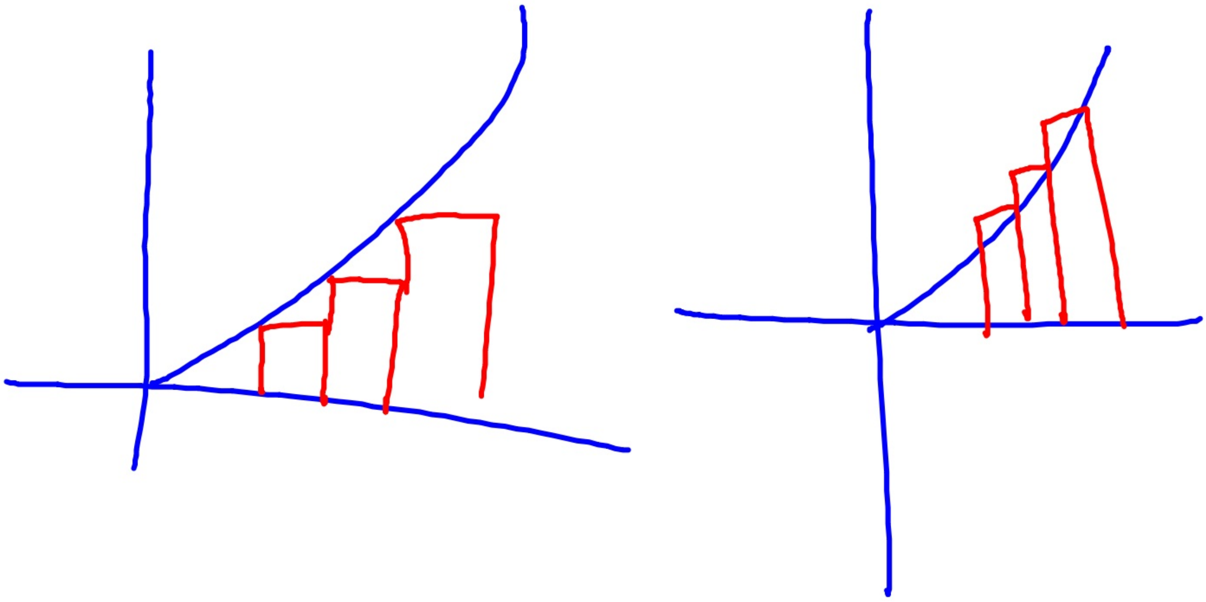
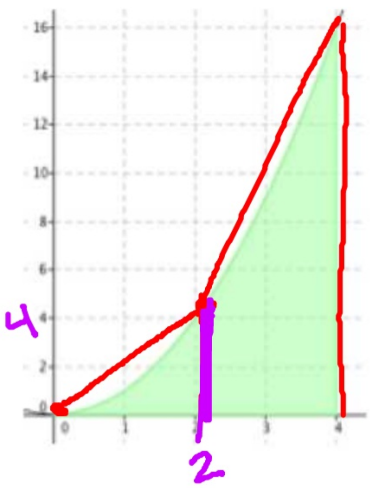


## Riemann Approximations - cont.

$\frac{1}{2}h(b_1+b_2)$

ex: Approximate the integral $\int_0^4 x^2\,dx$ using a trapezoidal approximation with two equal subdivisions. Then determine if the approximation is an over or under estimate.

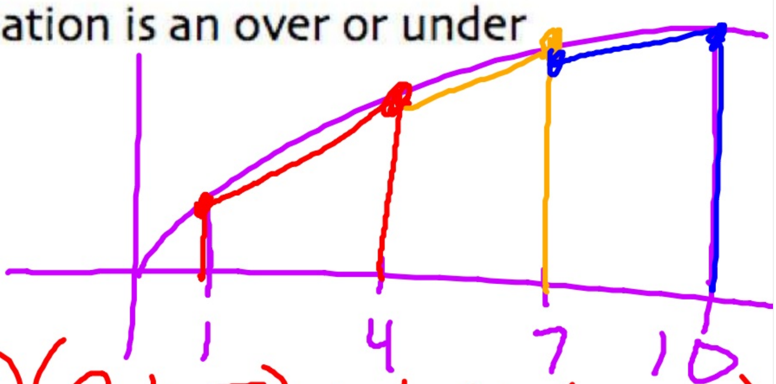



$\frac{1}{2}(2)(0+4)+\frac{1}{2}(2)(4+16)$

$4 + 20$

$24$

ex: Approximate the integral $\int_1^{10} \sqrt{x}\,dx$ using a trapezoidal approximation with three equal subdivisions. Then determine if the approximation is an over or under estimate.




$$\frac{1}{2}(3)(1+2) + \frac{1}{2}(3)(2+\sqrt{7}) + \frac{1}{2}(3)(\sqrt{7}+\sqrt{10})$$

$$\frac{1}{2}(3)\left(1+2(2)+2(\sqrt{7})+\sqrt{10}\right)$$

$$20.180067 \rightarrow 20.181 \text{ or } 20.18$$

ex: Approximate the integral $\int_1^5 \ln x dx$ using a trapezoidal approximation with four equal subdivisions. Then determine if the approximation is an over or under estimate.

ex: Approximate the integral $\int_2^{14} \frac{1}{x} dx$ using a trapezoidal approximation with three equal subdivisions. Then determine if the approximation is an over or under estimate.

**THEOREM 4.9 The Trapezoidal Rule**

Let $f$ be continuous on $[a, b]$. The Trapezoidal Rule for approximating $\int_a^b f(x)\,dx$ is

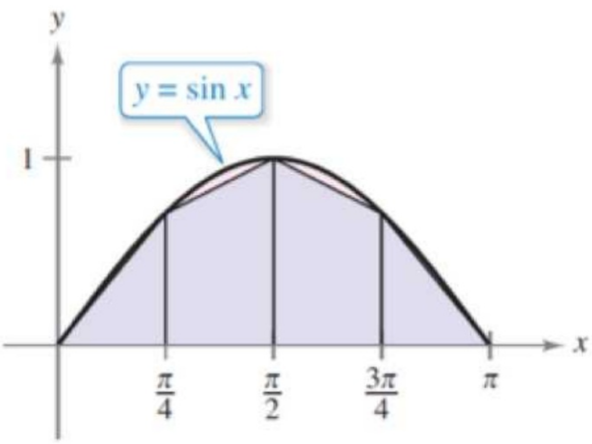
$$\int_a^b f(x)\,dx \approx \frac{b-a}{2n}[f(x_0) + 2f(x_1) + 2f(x_2) + \cdot \cdot \cdot + 2f(x_{n-1}) + f(x_n)].$$

Moreover, as $n \to \infty$, the right-hand side approaches $\int_a^b f(x)\,dx$.

## Remark

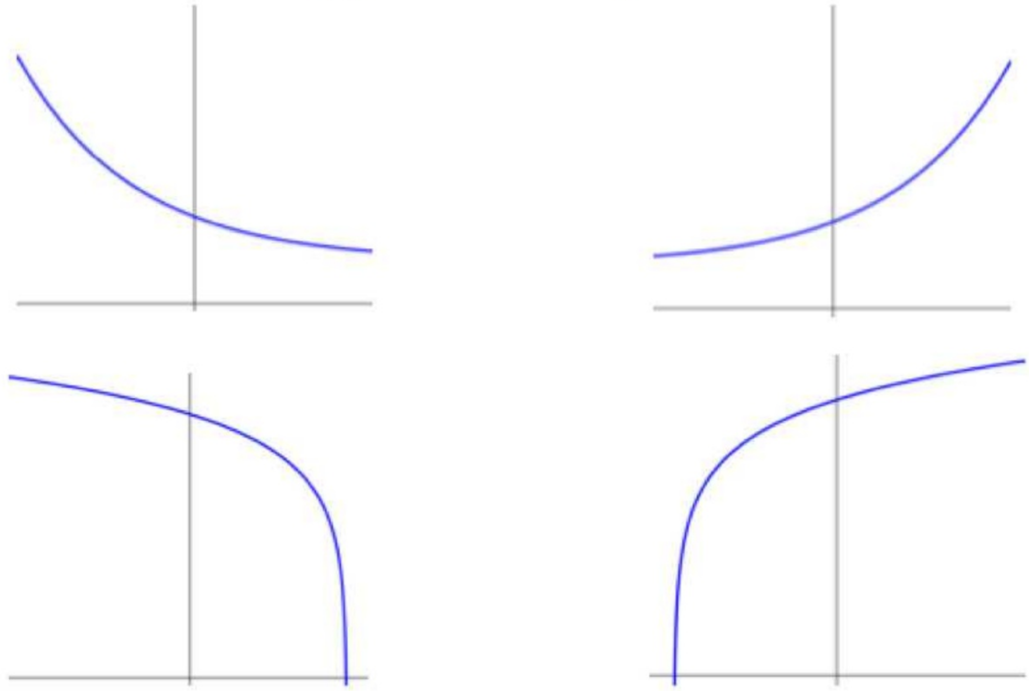
Observe that the coefficients in the Trapezoidal Rule have the following pattern.

1 2 2 2 . . . 2 2 1



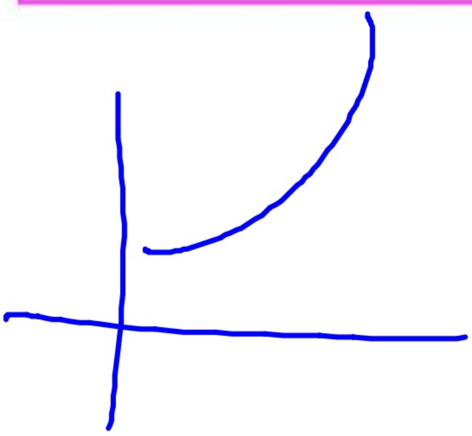
- Over and Under Estimates

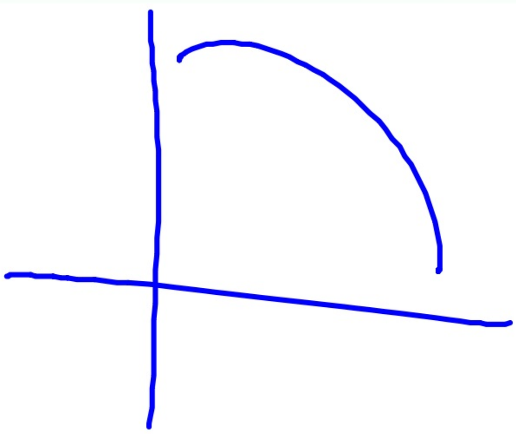
ex: Determine if Trapezoid Approximation would yield an over or under approximations.



When estimating an integral value using a Trapezoidal Approximation, the approximation will be an over or underestimate depending on whether the curve is

concave up or concave down.





- Riemann Approximations and Tabular Data

ex: Estimate the value of the integral using the indicated method and n subdivisions indicated by the data.

| $x$ | 2 | 4 | 6 | 8 | 10 |
|---|---|---|---|---|---|
| $f(x)$ | 17 | 1 | -2 | 8 | 7 |

a) $\int_{2}^{10} f(x)dx$ Left Riemann, n=4

$2(17) + 2(1) + 2(-2) + 2(8)$

$2(17 + 1 + -2 + 8)$

48

| $x$ | 2 | 4 | 6 | 8 | 10 |
|---|---|---|---|---|---|
| $f(x)$ | 17 | 1 | -2 | 8 | 7 |

b) $\int_{2}^{10} f(x)dx$ Right Riemann, n=4

$2(7+8+-2+1)$

$28$

| $x$ | 2 | 4 | 6 | 8 | 10 |
|---|---|---|---|---|---|
| $f(x)$ | 17 | 1 | -2 | 8 | 7 |

c) $\int_2^{10} f(x)dx$    Midpoint Riemann, n=2

$4(1 + 8)$

$36$

| $x$ | 2 | 4 | 6 | 8 | 10 |
|---|---|---|---|---|---|
| $f(x)$ | 17 | 1 | -2 | 8 | 7 |

d) $\int_{2}^{10} f(x)dx$ Trapezoid Approximation, n=4

$$\frac{1}{2}\cdot 2\left(\underbrace{17+1}+\underbrace{1+-2}+\underbrace{-2+8}+\underbrace{8+7}\right)$$

$38$

ex: Estimate the value of the integral using the indicated method and n subdivisions indicated by the data.

| $x$ | 1 | 3 | 9 | 12 | 21 |
|---|---|---|---|---|---|
| $f(x)$ | 2 | -10 | 11 | 5 | 6 |

a) $\int_{1}^{21} f(x)dx$ Right Riemann, n=4

$9(6)+3(5)+6(11)+2(-10)$

$115$

| $x$ | 1 | 3 | 9 | 12 | 21 |
| --- | --- | --- | --- | --- | --- |
| $f(x)$ | 2 | -10 | 11 | 5 | 6 |

b) $\int_{1}^{21} f(x)dx$ Trapezoid Approximation, n=2

$\frac{1}{2}\left[8(2+11)+12(11+6)\right]$

154

| $x$ | 1 | 3 | 9 | 12 | 21 |
| --- | --- | --- | --- | --- | --- |
| $f(x)$ | 2 | -10 | 11 | 5 | 6 |

+10

c) $\int_1^9 |f(x)|dx$ Left Riemann, n=2

$2(2)+6(10)$

$64$