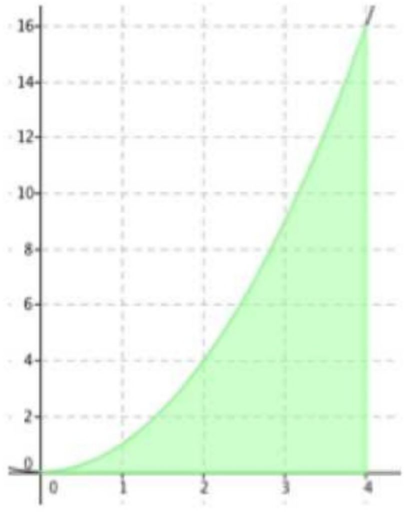
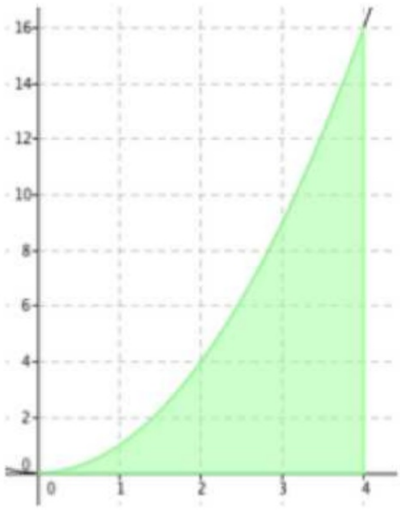
Riemann Approximations

ex: Evaluate: $\int_0^4 x^2 dx$



Since we can't find the exact area using "shapes", we will approximate the area using a Riemann Approximation.



rectangles

Approximation Techniques:

1. Left Riemann
2. Right Riemann
3. Midpoint Riemann
4. Trapezoidal

If there is a constant width, the width can be calculated by:

Width = (b - a)/n

$$\int_{0}^{10} f(x)\,dx$$

$n=5$ rectangles

$w = 2$

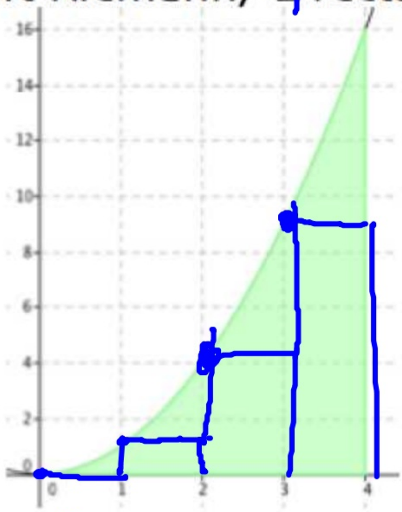
$$\int_{1}^{11} f(x)\,dx$$

$n=4$ rectangles

$w = 2.5$

ex: Approximate the integral $\int_0^4 x^2\,dx$ using the indicated technique. Then determine if the approximation is an over or under estimate.

a) Left Riemann, ~~2~~ 4 rectangles



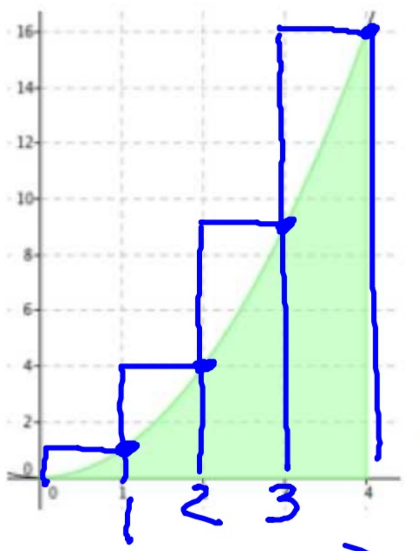
$1\cdot 1 + 4\cdot 1 + 9\cdot 1$

14

$w = \frac{4-0}{4} = 1$

b) Right Riemann, ~~2~~ 4 rectangles

$$\int_0^4 x^2\,dx$$



$16\cdot 1 + 9\cdot 1 + 4\cdot 1 + 1\cdot 1$

or

$1(16+9+4+1)$

30

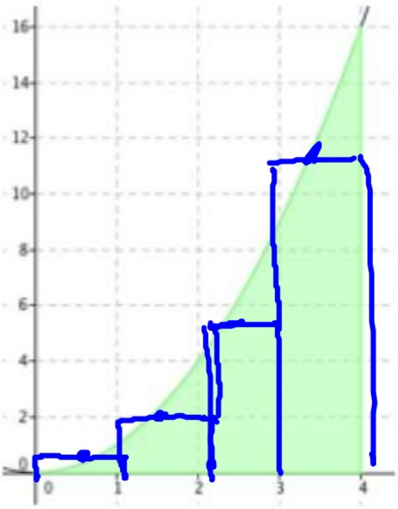
(over)

$\Delta x \to 0$

$n \to \infty$

c) Midpoint Riemann, ~~2~~ 4 rectangles

$$\int_0^4 x^2\,dx$$



21

$$1\left(\left(\frac{1}{2}\right)^2+\left(\frac{3}{2}\right)^2+\left(\frac{5}{2}\right)^2+\left(\frac{7}{2}\right)^2\right)$$

ex: Approximate the integral $\int_{1}^{10} \sqrt{x}dx$ using the indicated technique. Then determine if the approximation is an over or under estimate.

a) Left Riemann, 3 rectangles

16.937 (under approx)

ex: Approximate the integral $\int_{1}^{10} \sqrt{x}\,dx$ using the indicated technique. Then determine if the approximation is an over or under estimate.

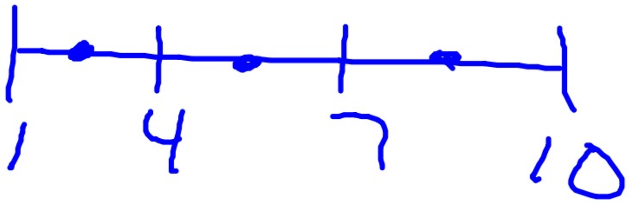
b) Right Riemann, 3 rectangles

23.424 (over approx)

ex: Approximate the integral $\int_{1}^{10} \sqrt{x}\,dx$ using the indicated technique. Then determine if the approximation is an over or under estimate.

c) Midpoint Riemann, 3 rectangles

$3\left(\sqrt{2.5}+\sqrt{5.5}+\sqrt{8.5}\right)$

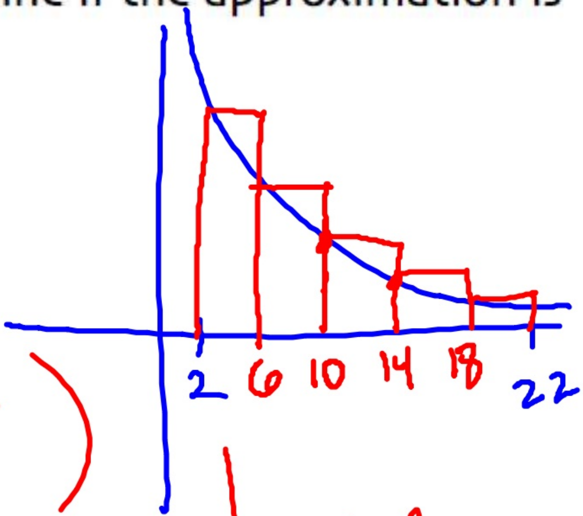


20.525

over

ex: Approximate the integral $\int_{2}^{22} \frac{1}{x}\,dx$ using the indicated technique. Then determine if the approximation is an over or under estimate.

a) Left Riemann, 5 rectangles



$w = 4$

$4\left(\frac{1}{2} + \frac{1}{6} + \frac{1}{10} + \frac{1}{14} + \frac{1}{18}\right)$

3.574

3.575

ex: Approximate the integral $\int_2^{22} \frac{1}{x} dx$ using the indicated technique. Then determine if the approximation is an over or under estimate.

b) Right Riemann, 5 rectangles

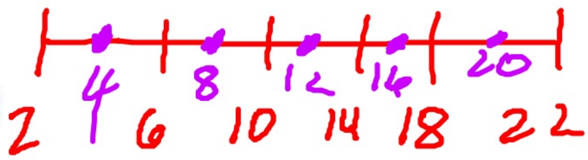
$$4\left(\frac{1}{22}+\frac{1}{18}+\frac{1}{14}+\frac{1}{10}+\frac{1}{6}\right)$$

$$1.756$$

ex: Approximate the integral $\int_{2}^{22} \frac{1}{x}\,dx$ using the indicated technique. Then determine if the approximation is an over or under estimate.

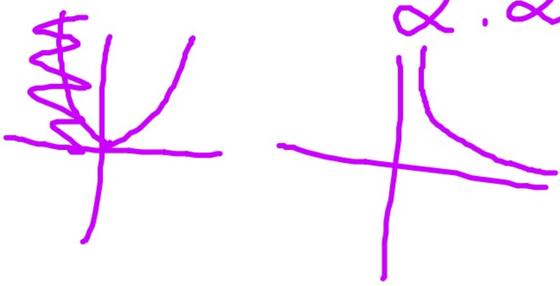
actual 2.398

c) Midpoint Riemann, 5 rectangles



$$4\left(\frac{1}{4}+\frac{1}{8}+\frac{1}{12}+\frac{1}{16}+\frac{1}{20}\right)$$

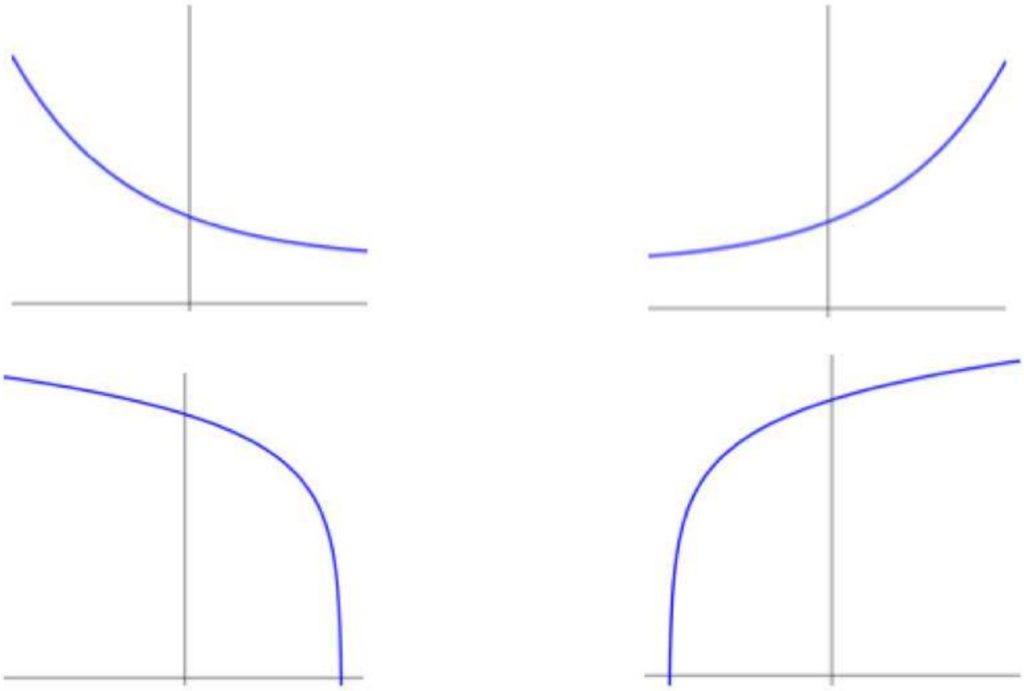
2.283



under

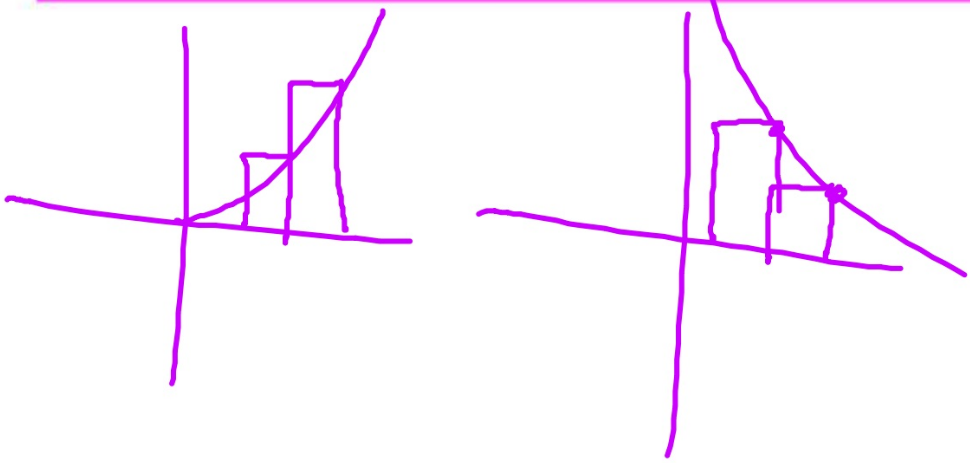
- Over and Under Estimates

ex: Determine if the Left and Right estimates are over or under approximations.

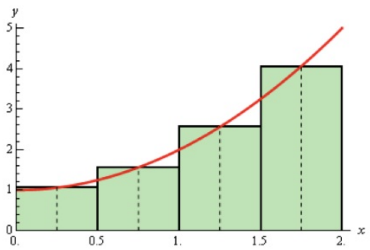


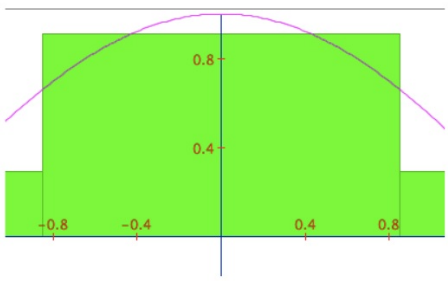
When estimating an integral value using a left or right Riemann Approximation, the approximation will be an over or underestimate depending on whether the curve is

incr. or decr.



For Midpoints: Will the estimate be an under or over approximation for Concave Up/Concave Down functions





Concave up
Under approx.
more area under
vs. over

Concave down
Over approx.
More area over
vs. under