41.) $g(t) = \tan(\arcsin t)$

$$g'(t) = \underline{\sec^2(\arcsin t)} \cdot \frac{1}{\sqrt{1-t^2}}$$

$$= \left(\frac{1}{(1-t^2)^1}\right)\left(\frac{1}{\sqrt{1-t^2}}\right)$$

$$= \frac{1}{(1-t^2)^{3/2}}$$

4.) $f(x) = x^3 - 3x^2 + 8x + 5$ $\quad g'(5)$

5

$0 = x$

$f : (0, 5)$

$g : (5, 0)$

$f'(x) = 3x^2 - 6x + 8$

$f'(0) = 8$

3.) $f(x) = \frac{1}{4}x^3 + x - 1$

3

$\underline{2} = x$

$f'(x) = \frac{3}{4}x^2 + 1$

$f'(2) = 4$

$(f^{-1})'(3) = \frac{1}{4}$

$f: (2, 3)$

$f^{-1}(3, 2)$

89.) $f(x) = 2x\sqrt{x-6}$ $(f^{-1})'(40)$

$40 = 2x\sqrt{x-6}$ $x=10$

$40 = 2(10)\sqrt{4}$ ✓

$f:(10, 40)$

$f^{-1}(40, 10)$

$f'(x) = \cancel{2}x \cdot \frac{1}{\cancel{2}}(x-6)^{-1/2} + (x-6)^{1/2} \cdot 2$

$f'(10) = \frac{10}{2} + 4$

$9 \longrightarrow \frac{1}{9}$

54.) $y = \operatorname{arcsec} 4x$ $\left(\frac{\sqrt{2}}{4}, \frac{\pi}{4}\right)$

$$y' = \frac{4}{|4x|\sqrt{16x^2-1}}$$

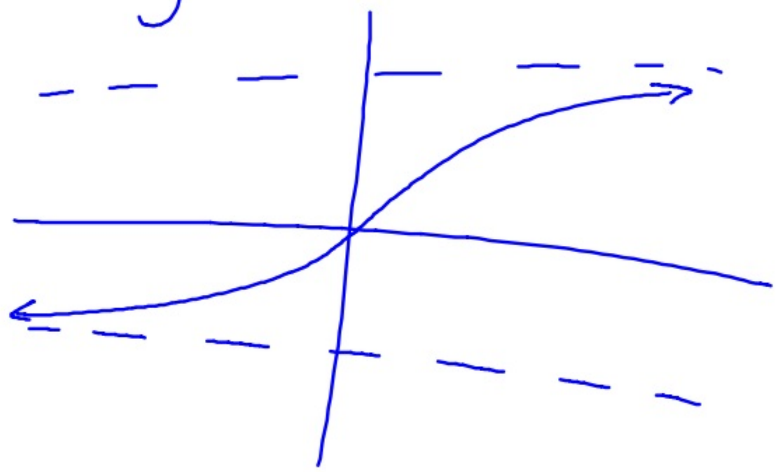
$$y' = \frac{1}{|x|\sqrt{16x^2-1}}$$

$$y'\left(\frac{\sqrt{2}}{4}\right) = \frac{4}{\sqrt{2}\sqrt{16\left(\frac{2}{16}\right)-1}}$$

$$= \frac{4}{\sqrt{2}}$$

$$y - \frac{\pi}{4} = \frac{4}{\sqrt{2}}\left(x - \frac{\sqrt{2}}{4}\right)$$

$y = \arctan x$



25.) $g(x) = 3\arccos\frac{x}{2}$

$$g'(x) = 3\left(\frac{-\frac{1}{2}}{\sqrt{1-\frac{x^2}{4}}}\right)$$

$$= \frac{-\frac{3}{\cancel{2}}}{\sqrt{\frac{4-x^2}{\cancel{4}}}} \quad \frac{-3}{\sqrt{4-x^2}}$$

57.) $f(x) = \arccos x$

$$f'(x) = \frac{-1}{\sqrt{1-x^2}}$$

$$+2 = \frac{+1}{\sqrt{1-x^2}}$$

$$2\sqrt{1-x^2} = 1$$

$$\sqrt{1-x^2} = \frac{1}{2}$$

$$1 - x^2 = \frac{1}{4}$$

$$\frac{3}{4} = x^2$$

$$\pm\frac{\sqrt{3}}{2} = x$$

$\left(\frac{\sqrt{3}}{2}, \frac{\pi}{6}\right)$	$y - \frac{\pi}{6} = -2\left(x - \frac{\sqrt{3}}{2}\right)$
$\left(-\frac{\sqrt{3}}{2}, \frac{5\pi}{6}\right)$	$y - \frac{5\pi}{6} = -2\left(x + \frac{\sqrt{3}}{2}\right)$

7a.) $f(x)$; $(3, 10)$ $m=4$

$y-10=4(x-3)$

7b.) $f\ (1,\ 3)$ $m=2$

$f^{-1}(3,\ 1)$

$y-1=\frac{1}{2}(x-3)$

51.) $y = 2\arcsin x \quad \left(\frac{1}{2}, \frac{\pi}{3}\right)$

$y' = 2 \cdot \frac{1}{\sqrt{1-x^2}}$ $\qquad y - \frac{\pi}{3} = \frac{4}{\sqrt{3}}\left(x - \frac{1}{2}\right)$

$y'\left(\frac{1}{2}\right) = 2 \cdot \frac{1}{\sqrt{1-\frac{1}{4}}}$

$= 2 \frac{1}{\sqrt{\frac{3}{4}}} = \frac{2}{\frac{\sqrt{3}}{2}} = \frac{4}{\sqrt{3}} = m$

5.) $f(x) = 2x^3 - 3x \ , \ x > \frac{\sqrt{2}}{2}$

$h: (-1, 1)$

$f: (1, -1)$

$-1 = 2x^3 - 3x$

$1 = x$

$f'(x) = 6x^2 - 3$

$f'(1) = 3$

$\boxed{\frac{1}{3}}$

Rates of Change and Rectilinear Motion Notes

- Application of the Derivative: slope, rates of change, slope of tangent lines.

- Rate of Change(definition): (change in y) ÷ (change in x)

Examples:

- ______
- ______
- ______
- ______
- ______

Ex 1: The length of a rectangle is given by $3t+2$ meters. The width is 3 times the length. Find the rate of change of the area with respect to time at t = 1hr.

$w = 3l$

$w = 3(3t+2)$

$w = 9t + 6$

$A = (3t+2)(9t+6)$

$A'(t) = (3t+2)9 + (9t+6)\cdot 3$

$A'(1) = 45 + 45 = 90 \text{ m/hr}$

Ex 2: A population of 500 bacteria is introduced into a culture and grows in numbers according to the equation $P(t) = 500\left(1 + \frac{4t}{50+t^2}\right)$, where t is measured in hours. Find the rate at which the population is growing when $t = 2$.

$P'(2) = 31.55$ bacteria/hr

- Rectilinear Motion Problems – When we talk about these types of problems, we often talk about three types of functions

1. Position:
   Notation: $s(t)$, $y(t)$, $x(t)$
   Application(s): position at $t = 3$; displacement meters
2. Velocity:
   Notation: $s'(t)$, $x'(t)$, $v(t)$
   - (rate of change of position) m/sec
   Application(s): find velocity at $t = 1$, instantaneous velocity
3. Acceleration:
   Notation: $s''(t)$, $v'(t)$, $a(t)$
   - rate of change of velocity m/sec^2
   Application(s): ______

"stature"

Ex 3: At time $t=0$ seconds a diver jumps from a platform diving board that is 32 feet above the water. The position of the diver is given by $s(t) = -16t^2 + 16t + 32$.

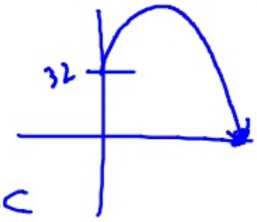
P
V
A
J

a) When does the diver hit the water?

$0 = -16t^2 + 16t + 32$

$0 = -16(t^2 - t - 2)$

$= -16(t-2)(t+1)$ $\quad t = 2\text{sec}$



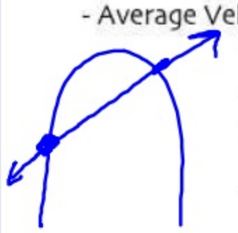
b) What is the diver's velocity at impact?

$s'(t) = -32t + 16$

$s'(2) = -64 + 16$

$= -48 \text{ m/sec}$

- Average Velocity vs. Instantaneous Velocity



- Average Velocity: rate of change on an interval

Formula: $\dfrac{s(b)-s(a)}{b-a}$ $\quad (m_{sec})$

- Instantaneous Velocity: rate of change at a given point

Formula: $s'(t) = v(t)$ $\quad (m_{tan})$

- Speed: $|v(t)|$

Justifications

- Rest: $v(t) = 0$

- Left and Right Motion:
  - Left: $v(t) < 0$
  - Right: $v(t) > 0$
  - Changes Direction: $v(t) = 0$ and $v(t)$ changes signs

Ex 4: A billiard ball is dropped from a height of 100 ft, its height s at time t is given by the position function $s(t) = -16t^2 + 100$, where s is measured in feet and t is measured in seconds.

a) Find the average velocity over time interval [1, 2].

$$\frac{s(2)-s(1)}{2-1} = \frac{36-84}{1} = -48 \text{ ft/sec}$$

b) Find the Instantaneous velocity at the endpoints of the interval.

$s'(t) = -32t$

$s'(1) = -32 \text{ ft/sec}$

$s'(2) = -64 \text{ ft/sec}$

c) Find the speed at the endpoints of the interval.

$|v(1)| = 32 \text{ ft/sec}$

$|v(2)| = 64 \text{ ft/sec}$

Ex 5: A particle starts at time t=0 and moves along the x-axis so that its position at any time $t \geq 0$ is given by $x(t) = (t-1)^3(2t-3)$.

a) Find the velocity of the particle at any time $t \geq 0$. Simplify.

$$x'(t) = v(t) = (t-1)^3(2) + (2t-3)\cdot 3(t-1)^2$$

$$= (t-1)^2\left[2(t-1) + 3(2t-3)\right]$$

$$= (t-1)^2(8t-11)$$

b) Determine the values of t for which the particle is at rest.

$v(t) = 0$

$t = 1, \frac{11}{8}$ sec

c) Determine the values of t for which the particle is moving to the left. JYA.

$v(t)$: 0 (−) 1 (−) $\frac{11}{8}$ (+)

Moving to left on (0, 11/8) because $v(t) \leqslant 0$ on this interval

d) Determine the values of t for which the particle is moving to the right. JYA.

Moving to the right on (11/8, inf) because $v(t)>0$ on this interval

e) Determine the values of t for which the particle changes direction. JYA.

The particle changed direction at t = 11/8 because v(11/8) = 0 and v(t) changes signs at this time.

Ex 6: A particle starts at time $t=0$ and moves along the x-axis so that its [redacted] at any time $t \geq 0$ is given by ~~$v(t)=2(t-3)(t+2)$~~ $x(t)=t^2-6t+3$ position

a) Find the time(s) when the particle is at rest.

$x'(t)=2t-6$ $\quad t=3$

$0=2t-6$

b) Find the displacement of the particle at t = 5.

$x(5)-x(0)$

$x(5)=-2$ $\quad -2-3$

$x(0)=3$

-5

Ex 7: Let y(t) represent the temperature of a pie that has been removed from a 450 degree oven and left to cool in a room with a temperature of 72 degrees.

t (min)	0	5	10	15	20	25	30
$y(t)$ (°F)	450	388	338	292	257	226	200

-7 °F/min

a) Estimate the rate of change of the temperature of the pie at t = 17 minutes.

$$\frac{257-292}{20-15}$$

b) Find the average rate of change of the temperature of the pie from t = 0 to t = 30. Include units of measure.

$$\frac{y(30)-y(0)}{30-0} = \frac{200-450}{30} = \frac{-25}{3} \text{ °F/min}$$