3.6—Optimization

Steps to follow

Guidelines for Solving Optimization Word Problems

1. Identify all given quantities and quantities to be determined.
2. If possible, draw a sketch.
3. Write a primary equation for the quantity that is to be maximized or minimized.
4. If possible, write a secondary equation relating the variables of the primary equation. Using the secondary equation, use substitution to reduce the primary equation to one variable.
5. Determine a feasible domain for the unknown quantities.
6. Determine the desired maximum or minimum values by calculus techniques learned in this chapter. (i.e. find a DERIVATIVE, etc)

Example 1:
Find two positive numbers whose sum is 60 and whose product of one times the square of the other is a maximum. Don't get your toga in a knot!!

Example 2:
An 8-inch long pipe cleaner (used to clean pipes and form rectangles for a calculus class example) is bent into the shape of a rectangle. What dimensions of the rectangle will produce the rectangle with maximum area?

Calculus Ninja (Another useful application of a pipe cleaner)

Example 3:

A ladybug farmer has 500 inches of fencing and wants to fence off a rectangular field that borders on a straight river (to enclose his grazing ladybugs). He needs no fence along the river (ladybugs can't swim, and he has clipped their wings). What are the dimensions of the field that has the largest grazing area for his hungry ladybugs?


Example 4:

The same ladybug farmer has purchase some expensive, extraordinary diva ladybugs who require *exactly* 10,000 square inches of grazing in order to be at their optimal "ladybug-like" state. What is the least amount of fencing required to get these diva ladybugs at their optimal state if the farmer is still allowed to build along the very straight river?? (Don't think diva lady bugs can swim and don't still have their wings clipped).


Example 5:
Find the point on the curve $y = x^2$ closest to $(3,0)$.

Because d is the smallest when the expression inside the radical is smallest, you need only find the critical numbers of the radicand when maximizing or minimizing distance.

Example 6:

A manufacturer wants to design an open box having a square base with a surface area of 108 square inches. What dimensions will produce a maximum volume?

Example 7:
Find the dimensions of the largest rectangle that can be inscribed under the curve $y = 16 - x^2$ in the first quadrant.

Example 8:
An open-top box is to be constructed from a square piece of cardboard, 3 feet wide, by cutting out a square from each corner and bending up the sides. What is the largest volume that the box can have?

WAYS TO JUSTIFY AN ABSOLUTE EXTREMA (in absence of endpoints)

Method 2: 1^{st} Derivative Test for Relative Extrema modified for Absolute Extrema
We will use this method when we don't have both endpoints at a closed interval, but instead have either a half-open interval or open interval. It does require a continuous function, though.

Method 3: 2^{nd} Derivative Test for Relative Extrema modified for Absolute Extrema
This method is preferred to method 2 when the 2^{nd} derivative is easy to obtain.

**In either case, it is important to show the test works FOR ALL values in the relevant domain. This transforms each test from a local argument to a global argument.