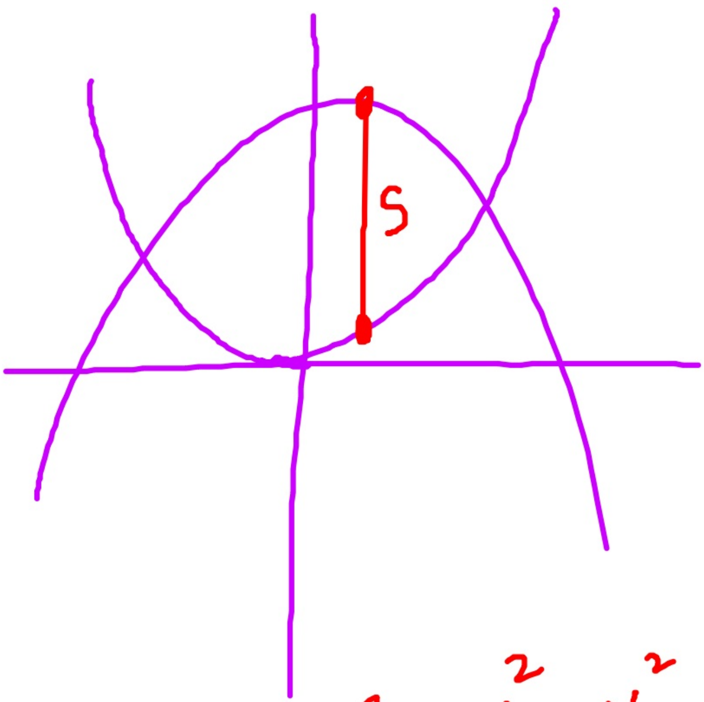


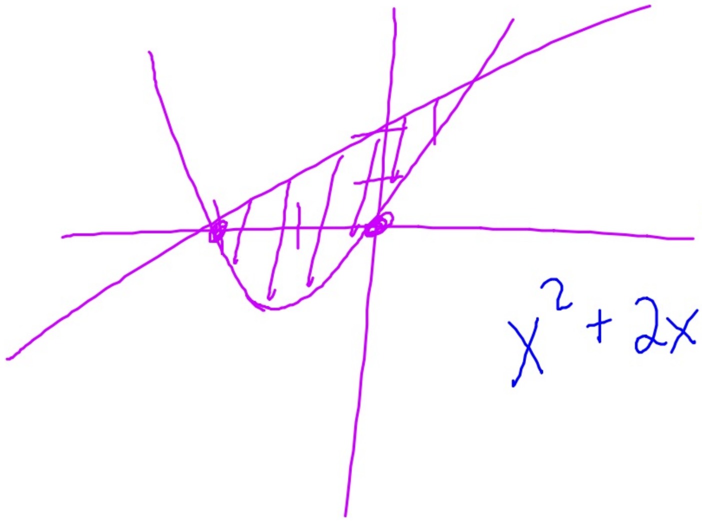

$$\int s^2\,dx$$

$$\int_{-2}^{2} (8-2x^2)^2\,dx$$

$$s = 8 - x^2 - x^2$$
$$= 8 - 2x^2$$

21.) $f(x) = x^2 + 2x = x(x+2)$

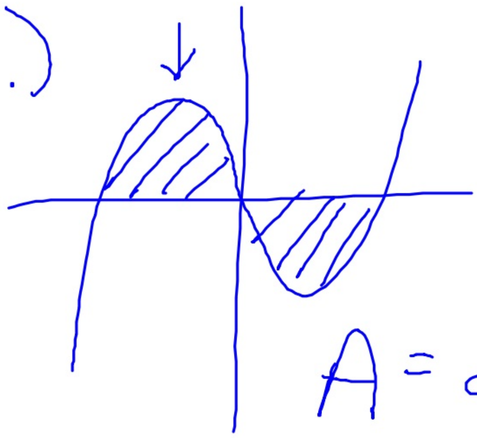
$g(x) = x + 2$



$$\int_{-2}^{1} \left(x + 2 - (x^2 + 2x)\right) dx$$

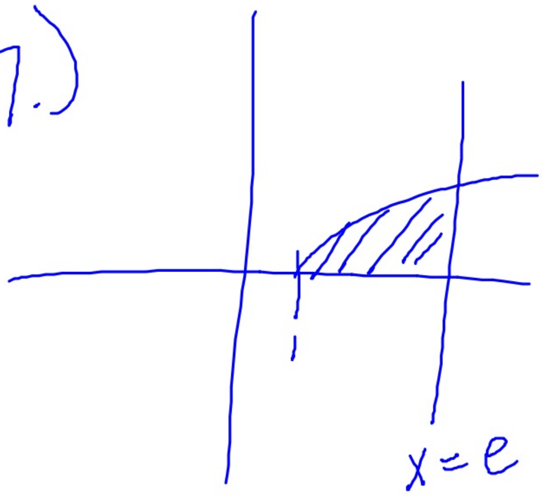
$x^2 + 2x = x + 2$

5.)



$$A = 2\int_{-1}^{0} 3(x^3 - x)\,dx$$

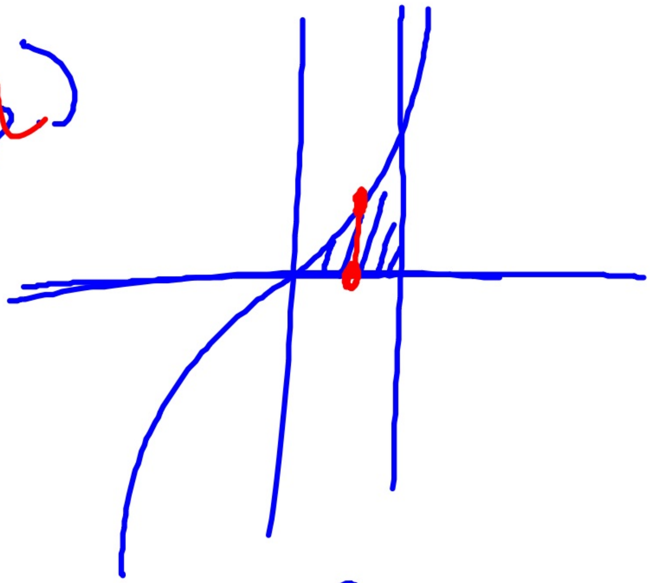
7.)




$$\int_1^e \left(\sqrt{\ln x}\right)^2 dx$$

$$\int_1^e \ln x \, dx$$

4b)



$$\frac{\pi}{8}\int_0^2 \left(\frac{1}{2}x^3\right)^2 dx$$

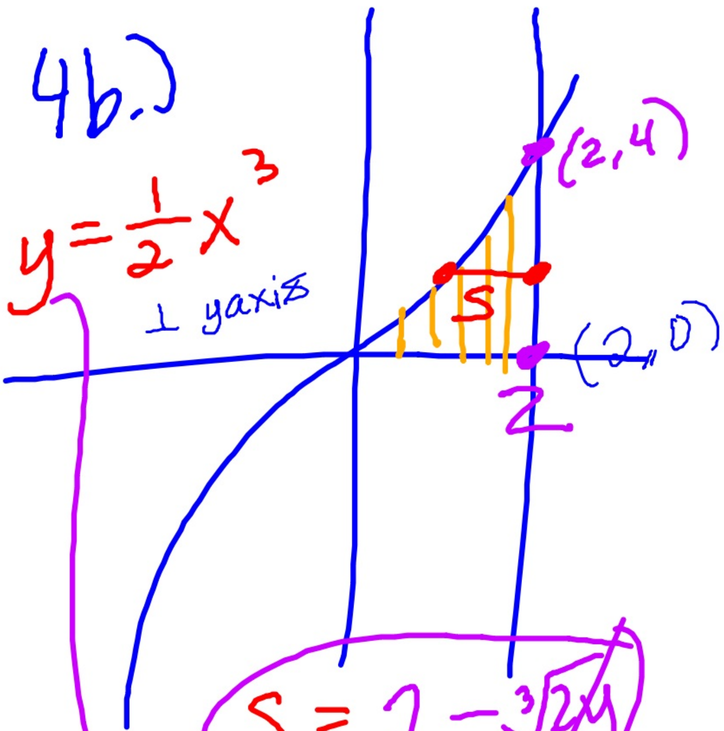
$$\frac{\pi}{8} S^2$$

$$S = \frac{1}{2}x^3$$

4b.)

$y = \frac{1}{2}x^3$

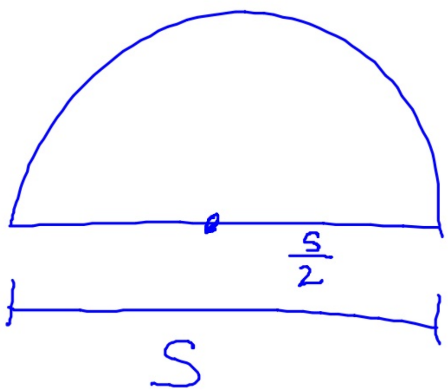
$\perp$ y axis




$S = 2 - \sqrt[3]{2y}$

$$V = \frac{\sqrt{3}}{4}\int s^2\,dy$$

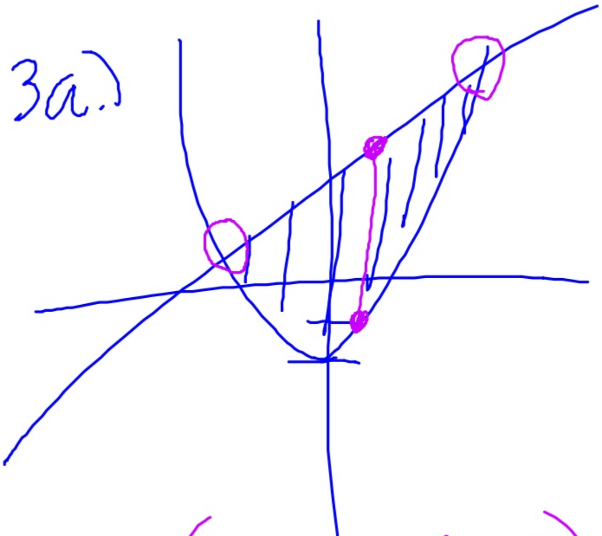
$$= \frac{\sqrt{3}}{4}\int_0^4 \left(2 - \sqrt[3]{2y}\right)^2 dy$$




$$\frac{1}{2}\pi r^2$$

$$\frac{1}{2}\pi\left(\frac{s}{2}\right)^2 \rightarrow \frac{\pi}{8}s^2$$

3a.)

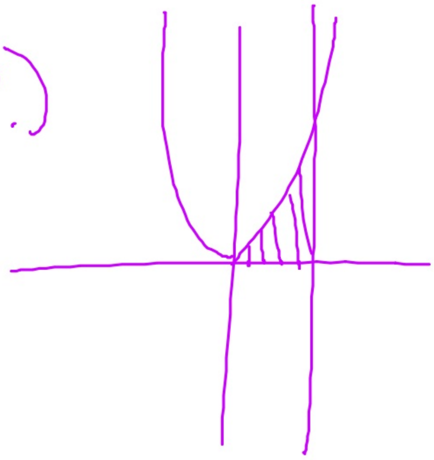


$$\frac{\pi}{8}\int_{-2}^{5}\left(-x^2+3x+10\right)^2 dx$$

$$S=\left(3x+8-\left(x^2-2\right)\right)$$

$$S=\left(-x^2+3x+10\right)$$

8.)



$S = 4x^2 - 0$

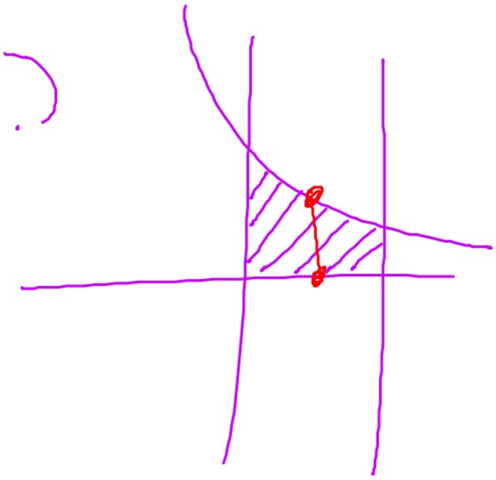
$$\int_0^1 S^2 \, dx$$

$$\int_0^1 16x^4 \, dx$$

$$\frac{16x^5}{5}\Big|_0^1$$

$0$

5.)



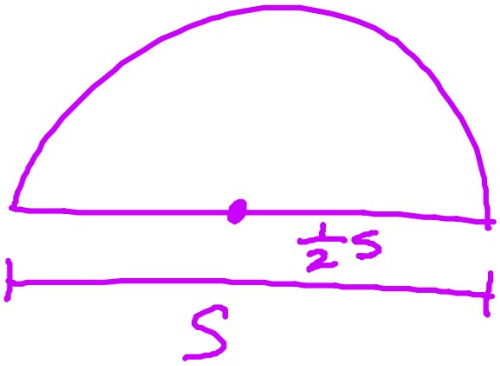
$$S = e^{-x} - 0$$

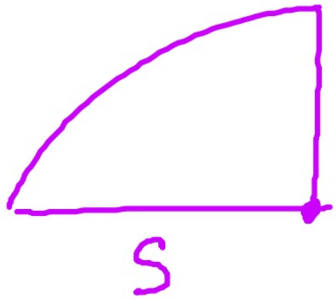
$$\int_0^3 (e^{-x})^2 \, dx$$

$$\int_0^3 e^{-2x} \, dx$$

$$-\frac{1}{2} e^{-2x} \Big|_0^3$$

$$-\frac{1}{2}\left(e^{-6} - 1\right)$$



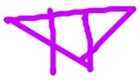




$$\frac{1}{2}\pi r^2$$

$$\frac{1}{2}\pi\left(\frac{1}{2}s\right)^2 =$$

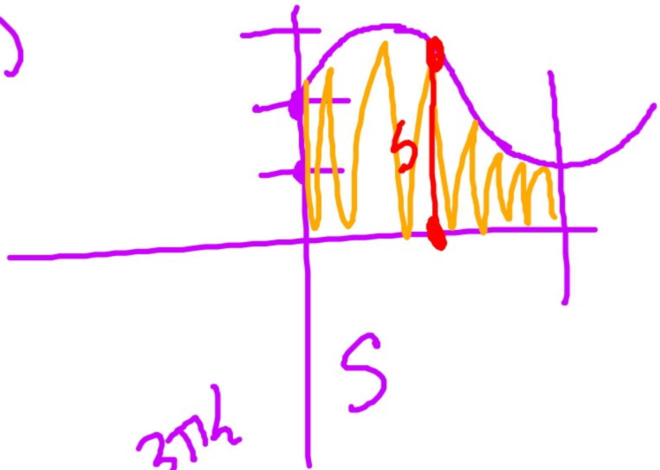
$$\frac{1}{4}\pi r^2$$

$$\frac{1}{4}\pi s^2$$



$\frac{\pi}{4} S^2$

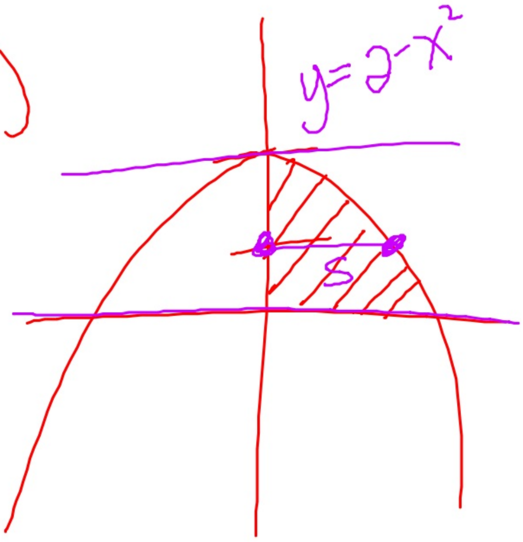
2.)



$$\int_0^{3\pi/2} \frac{\pi}{4} S^2 \, dx$$

$$\frac{\pi}{4} \int_0^{3\pi/2} (2+\sin x)^2 \, dx$$

9.)



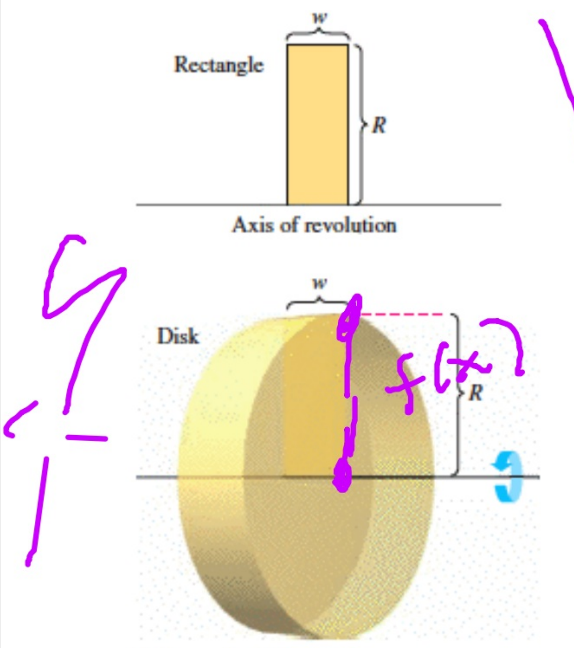

$$S = R - L$$
$$S = \sqrt{2-y} - 0$$

$$\int_0^2 S^2\,dy$$
$$\int_0^2 (2-y)\,dy$$

$$y = 2 - x^2$$
$$x = \sqrt{2-y}$$

# 7.2 Volume: The Disk Method

- Find the volume of a solid of revolution using the disk method.
- Find the volume of a solid of revolution using the washer method.




Volume of a disk: $\pi R^2 w$

$$V = \int_a^b \pi r^2 h$$

$$A_{\text{circle}} = \pi r^2$$

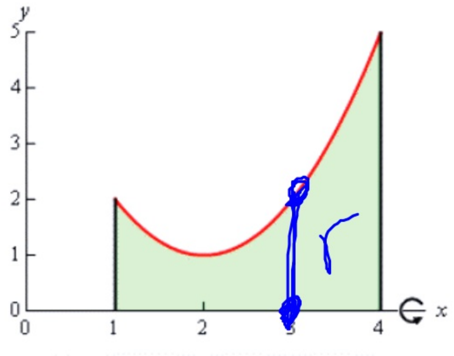
$$V_{\text{cylinder}} = \pi r^2 h$$

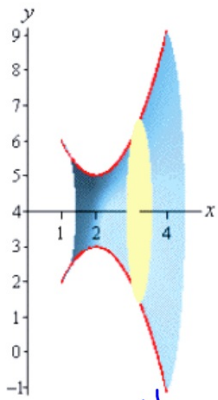
$$= \int_a^b \pi (f(x))^2 \, dx$$

# Volume by Revolution (Disk Method)

***Example 1*** Determine the volume of the solid obtained by rotating the region bounded by $y = x^2 - 4x + 5$, $x = 1$, $x = 4$, and the $x$-axis about the $x$-axis.

dx

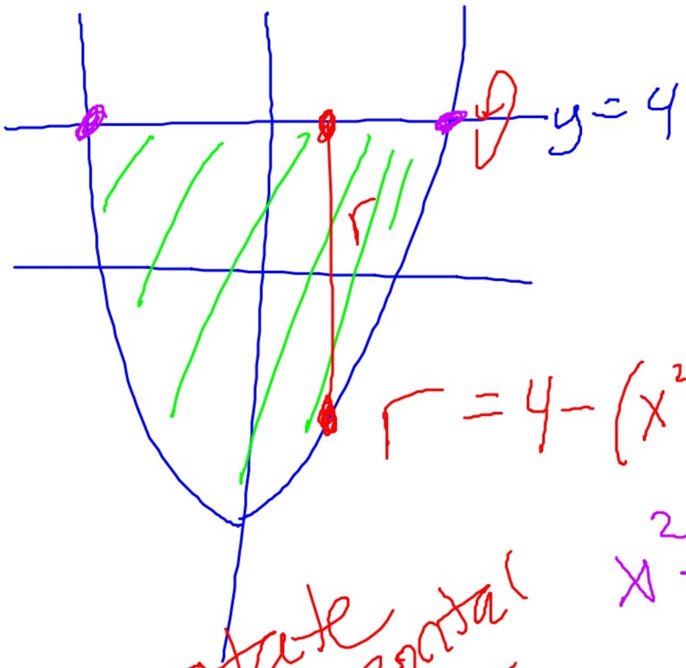



$y=x^2 - 4x + 5$
$x = 1, x = 4,$
rotate about the x-axis

$$V = \int \pi r^2\,dx = \pi \int_1^4 (x^2 - 4x + 5)^2\,dx$$

#2 Find the volume of the solid that is formed by $y = x^2 - 9$, $y = 4$; rotated about $y = 4$.




$$\pi \int_{-\sqrt{13}}^{\sqrt{13}} \left(4 - x^2 + 9\right)^2 \, dx$$

2041.912

$r = 4 - (x^2 - 9)$

rotate horizontal

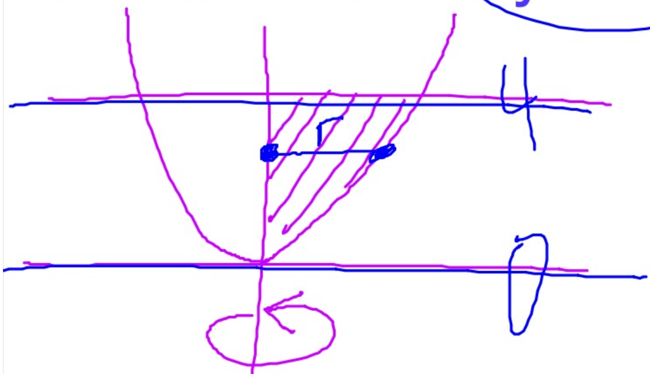
r = top - bottom

$x^2 - 9 = 4$

$x^2 = 13$

$x = \pm\sqrt{13}$

#3 Region: $y = x^2$, $x = 0$, $y = 4$ (1st quad) rotated about y-axis




$$V = \pi \int_0^4 \left(\sqrt{y}\right)^2 dy$$

$$= \pi \int_0^4 y\, dy$$

$$8\pi$$

$r = \text{Right} - \text{left}$

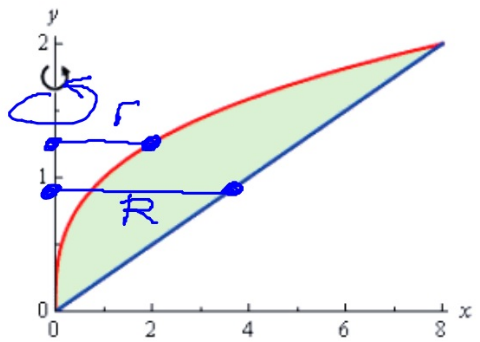
$r = \sqrt{y} - 0$

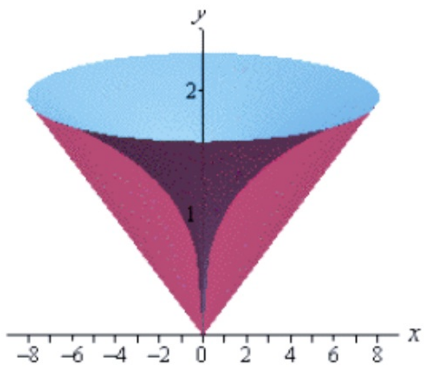
# Volume by Revolution (Washer Method)

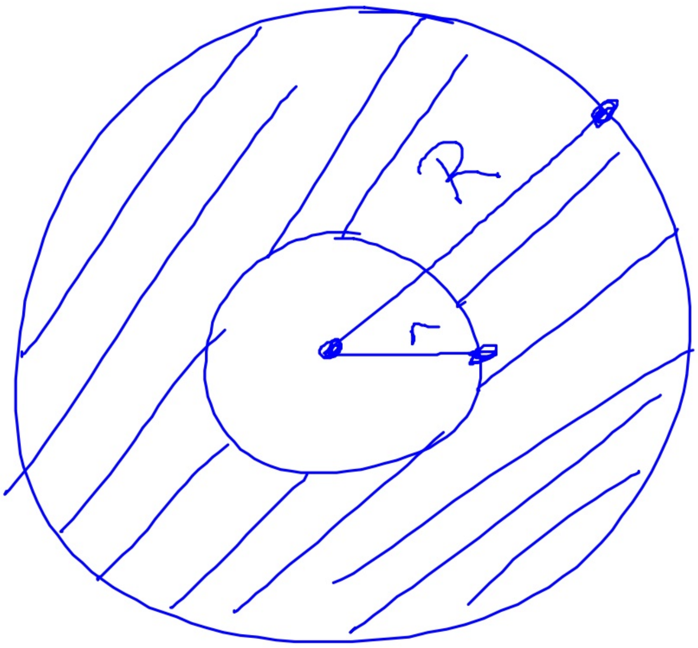
$$V=\pi\int_{a}^{b}\left[R(x)\right]^{2}-\left[r(x)\right]^{2}dx$$

$$V=\pi\int_{a}^{b}\left[(outer-axis)^{2}-(inner-axis)^{2}\right]dx$$

***Example 2*** Determine the volume of the solid obtained by rotating the portion of the region bounded by $y=\sqrt[3]{x}$ and $y=\dfrac{x}{4}$ that lies in the first quadrant about the $y$-axis.

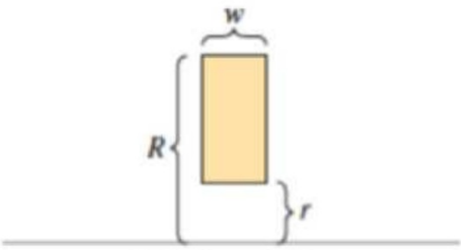


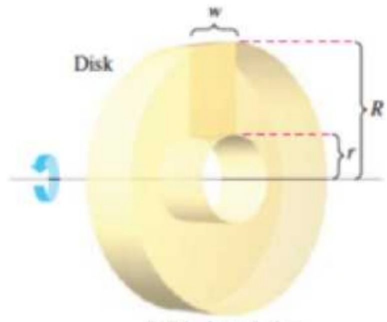






$\pi R^2 - \pi r^2$

## Washer Method




$$A_{\text{washer}} = \pi R^2 - \pi r^2 = \pi\left(R^2 - r^2\right)$$

$$V_{\text{cylinder}} = \pi R^2 h - \pi r^2 h = \pi\left(R^2 - r^2\right)h$$

## Washer Method

Horizontal Axis of Revolution

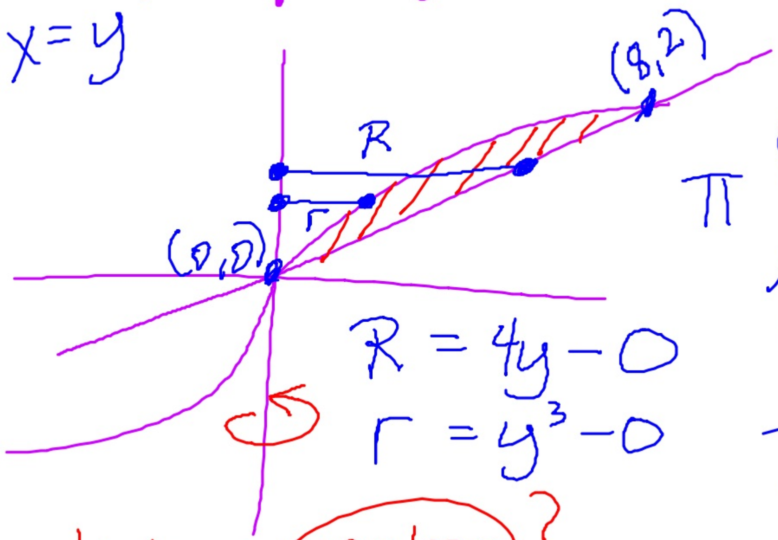
$$V = \pi \int_a^b \left( \left[ R(x) \right]^2 - \left[ r(x) \right]^2 \right) dx$$

Vertical Axis of Revolution

$$V = \pi \int_a^b \left( \left[ R(y) \right]^2 - \left[ r(y) \right]^2 \right) dy$$

#4 $y=\sqrt[3]{x}$ $\quad y=\frac{x}{4}$ $\quad$ rotate : y-axis

$x=y^3$ $\quad$ 1st Quad. $\longrightarrow$ $x=4y$




$y^3=4y$

$y^3-4y=0$

$y(y^2-2)=0$

$$\pi\int_0^2 R^2-r^2$$

$R=4y-0$

$r=y^3-0$

$$\pi\int_0^2\left((4y)^2-(y^3)^2\right)dy$$

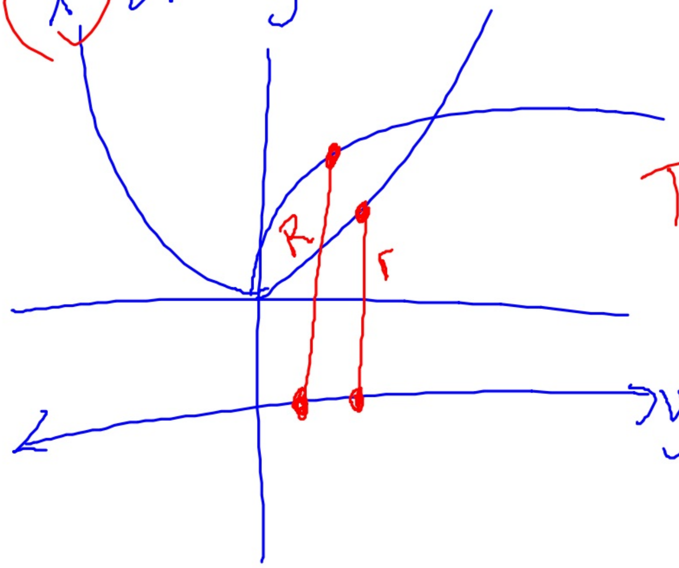
disk or washer?

x or y

#5 *Find the volume of the solid obtained by rotating the region bounded by the curve $y = x^2$ and $y = \sqrt{x}$ about the line $y = -1$.*

disk or washer ?

x or y ?




$$R = \sqrt{x} - (-1) = \sqrt{x} + 1$$

$$r = x^2 - (-1) = x^2 + 1$$

$$\pi \int_0^1 \left( (\sqrt{x}+1)^2 - (x^2+1)^2 \right) dx$$

#6 *Find the volume of the solid obtained by rotating the region bounded by* $x = y^2 - 2$ *and* $y = x$ *about the line* x=4

# AP Practice Question

Let $R$ be the shaded region bounded by the graphs of $y = \sqrt{x}$ and $y = e^{-3x}$ and the vertical line $x = 1$, as shown in the figure above.

$\int\left(\sqrt{x} - e^{-3x}\right)dx$

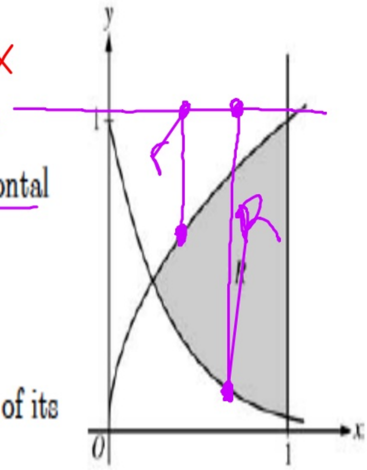
$.239$

(a) Find the area of $R$.

(b) Find the volume of the solid generated when $R$ is revolved about the horizontal line $y = 1$.

$\pi \int \Big( \qquad \Big) dx$

(c) The region $R$ is the base of a solid. For this solid, each cross section perpendicular to the $x$-axis is a rectangle whose height is 5 times the length of its base in region $R$. Find the volume of this solid.



# Answers to AP Question

| | |
|---|---|
| Point of intersection<br>$e^{-3x} = \sqrt{x}$ at $(T, S) = (0.238734, 0.488604)$ | 1: Correct limits in an integral in (a), (b), or (c) |
| (a) Area $= \int_T^1 \left(\sqrt{x} - e^{-3x}\right) dx$<br>$= 0.442$ or $0.443$ | $2: \begin{cases} 1: \text{integrand} \\ 1: \text{answer} \end{cases}$ |
| (b) Volume $= \pi \int_T^1 \left( \left(1 - e^{-3x}\right)^2 - \left(1 - \sqrt{x}\right)^2 \right) dx$<br>$= 0.453\pi$ or $1.423$ or $1.424$ | $3: \begin{cases} 2: \text{integrand} \\ \quad < -1 > \ \text{reversal} \\ \quad < -1 > \ \text{error with constant} \\ \quad < -1 > \ \text{omits 1 in one radius} \\ \quad < -2 > \ \text{other errors} \\ 1: \text{answer} \end{cases}$ |
| (c) Length $= \sqrt{x} - e^{-3x}$<br>Height $= 5\left(\sqrt{x} - e^{-3x}\right)$<br>Volume $= \int_T^1 5\left(\sqrt{x} - e^{-3x}\right)^2 dx = 1.554$ | $3: \begin{cases} 2: \text{integrand} \\ \quad < -1 > \ \text{incorrect but has} \\ \qquad \sqrt{x} - e^{-3x} \\ \qquad \text{as a factor} \\ 1: \text{answer} \end{cases}$ |