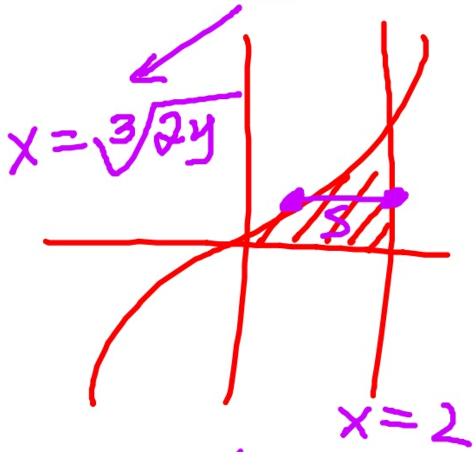
4.) $y=\frac{1}{2}x^3 \quad y=0 \quad x=2$

b.) $\perp$ y-axis

equil $\triangle$

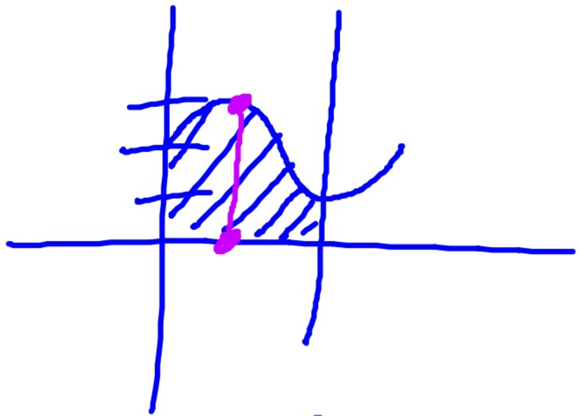


$s=\left(2-\sqrt[3]{2y}\right)$

$$\frac{\sqrt{3}}{4}\int_0^4 \left(2-\sqrt[3]{2y}\right)^2 dy$$

2.) $y = 2 + \sin x$ y-axis, x-axis

$x = \frac{3\pi}{2}$



$$\frac{1}{4}\int_0^{3\pi/2} s^2 \, dx$$

$s = 2 + \sin x$

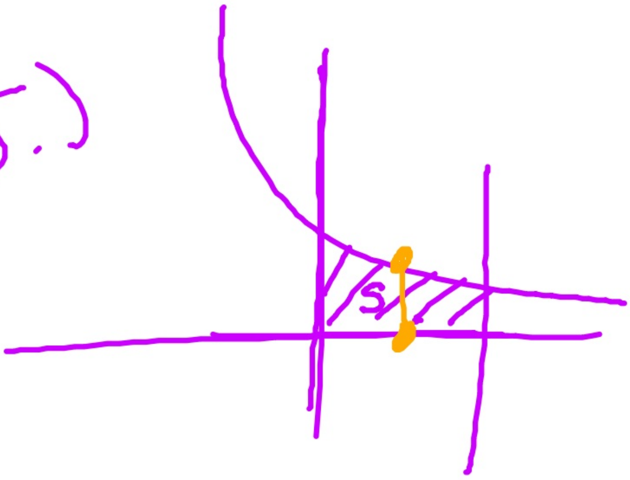
7.) $y = \sqrt{\ln x}$

$x = e$

$$s = \left(\sqrt{\ln x} - 0\right)$$

$$\int_1^e s^2 \, dx = \int_1^e \ln x \, dx$$

5.)

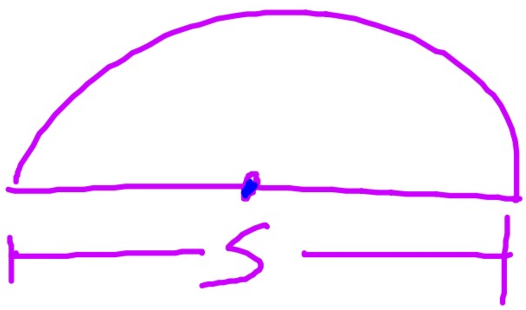


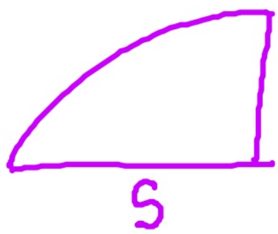
$$s = (e^{-x} - 0)$$

$$\int_0^3 (e^{-x})^2 \, dx$$

$$\int_0^3 e^{-2x} \, dx$$

$$-\frac{1}{2} e^{-2x} \Big]_0^3$$





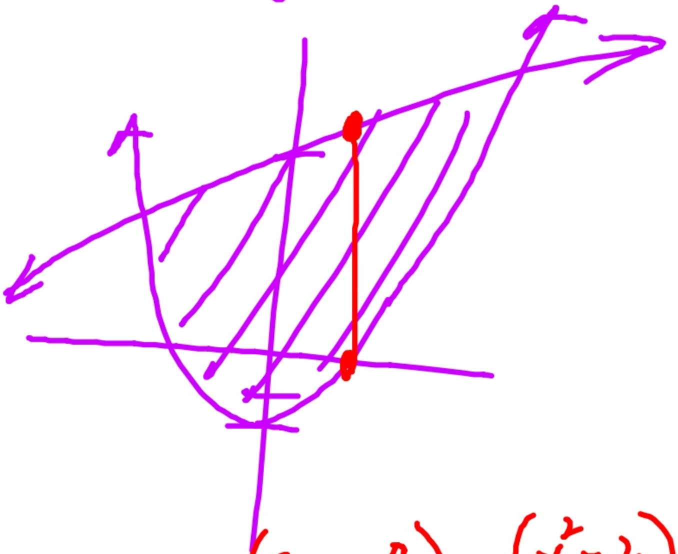
$\frac{\pi}{8}s^2$

$\frac{1}{4}\pi s^2$

$\frac{1}{2}\pi r^2$

$\frac{s}{2}$

3a.) $y = x^2 - 2$ $\quad y = 3x + 8$ $\quad \perp$ x axis semicircles



$$\frac{\pi}{8}\int_{-2}^{5} s^2\,dx$$

$$\frac{\pi}{8}\int_{-2}^{5} \left(3x+8-x^2+2\right)^2 dx$$

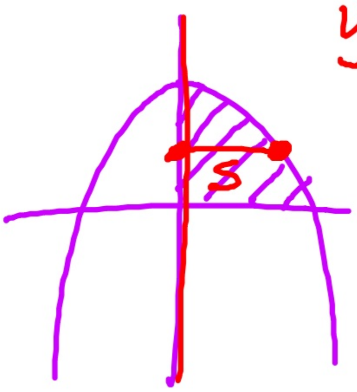
$S = (3x+8) - (x^2-2)$

$y = -x^2 + 2 \quad \perp \text{ y axis}$

0

$y = 2$

$x = \sqrt{2-y}$



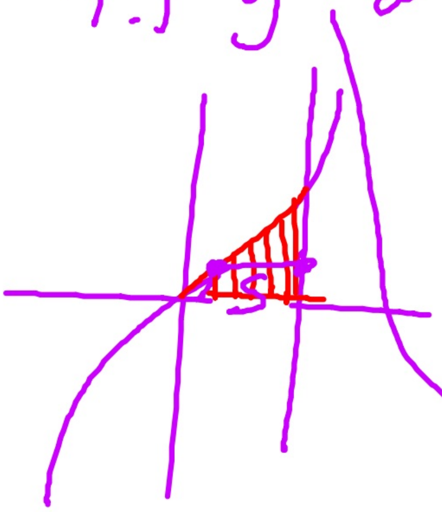
$$\int_0^2 \left(\sqrt{2-y} - 0\right)^2 dy$$

$$\int_0^2 (2-y)\, dy$$

$S = \text{right} - \text{left}$

4.) $y=\frac{1}{2}x^3$ $y=0$ $x=2$

b.) equil. $\Delta \perp$ y-axis



$$\int_0^4 \frac{\sqrt{3}}{4}\left(2-\sqrt[3]{2y}\right)^2 dy$$

$S =$ right $-$ left

$s = 2 - \sqrt[3]{2y}$

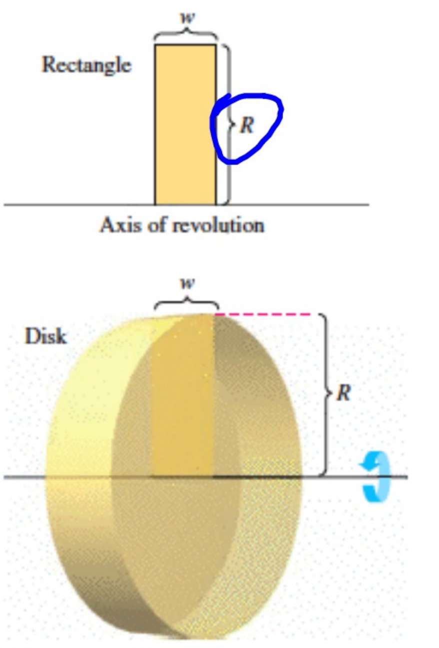
$y=\frac{1}{2}x^3$

$2y = x^3$

$= x$

7.2 Volume: The Disk Method

- Find the volume of a solid of revolution using the disk method.
- Find the volume of a solid of revolution using the washer method.



Volume of a disk: $\pi R^2 w$

$$\int \pi r^2 h = \int_a^b \pi r^2\,dx$$

$h \to dx$

$$A_{\text{circle}} = \pi r^2$$

$$V_{\text{cylinder}} = \pi r^2 h$$

$$\pi \int_a^b r^2\,dx$$

$(7-4)^2$ $(x-5)^2$

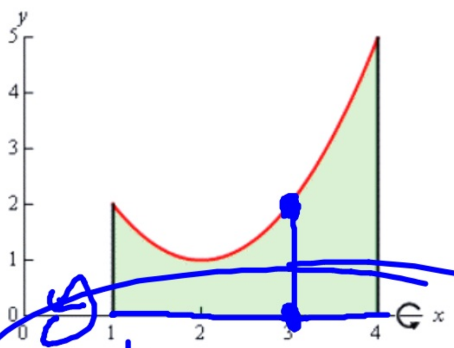
$(4-7)^2$ $(5-x)^2$

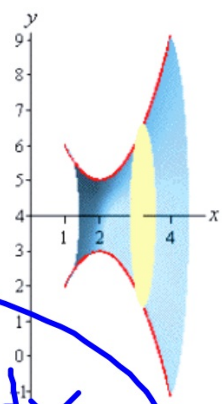
$$(x-4)^2 = x^2 - 8x + 16$$

$$(4-x)^2 = 16 - 8x + x^2$$

Volume by Revolution
(Disk Method)

Example 1 Determine the volume of the solid obtained by rotating the region bounded by $y = x^2 - 4x + 5$, $x = 1$, $x = 4$, and the x-axis about the x-axis.





$y=x^2 - 4x + 5$
$x = 1, x = 4,$
rotate about the x-axis

disk

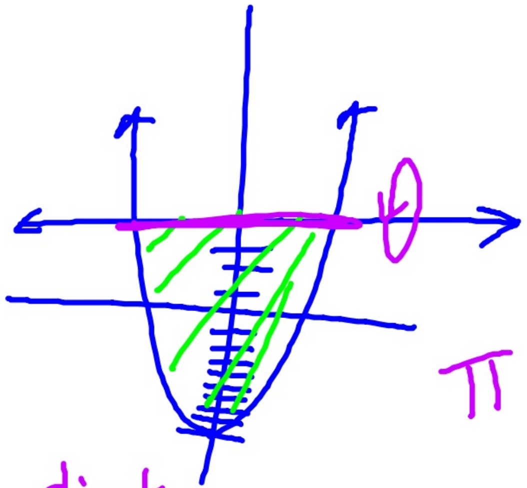
$$\pi \int_a^b (\text{funct.} - \text{axis})^2\,dx$$

$$\pi \int_1^4 (x^2-4x-5-0)^2\,dx$$

#2 Find the volume of the solid that is formed by $y = x^2 - 9$, $y = 4$; rotated about $y = 4$.

horiz.

dx

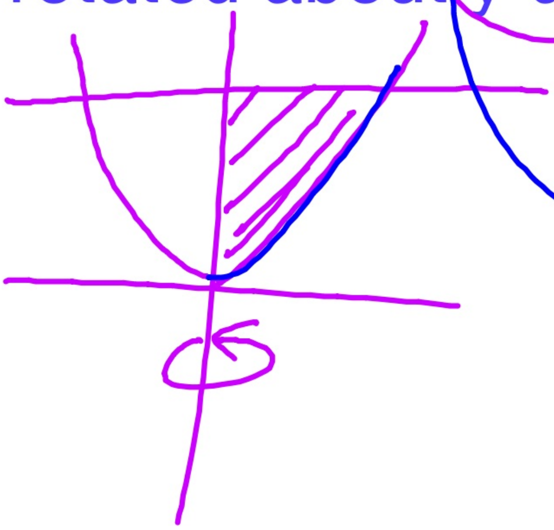


disk

$$\pi \int_{-\sqrt{13}}^{\sqrt{13}} \left(x^2 - 9 - 4\right)^2 dx$$

#3 Region: $y = x^2$, $x = 0$, $y = 4$ (1st quad)
rotated about y-axis

vertical (y's)



$$y = x^2$$
$$\sqrt{y} = x$$

$$\pi \int_0^4 (\sqrt{y} - 0)^2 \, dy$$

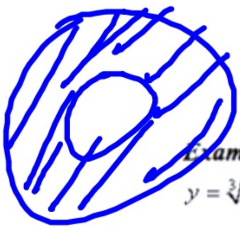
$$\pi \int_0^4 y \, dy = 8\pi$$

Volume by Revolution (Washer Method)

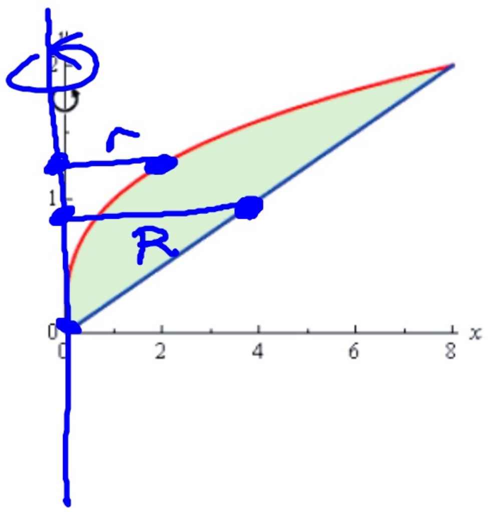
$$V = \pi \int_a^b \left[R(x)\right]^2 - \left[r(x)\right]^2 dx$$

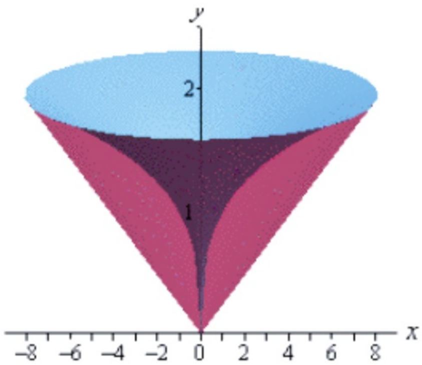
$$V = \pi \int^{b} \left[(outer - axis)^2 - (inner - axis)^2 \right] dx$$

R(x) r(x)

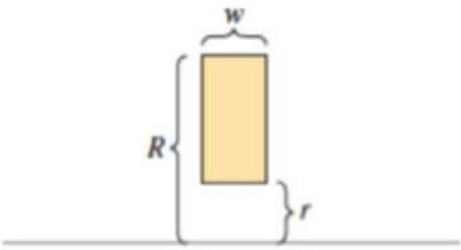


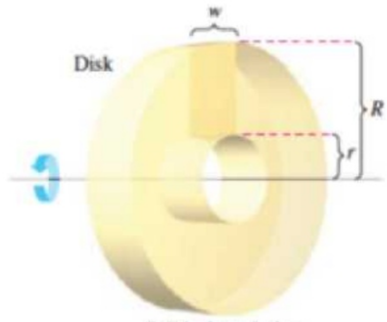
Example 2 Determine the volume of the solid obtained by rotating the portion of the region bounded by $y = \sqrt[3]{x}$ and $y = \frac{x}{4}$ that lies in the first quadrant about the y-axis.





Washer Method





$$A_{\text{washer}} = \pi R^2 - \pi r^2 = \pi\left(R^2 - r^2\right)$$

$$V_{\text{cylinder}} = \pi R^2 h - \pi r^2 h = \pi\left(R^2 - r^2\right)h$$

Washer Method

Horizontal Axis of Revolution

$$V = \pi \int_a^b \left([R(x)]^2 - [r(x)]^2 \right) dx$$

Vertical Axis of Revolution

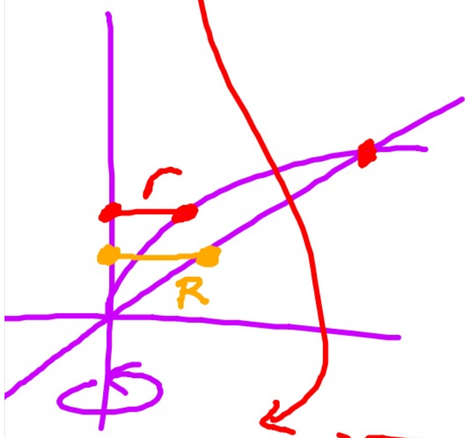
$$V = \pi \int_a^b \left([R(y)]^2 - [r(y)]^2 \right) dy$$

#4 $y=\sqrt[3]{x}$ $\quad y=\frac{x}{4}$ $\quad$ rotate : y-axis

1^{st} Quad.

disk or washer

x's or y's



$y=\frac{x}{4}$

$4y=x$

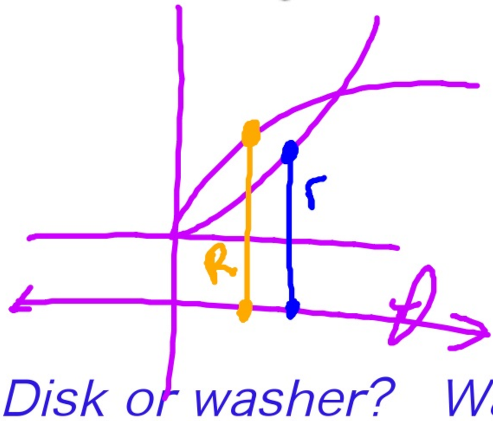
$$\pi\int_0^2\left[(4y-0)^2-(y^3-0)^2\right]dy$$

$$\pi\int_0^2\left(16y^2-y^6\right)dy$$

$y=\sqrt[3]{x}$

$y^3=x$

#5 *Find the volume of the solid obtained by rotating the region bounded by the curve $y = x^2$ and $y = \sqrt{x}$ about the line $y = -1$.*



$$\pi \int_0^1 \left(\left(\sqrt{x} - -1\right)^2 - \left(x^2 - -1\right)^2 \right) dx$$

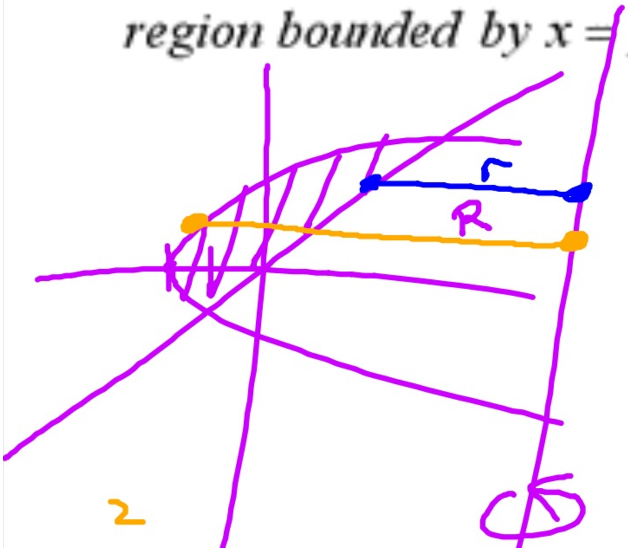
$$\pi \int_0^1 \left(\left(\sqrt{x} + 1\right)^2 - \left(x^2 + 1\right)^2 \right) dx$$

Disk or washer? Washer!

dx or dy? dx (horizontal axis of revolution)

#6

Find the volume of the solid obtained by rotating the region bounded by $x = y^2 - 2$ *and* $y = x$ *about the line* x=4



$y^2-2=y$

$x=4$

disk or washer?

x's or y's?

$$\pi\int_{-1}^{2}\left(\left(y^2-2-4\right)^2-\left(y-4\right)^2\right)dy$$

AP Practice Question

be the shaded region bounded by the graphs of $y = \sqrt{x}$ and $y = e^{-3x}$ and

tical line $x = 1$, as shown in the figure above. $y=\sqrt{x}$ $y=e^{-3x}$

nd the area of R.

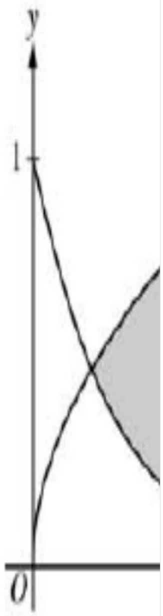
nd the volume of the solid generated when R is revolved about the horizontal

e $y = 1$.

e region R is the base of a solid. For this solid, each cross section

rpendicular to the x-axis is a rectangle whose height is 5 times the length of its

se in region R. Find the volume of this solid.



Answers to AP Question

Point of intersection

$e^{-3x} = \sqrt{x}$ at $(T, S) = (0.238734, 0.488604)$

(a) Area $= \int_T^1 \left(\sqrt{x} - e^{-3x}\right) dx$

$= 0.442$ or 0.443

(b) Volume $= \pi \int_T^1 \left(\left(1 - e^{-3x}\right)^2 - \left(1 - \sqrt{x}\right)^2\right) dx$

$= 0.453\pi$ or 1.423 or 1.424

(c) Length $= \sqrt{x} - e^{-3x}$

Height $= 5\left(\sqrt{x} - e^{-3x}\right)$

Volume $= \int_T^1 5\left(\sqrt{x} - e^{-3x}\right)^2 dx = 1.554$

1: Correct limits in an integral in (a), (b), or (c)

$2: \begin{cases} 1: \text{integrand} \\ 1: \text{answer} \end{cases}$

$3: \begin{cases} 2: \text{integrand} \\ \quad < -1 > \text{ reversal} \\ \quad < -1 > \text{ error with constant} \\ \quad < -1 > \text{ omits 1 in one radius} \\ \quad < -2 > \text{ other errors} \\ 1: \text{answer} \end{cases}$

$3: \begin{cases} 2: \text{integrand} \\ \quad < -1 > \text{ incorrect but has } \sqrt{x} - e^{-3x} \text{ as a factor} \\ 1: \text{answer} \end{cases}$